Abstract. This paper constructs a two-country trade model to examine the optimal policies on the exports of final and intermediate products under Cournot as well as Bertrand competition when firms engage in symbiotic production internationally. The paper shows that given linear demand for the final product, the optimal export policies are to tax the exports of both the final and intermediate goods under symbiotic production, no matter whether firms engage in Cournot or Bertrand competition in the final good market, which is contrary to the conventional wisdom.
1. Introduction

Since the early 1980s, there has been a growing interest in the study of strategic trade policy in the new IO and trade literature. In their pioneering paper, Brander and Spencer (1985) incorporate an international Cournot duopoly into a “third-country” model in which one domestic firm and one foreign firm produce a homogeneous final product and compete in a third-country market. They find that under Cournot competition, an export subsidy is optimal. Nevertheless, Eaton and Grossman (1986) show that if both firms play a Bertrand game, the optimal policy is an export tax. One important implication of the theory of strategic trade policy is that the direction of optimal trade policy on the export of the final product is critically dependent upon the mode of competition.¹ In other words, optimal trade policy by the exporting country is fundamentally affected by whether the final products produced by each firm are strategic substitutes (as in Cournot competition) or complements (as in Bertrand competition).² This seems to weaken the merit of the theory of strategic trade policy.

In a seminal paper, Spencer and Jones (1991) introduce a key intermediate input in the strategic trade policy by developing a “two-country” model in which one

---


² See Bulow et al. (1985)
vertically integrated domestic firm exports both the intermediate and final products to a foreign country. The domestic and foreign firms compete in the foreign-country market. They find that under Cournot competition, it is optimal for the exporting country to tax, subsidize or not intervene in the exports of both products if the difference in profit margins from the sales of the intermediate and final products of domestic firm is greater than, less than or equal to zero, respectively. Moreover, under Bertrand competition, the optimal export policy is that if the exporting country subsidizes the exports of the final good, then it should tax the exports of the intermediate good, and *vice versa.* This result seems to indicate that the direction of optimal trade policy is not directly related to the choice of competition mode.

As Bond (2001) and Chen *et al.* (2004) argued that there is abundant evidence that the production process of firms is becoming increasingly outsourcing and/or fragmented internationally, in the sense that a final manufactured good will consist of parts that have been manufactured in a variety of different countries. For example, Feenstra and Hanson (1996) found that between 1972 and 1990, imported

---

3 Other studies dealing with some strategic aspects in the vertically related markets include Spencer and Jones (1992), Ishikawa and Lee (1997), Ishikawa and Spencer (1999), and Shy and Stenbacka (2003), *etc.*
intermediate inputs of the U.S. increased from 5.3 percent of materials to 11.6 percent. Casual observation indicates that telephone services and other forms of telecommunications services, such as telex, telegrams, and data exchanges, involve the so-called *symbiotic production* internationally.\(^4\) This form of production has the following characteristics: firstly, each produces both an intermediate and a final good; secondly, each producer must purchase the intermediate good from the other producer. This kind of production also applies to complementary components, such as mutually compatible hardware and software, e.g., personal computers, software, VCRs and video tapes, etc. (Economides and Salop (1992)). For instance, Taiwanese PC companies purchase CPU chips and software from American companies to produce desktop and laptop PCs, while exporting PCs and parts to the U.S. in competition with American companies in the PC market of the U.S. Mitsubishi is poised to sign a series of deals with Fiat Auto and it is likely that Mitsubishi’s GDI (gasoline direct injection) engines would power several new Fiat models.\(^5\) Taiwanese LCD TV companies pay technology licensing fees to Japanese companies, while export LCD to Japan. Taiwanese cell phone companies sell chip set to Korean cell phone companies,

----
\(^4\) See Carter and Wright (1994). The history of symbiotic production can be traced back to the classic writing of Cournot (1838) which develops a simple model of complementary goods to examine the merger of two monopolists that produce complementary goods (zinc and copper) into a single (fused) monopolist that produces a combination of the two complementary goods (brass).

while buy passive components from them.

Unfortunately, to the best of our knowledge, there are no studies in the literature that explore the strategic trade policy implications of having the form of symbiotic production. The main concern considered in this paper is an export subsidy applied to the final product, but attention is also given to an export tax applied to the intermediate product. More specifically, the paper intends to examine the two issues: (i) Whether the direction of optimal trade policy is directly related to the choice of competition mode in the presence of symbiotic production; (ii) What are the optimal policies on the exports of the intermediate and final products when symbiotic production arises internationally, as compared with those of Spencer and Jones (1991)? Although the basic market structure is similar to Spencer and Jones (1991), our analysis is distinct from theirs. The major difference between this paper and Spencer and Jones (1991) is in the intermediate market. Spencer and Jones (1991) assume that there is only one intermediate good used in the production of the final product, while this paper assumes that each firm produces a distinct intermediate good that is vital in the production of its rival’s final product.

The remainder of the paper is organized as follows. Section Π sets up a two-country trade model and explores firms’ equilibrium quantities and prices of final and intermediate products under Cournot competition. Section ΠΠ analyzes the
optimal policies on the exports of intermediate and the final product. Section IV considers the optimal trade policies under Bertrand competition. Section V examines the non-cooperative Nash trade policy equilibrium under both Cournot and Bertrand competitions where the foreign government’s trade policies are also considered. The final section concludes the paper.

2. The Model under Cournot Competition

Consider two vertically integrated firms, $d$ and $f$, who are located in the domestic country $d$ and the foreign country $f$, respectively. These two firms produce one homogenous product, whose production function is in Leontief type, in which one unit production of the final good needs to employ one unit of the intermediate goods $x_d$ and $x_f$. Suppose that the intermediate goods $x_d$ and $x_f$ are independent and produced by firms $d$ and $f$ with constant marginal costs $c_d$ and $c_f$, respectively. Thus, each firm needs to use an intermediate good provided by the other firm for the production of its final product. We refer to this kind of production as symbiotic production. Suppose that country $d$ imposes a specific tax and a specific subsidy on the exports of the intermediate and final goods, respectively. Following Spencer and Jones (1991), the final product is sold in a foreign country only. The price of the final product in the importing country, $p$, is given by the inverse demand function $p = p(Y)$, where
\( p'(Y) < 0 \) and \( Y = y^d + y^f \) represents aggregate output, and \( y^d \) and \( y^f \) are quantities produced by firms \( d \) and \( f \), respectively. The profit function of each firm can be described as follows:

\[
(1.1) \quad \pi^d = \pi^{id} + \pi^{fd} = \{(m^d - c^d - v)y^f\} + \{(p - m^f - c^d + s)y^d\},
\]

\[
(1.2) \quad \pi^f = \pi^{if} + \pi^{ff} = \{(m^f - c^f)y^d\} + \{(p - m^d - c^f)y^f\},
\]

where \( \pi^j, j = I, F, i = d, f \), denotes the profit of firm \( i \) earned from the sales of \( j \) good; \( m^i, (i = d, f) \), represents the price of the export of the intermediate good \( i \); and \( v \) and \( s \) are the specific tax and the specific subsidy on the exports of the intermediate and final goods imposed by the domestic country, respectively.

The first term on the right-hand side of eq. (1) denotes the profit earned from the sales of the intermediate good, while the second term represents the profit from the sales of the final good.

The model involves three stages of decision. In stage 1, the domestic government commits to the values of its trade policy instruments. Next, in stage 2, firms simultaneously determine their intermediate good price in the intermediate good markets. In stage 3, firms select their final output under Cournot competition or final good price under Bertrand competition in the final good market. As usual, we solve the stage 3 problem first by backward induction.

In stage 3, firms play Cournot competition in the final good market.
Differentiating (1) with respect to each firm's output yields the profit-maximizing conditions as follows:

\begin{align}
(2.1) \quad & \frac{d\pi^d}{dy^d} = p'y^d + (p - m^d - c^d + s) = 0, \\
(2.2) \quad & \frac{d\pi^f}{dy^f} = p'y^f + (p - m^d - c^f) = 0, 
\end{align}

The second and stability conditions are derivable as follows:

\begin{align}
(3) \quad & 2p' + y^d p'' < 0, \quad 2p'^f + y^f p'' < 0, \quad \text{and} \\
& \quad H = (2p' + y^d p'')(2p'^f + y^f p'') - (p' + y^d p^*)(p'^f + y^f p^*) > 0.
\end{align}

Solving (2) simultaneously, we can derive the levels of Cournot output as follows:

\begin{align}
(4.1) \quad & y^d = y^d(m^d, m^f, s), \\
(4.2) \quad & y^f = y^f(m^d, m^f, s).
\end{align}

In considering the comparative statics of the Cournot output, we assume that own marginal profit declines with an increase in the output of the other firm, which implies the reaction curves in output space are negatively sloping, and outputs of the two firms are strategic substitutes. That is,

\begin{align}
(5) \quad & \pi^d_{12} = p' + y^d p^* < 0, \quad \text{and} \quad \pi^f_{21} = p'^f + y^f p^* < 0.
\end{align}

Totally differentiating (2), and using (3) and (5), we can derive the comparative static effects of the intermediate good price and trade policy on each firm’s output as follows:

\begin{align}
(6.1) \quad & \frac{dy^d}{dm^d} = \frac{-1}{H}(p' + y^d p^*) > 0,
\end{align}
We see from (6) that a rise in the intermediate good price provided by the domestic firm would increase the foreign firm’s cost, and then increase (decrease) the output of the domestic (foreign) firm. On the contrary, a rise in the intermediate good price provided by the foreign firm has the opposite effect on the output of each firm. In addition, a rise in the specific subsidy of the export on the final good decreases the cost of the domestic firm, and then increases (decreases) the output of the domestic (foreign) firm.

We now turn to stage 2 to determine the intermediate good price provided by each firm. Suppose that firms engage in a non-cooperative game, which means that each firm will charge a monopoly price in the intermediate good market due to having monopoly power while selling its own intermediate good.

Substituting (4) into (1) yields the reduced profit functions as function of intermediate good price and subsidy rate as follows:

\[
(7.1) \pi^d(m^d, m^f, s) = \{(m^d - c^d - v)y^f(m^d, m^f, s)\} + \{(p - m^f - c^d + s)y^d(m^d, m^f, s)\},
\]
Differentiating (7) with respect to the price of each firm’s intermediate good and considering the profit-maximizing conditions for each firm’s output, i.e., eq. (2), we can obtain the first derivatives of the profit function with respect to the price of intermediate good as follows:

\[
(8.1) \quad \frac{d\pi^d}{dm^d} = (d\pi^d / dy^f)(dy^f / dm^d) + (\partial \pi^d / \partial m^d),
\]

\[
(8.2) \quad \frac{d\pi^f}{dm^f} = (d\pi^f / dy^d)(dy^d / dm^d) + (\partial \pi^f / \partial m^f).
\]

The first term on the right-hand side of eq. (8) denotes the strategic effect, while the second term represents the direct effect. The direct effect is definitely positive because a rise in the price of intermediate good increases its rival’s marginal cost. This can be evidenced by the signs \((\partial \pi^i / \partial m^i) = y^i > 0\), and \(i = d, f\). However, the strategic effect is ambiguous. A rise in the price of firm \(d\)’s intermediate good decreases the foreign firm’s output, but the effect of the reduced output of the foreign firm on the domestic firm’s profit is indeterminate. This leads to an ambiguous strategic effect. The same result applies to the foreign firm’s strategic effect. Consequently, of importance in what follows is the sign of \((d\pi^d / dy^f)\) or \((d\pi^f / dy^d)\), that is, the effect of output on the rival firm’s profit. This effect can be derived by differentiating (1) with respect to the rival firm’s output as follows:

\[
(7.2) \quad \pi^f(m^d, m^f, s) = \{(m^f - c^f)y^d(m^d, m^f, s)\} + \{(p - m^d - c^f)y^f(m^d, m^f, s)\}.
\]
\[
\begin{align*}
    d\pi^d / dy^f &= m^d - c^d - v + y^d p', \\
    d\pi^f / dy^d &= m^f - c^f + y^f p',
\end{align*}
\]

Substituting (2) into the above equations yields:

\[
\begin{align*}
    (9.1) \quad &M^d(m^d, m^f, v, s) = d\pi^d / dy^f = \{m^d - c^d - v\} - \{p - m^f - c^d + s\}, \\
    (9.2) \quad &M^f(m^d, m^f, v, s) = d\pi^f / dy^d = \{m^f - c^f\} - \{p - m^d - c^f\},
\end{align*}
\]

where \(M^i, i = d, f\), denotes the difference in profit margins from the sales of the intermediate and final products of firm \(i\), which is defined by Spencer and Jones (1991). This difference in profit margins is positive if firm \(i\)'s profits are increased by a rise in its rival’s output whenever the prices of intermediate good and its own output remain unchanged.

Recalling that \(\partial \pi^i / \partial m^i = y^i > 0\), and \(dy^i / dm^i < 0, i = d, f\), we can then derive the following results by considering (8) and (9).

\[
\begin{align*}
    (10.1) \quad &d\pi^d / dm^d = (d\pi^d / dy^f)(dy^f / dm^d) + (\partial \pi^d / \partial m^d), \\
    &\begin{cases}
            > 0 & \text{if } M^d \leq 0, \\
            = 0 & \text{if } M^d > 0,
        \end{cases}
    \\
    (10.2) \quad &d\pi^f / dm^f = (d\pi^f / dy^d)(dy^d / dm^d) + (\partial \pi^f / \partial m^f), \\
    &\begin{cases}
            > 0 & \text{if } M^f \leq 0, \\
            = 0 & \text{if } M^f > 0.
        \end{cases}
\end{align*}
\]

We find from (10) that a firm will charge a price of intermediate good as high as possible to foreclose its rival if the difference in profit margins is no greater than zero. Thus, the profit-maximizing conditions give rise to corner solutions so that vertical foreclosure may occur. This is an interesting result. However, with symbiotic
production, the possibility of vertical foreclosure should be ruled out. This is because each firm produces an intermediate good, which is vital for its rival. Each firm would be unable to compete with the other in the final good market via stopping the supply of the vital intermediate good as long as the profit margin from the sales of the final good is greater than that from the intermediate good. Thus, we will not pursue this problem further. On the other hand, a firm is willing to export its intermediate good to its rival by offering the profit-maximizing price when the difference in profit margins is greater than zero (i.e., $M^d > 0$ and $M^f > 0$). If this is the case, we can solve (10) simultaneously and express the optimal prices of intermediate good as follows:

\begin{align}
(11.1) \quad & m^d = m^d(s, v) \text{ if } M^d > 0, \\
(11.2) \quad & m^f = m^f(s, v) \text{ if } M^f > 0,
\end{align}

Accordingly, we can establish:

**Proposition 1.** In the case of Cournot competition, symbiotic production exists only if the difference in profit margins is greater than zero, which in turn excludes the possibility of vertical foreclosure.

3. Optimal Export Policies

We now move on to investigate the optimal export policies imposed by the domestic country in stage 1. Recall that we consider a subsidy $s$ on the exports of the final
product as well as a tax $v$ on the exports of the intermediate good. The welfare function of the domestic country, $W^d$, is defined as follows:

$$W^d = \pi^d + vy^f - sy^d.$$  \hspace{1cm} (12)

In order to derive clear-cut outcomes, we assume throughout the rest of the paper that the demand function for the final product under consideration is linear: $p = \alpha - \beta(y^d + y^f)$. Thus, the difference in profit margins can be derivable as $M^d = (1/3)\{2m^d + 2m^f - \alpha - c^d - c^f - 2s - 3\nu\}$ and $M^f = (1/3)\{2m^d + 2m^f - \alpha - c^d - c^f + 3s\}$.

Totally differentiating (12) with respect to $s$ and $v$, applying the Envelope theorem, and then solving the welfare-maximizing conditions simultaneously, we can calculate the reduced-form solutions of the optimal values of export policies as follows: \hspace{1cm} (13)

$$s^* = (-7/22)(\alpha - c^d - c^f) < 0, \text{ and}$$

$$v^* = (2/11)(\alpha - c^d - c^f) > 0.$$  

It follows from (13) that the optimal export policies of exporting country are to tax both the exports of the final and the intermediate products. Intuitively, the effects

---

6 Brander and Spencer (1985) impose a fairly standard regularity condition in non-cooperative models that demand is not likely to be highly convex. Obviously, a linear demand belongs to this class of demand.

7 Note that the autonomous demand has to be greater than the marginal costs, i.e., $\alpha - c^d - c^f > 0$. We owe these solutions to an anonymous referee.
that determine the optimal export policies of the domestic country consist of two effects: the strategic effect and the symbiotic effect. The strategic effect tends to attract the domestic government to impose a tax on the exports of the final product for switching the resource from the export of the final product to the more profitable export market of the intermediate good. In addition, a rise in the subsidy to the export of the final product would also increase the price of rival firm’s intermediate good by increasing domestic final output caused by the reduction of the rival firm’s final output. The resulting increase in the price of intermediate good would decrease the domestic firm’s profit through the rise in its marginal cost. Thus, the symbiotic effect resulting from the existence of symbiotic production tends to impose a tax on the exports of the final good. Consequently, we can conclude that both the strategic and symbiotic effects attract the domestic government to set an optimal tax on the exports of the final product. Moreover, concerning the optimal tax on the exports of the intermediate good, as Spencer and Jones (1991) mentioned, the above tax on the final product creates a wedge between the exporting firm’s objective function and welfare in the domestic country. This distortion can be corrected if the domestic country also imposes a tax on exports of the intermediate good.

Note that based on the premise that firms are willing to export their intermediate good to their rival, the optimal export policies can be derived by (13). Thus, the
primitive conditions \( M^d > 0 \) and \( M^f > 0 \) in (10) should be regarded as constraints on the welfare optimization problem in stage 1. We now check whether or not the constraints can be fulfilled by calculating the value of the difference in profit margins \( M^d \) and \( M^f \). Substituting (13) into the difference in profit margins, we can calculate \( M^d = \frac{3}{11}(\alpha - c^d - c^d') > 0 \) and \( M^f = \frac{1}{6}(\alpha - c^d - c^d') > 0 \). Thus, the primitive conditions, \( M^d > 0 \) and \( M^f > 0 \) in (10) hold. As a result, we can ensure from (10) that firms are willing to export their intermediate good to their rival.

Based on the above analysis, we can establish the following proposition:

**Proposition 2.** Assuming a linear demand for the final product with Cournot competition, it is optimal for the exporting country to tax both the exports of the final and the intermediate products if the difference in profit margins is positive.

This result is significantly different from the one derived by Spencer and Jones (1991), who argue that under Cournot competition the optimal policy is to tax, subsidize, or not to intervene if the difference in profit margins is greater, less than or equal to zero respectively. The latter two results can never emerge in our model because firms would be unable to charge a prohibitively high price for the intermediate good to help compete in the final good market in these two cases. Moreover, there is an extra symbiotic effect, which tends to tax both products in our
4. The Model under Bertrand Competition

In this section, we explore optimal export policies, in which firms engage in Bertrand price competition in the final goods market. In order to simplify the analysis, we assume that the final goods produced by two countries are differentiated, and their inverse demand functions take a linear form as follows:

\begin{align}
(14.1) \quad & p^d = 1 - q^d - \gamma q^f, \\
(14.2) \quad & p^f = 1 - q^f - \gamma q^d,
\end{align}

where \( p^i (i = d, f) \) denotes price of final good \( i \); and \( 0 < \gamma < 1 \) denotes the measure of the degree of product differentiation, where the two products turn out to be homogeneous (independent) as \( \gamma \) approaches one (zero).\(^8\)

From (14), we can derive the demand functions for the two products as:

\begin{align}
(15.1) \quad & q^d = a - bp^d + cp^f, \\
(15.2) \quad & q^f = a - bp^f + cp^d,
\end{align}

where \( a = (1-\gamma)/\delta > 0, \ b = 1/\delta > 0, \ c = \gamma/\delta > 0, \ \delta = 1-\gamma^2 > 0, \) and \( b-c = (1-\gamma)/\delta > 0. \)

In stage 3, substituting (15) into (1), and then differentiating with respect to each firm’s final good price, respectively, yields the profit-maximizing conditions as

\(^8\) We do not discuss the case of \(-1 < \gamma < 0\), in which two products are complements.
follows: 

\[
\begin{align*}
(16.1) \quad \frac{d\pi^d}{dp^d} &= (p^d - m^d - c^d + s)(-b) + (m^d - c^d - \nu)c + (a - bp^d + cp^f) = 0, \\
(16.2) \quad \frac{d\pi^f}{dp^f} &= (p^f - m^d - c^f)(-b) + (m^f - c^f)c + (a - bp^f + cp^d) = 0,
\end{align*}
\]

where \( p^i, i = d, f \), represents the price of the final good \( i \).

The left-hand side of (16) demonstrates that there are three effects, denoted the own output effect, the cross output effect and the direct effect in that order in the determination of the optimal prices of the final goods. These effects work as follows: given a rise in the price of the final good, the own output effect decreases own firm’s profit via reducing the demand for its final good, the cross output effect increases own firm’s profit through raising the rival’s output and then the demand for own firm’s intermediate good, and the direct effect increases own firm’s profit by raising its price.

Assuming that both the second and stability conditions are met, we find from (16) that the reaction curve of each firm is positively sloping in the prices of the final goods, demonstrating that the final products are strategic complements.

Solving (16) simultaneously yields:

\[
\begin{align*}
(17.1) \quad p^d &= \{ a(2b + c) + 3bcm^d + (2b^2 + c^2)m^f + 2b(b - c)c^d \\
&\quad + c(b - c)c^f - 2b^2s - 2bc\nu \}/(4b^2 - c^2), \\
(17.2) \quad p^f &= \{ a(2b + c) + (2b^2 + c^2)m^d + 3bcm^f + c(b - c)c^d \\
&\quad + 2b(b - c)c^f - bcs - c^2\nu \}/(4b^2 - c^2).
\end{align*}
\]

\[9\] It can be calculated that the second-order and stability conditions are satisfied.
Recall that \((b - c) > 0\). We see from (17) that a rise in the prices of the domestic and foreign firms’ intermediate goods \(m^d\) and \(m^f\) increase the prices of both the domestic and foreign firms’ final products by raising their marginal costs. This result occurs because the two final products are strategic complements. We also find from (17) that a rise in the prices of the domestic and foreign firms’ final products decrease the prices of both the domestic and foreign firms’ final products.\(^{10}\) This result occurs because the rise in \(s\) strengthens the own output effect of the domestic firm, and the rise in \(\nu\) weakens the cross output effect of the domestic firm. Both impacts lower the price of the domestic final good, and then lower the foreign final good due to strategic complements. Moreover, the impact of the subsidy on the prices is stronger than that of the tax.\(^{11}\)

\(^{10}\) It is worthy of indicating that the tax on the exports of the intermediate good in the case of a homogeneous final product does not directly affect the levels of Cournot output as shown in (4). However, this appears to be not the case for the final products that are differentiated in this section. This difference occurs because firms play Bertrand price competition in the case of differentiated products, while play Cournot quantity competition in the case of the homogeneous product. Thus, a change in either price of the two differentiated products affects the output of both firms via (15). Consequently, the marginal profit for the intermediate good does not vanish in the case of differentiated products, while it is vanished in the case of homogeneous product, leading to the difference between the two cases.

\(^{11}\) This result can be verified by the following comparative statics:

\[
\frac{\partial p^d}{\partial s} = [\frac{2b^2}{(4b^2 - c^2)}] < \frac{\partial p^d}{\partial \nu} = [\frac{-2bc}{(4b^2 - c^2)}], \text{ and} \\
\frac{\partial p^f}{\partial s} = \frac{bc}{(4b^2 - c^2)} < \frac{\partial p^f}{\partial \nu} = \frac{-c^2}{(4b^2 - c^2)},
\]
We now proceed to stage 2. Substituting (15) and (17) into (1) gives the reduced form of profit functions. Differentiating these resulting profit functions with respect to the price of each firm’s intermediate good yields the profit-maximizing prices, respectively, as follows:  

\[(18.1) \quad m^d = \left(\frac{1}{2b}\Delta^m_{Bd}\right)\left\{-c\Delta^m_{Bd}m^f + \Delta^c_{Bd}c^d + \Delta^s_{Bd}s + \Delta^\nu_{Bd}\nu + \Delta^i_{Bd}\right\},\]

\[(18.2) \quad m^f = \left(\frac{1}{2b}\Delta^m_{Bf}\right)\left\{-c\Delta^m_{Bf}m^d + \Delta^c_{Bf}c^d + \Delta^s_{Bf}s + \Delta^\nu_{Bf}\nu + \Delta^i_{Bf}\right\},\]

where \(\Delta^m_{Bd} = (b-c)(8b^2 + c^2) > 0; \Delta^c_{Bd} = (b-c)(8b^3 + c^2 (2b-c)) > 0; \Delta^\nu_{Bd} = -8b^2(b-c)^2 < 0; \Delta^i_{Bd} = c^3(b-c) > 0; \Delta^s_{Bd} = 2b(4b^2+c^2)(b-c) > 0; \Delta^\nu_{Bd} = a(8b^3+c^3) > 0; \Delta^c_{Bf} = -8b^2(b-c)^2 < 0; \Delta^\nu_{Bf} = 8b^2c(b-c) > 0; \Delta^s_{Bf} = 8b^3(b-c) > 0; \Delta^i_{Bf} = bc(b-c) (4b^2+c^2) (2b-c) + \Delta^i_{Bd} > \Delta^i_{Bd} > 0.\]

It follows from (18) that the reaction curves of the domestic and foreign firms for the intermediate good are negatively sloping. In addition, we can also figure out that the slope of the domestic firm’s reaction curve is \((-c/2b)\), which is larger than the slope of the foreign firm’s reaction curve \((-2b/c)\).

Solving (18), we can obtain the equilibrium prices of the intermediate goods in stage 2 as follows:

\[(19.1) \quad m^{d*} = \left\{-2bc(b-c)s + 4b^2(b-c)\nu + 2b(2b + c)(b-c)c^d - (b-c)(4b^2 - 2bc + c^2)c^f + a(4b^2 - 2bc + c^2)\right\} / \Delta^m_{B},\]

\[\text{12 We can verify that the second-order and stability conditions are satisfied.}\]
Recall that $\Delta_{mB} > 0$ and $(b - c) > 0$. We find from (19) that a rise in $s$ decreases (increases) the price of the domestic (foreign) intermediate good, while a rise in $\nu$ increases this price of the domestic (foreign) intermediate good. Intuitively, a rise in $s$ lowers the price of the domestic firm’s intermediate good via decreasing the price of the final good. Moreover, a fall in the price of the domestic final good increases the demand for its exports of the final good. This will then increase the demand for the foreign firm’s intermediate good. Consequently, the price of the foreign firm’s intermediate good rises. Similarly, a rise in $\nu$ increases the price of export of the domestic firm’s intermediate good and then increases the price of the foreign firm’s final good. Since the final goods are strategic complements in price under Bertrand competition, the price of the domestic firm’s final good will also rise, with less magnitude. This leads to the result that the demand for the domestic final good rises and then increases the price of the foreign firm’s intermediate good.

We now turn to stage 1. Substituting (17) and (19) into (12), we can derive the reduced welfare function. Differentiating this welfare function with respect to $s$ and $\nu$, and solving these resulting equations simultaneously, we can obtain:\footnote{Assume that the second-order conditions are fulfilled.}

\begin{equation}
(20.1) \quad s^{a*} = (2b^2 - bc + c^2)\{(c^d + c^f)(b - c) - a\}/\Delta_{\nu} < 0,
\end{equation}
\[(20.2) \quad \nu^{B_*} = -c(b + c)\{(c^d + c^f)(b - c) - a\}/\Delta B > 0,\]

where \(\Delta B = 2(b-c)(3b^2+c^2) > 0\) and note that \([c^d+c^f(b-c)-a] < 0\) is required for the outputs of the final goods to be positive.\(^{14}\)

Thus, we find from (20) that the optimal export policies are to tax both the exports of the intermediate and final goods.

Intuitively, since the final goods are strategic complements in prices under Bertrand competition, it is optimal to impose a tax on the exports of the final good by increasing its price and then profit as well as domestic welfare.\(^{15}\) In addition, a rise in the tax of the exports of the intermediate good, \(\nu\), raises the price of the domestic intermediate good. Since domestic intermediate good is vital for the production of the foreign final good, the price of the foreign final good will thus rise. This will in turn increase the domestic firm’s profit and welfare by raising the price of domestic final

\(^{14}\) Substituting (17), (19) and (20) into (15) obtains:

\[q^d = \left(\frac{-1}{\Delta B}\right)\frac{2b^2 + c^2}{8b^2 + c^2}\left(b - c\right)\left(\frac{b}{b - c}\right)\left(c^d + c^f\right)(b - c - a),\]

\[q^f = \left(\frac{-1}{\Delta B}\right)\frac{2b^2 + c^2}{8b^2 + c^2}\left(b - c\right)\left(\frac{c}{b - c}\right)\left(c^d + c^f\right)(b - c - a).\]

Recalling that \(b - c > 0\) and \(\Delta B > 0\), it then follows that the outputs of the final goods are positive if the condition \([c^d+c^f(b-c)-a] < 0\).

\(^{15}\) Domestic welfare can be improved due to higher profits earned by the domestic firm and larger tax revenues.
product due to strategic complements in prices under Bertrand competition. In sum, we can conclude that it is optimal for the government to impose taxes on both the exports of the final and intermediate goods. Accordingly, we can establish the following proposition.

**Proposition 3.** Given a linear demand for the final product with Bertrand competition, the optimal export policies are to tax both the exports of the final and the intermediate products.

This result is in sharp contrast with the one obtained by Spencer and Jones (1991), in which the government should tax the exports of the intermediate good if it subsidizes the exports of the final good and *vice versa*, when firms engage in Bertrand competition in the final goods markets. This difference arises because there is an extra symbiotic effect in this analysis. As domestic intermediate good is vital for the production of the foreign final good, a rise in the tax of the exports of the intermediate good raises the price of foreign final product. This will in turn increase domestic firm’s profit and welfare caused by strategic complements of prices between the two products.
5. Two Governments: Nash Trade Policy Equilibrium

In this section, the foreign government’s trade policies in the policy game are also considered. Non-cooperative Nash equilibrium in trade policies can be derived, in which each country is assumed to choose its trade policies given the trade policies of the other country. To this end, we examine that the foreign government imposes a tax on the imports of the final product as well as a tax on the exports of the intermediate good. Let us consider firstly the equilibrium under Cournot competition.

The profit function of each firm can be rewritten as follows:

\[(21) \quad \pi^d = \{(m^d - c^d - v)y^f\} + \{(p - m^f - c^d + s - t)y^d\},\]

\[(22) \quad \pi^f = \{(m^f - c^f - \tau)y^d\} + \{(p - m^d - c^f)y^f\},\]

where \(\tau\) and \(t\) are the specific tax on the exports of the intermediate goods and the imports of the final goods imposed by the foreign government, respectively.

In addition to the welfare function of the domestic country as specified in equation (12), the welfare function of foreign country, \(W^f\), can be defined as follows:

\[(23) \quad W^f = \pi^f + (t + \tau)y^d.\]

In order to save space, we jump directly to report the optimal trade policies under Cournot competition as follows:

**Proposition 4.** Assuming a linear demand for the final product with Cournot
competition, we have the following non-cooperative Nash trade policy equilibria:

(i) it is optimal for the exporting country to tax both the exports of the final and the intermediate products, if the difference in profit margins is positive.

(ii) it is optimal for the importing country to impose a tax on the exports of the intermediate product. However, the optimal trade policy on the imports of the final good is ambiguous.

Proof: see Appendix A.

Finally, we proceed to explore the non-cooperative Nash trade policy equilibrium under Bertrand competition. Likewise, we jump directly to report the optimal trade policies under Bertrand competition as follows:

**Proposition 5.** Given a linear demand for the final product with Bertrand competition, we have the following non-cooperative Nash trade policy equilibria:

(i) it is optimal for the exporting country to tax both the exports of the final and the intermediate products.

(ii) the optimal trade policy for the importing country requires the sum of the import tariff on the final good and export tax on the intermediate product be less than zero.

Proof: see Appendix B.

It can be seen from Propositions 4 and 5 that the directions of the optimal export
policies for the exporting countries under both Cournot and Bertrand competitions remain unchanged, which are to tax both the exports of the final and the intermediate products, even if the optimal trade policies of the foreign government are also taken into consideration.

6. Concluding Remarks

This paper has constructed a two-country trade model with a three-stage game to examine the optimal export policies on the exports of final and intermediate goods under Cournot as well as Bertrand competition, when firms engage in symbiotic production internationally by using a vital intermediate good provided by its rival. Several striking results are derived.

First of all, a firm will charge its intermediate good price as high as possible to choose vertical foreclosure under Cournot competition if the difference in profit margins is no greater than zero. But the possibility of vertical foreclosure never happens once symbiotic production is taken into consideration. Secondly, under Cournot competition, given a linear demand for the final product, it is optimal for the exporting country to tax the exports of both products if the difference in profit margins is positive. Thirdly, given a linear demand for the final product with Bertrand competition, the optimal export policies are to tax both the exports of the final and the
intermediate products. Lastly, the results derived in the case where the optimal export policies imposed by the domestic country under both Cournot and Bertrand competitions carry over to the case where the optimal trade policies of foreign country are also considered. Based on the above results, we can conclude that under symbiotic production, the optimal export policies are to tax the exports of both intermediate and final goods, no matter whether firms engage in Cournot or Bertrand competition. We believe this is a striking result, which revives the contribution of the theory of strategic trade policy.

The potentially interesting extensions of the model of strategic trade policies under symbiotic production include two aspects: firstly, it would be interesting to explore the optimal policies with a general demand function. The curvature of the demand function would play an important role in the determination of optimal policies in this analysis; secondly, it would also be interesting to examine the case where the production function does not take the Leontief type. Nevertheless, these issues will be reserved for future research.

Finally, this paper may also be extended to the analysis of the optimal import policy. Both Hwang et al. (2003) and Hwang et al. (2007) all derived the result that when the derived demand for the intermediate good is very convex to the origin under Cournot competition, a tariff on the intermediate good will raise the price of the
intermediate good by more than the tariff rate. This will in turn decrease the profit of
the domestic firm and the welfare level of the domestic country. Therefore, the
optimal import policy is to subsidize the imports of the intermediate good. On the
other hand, the traditional profit-shifting result where the optimal import policy is to
impose a positive tariff on the imports occurs, when the derived demand for the
intermediate good is not very convex. In the present paper, the only imported good is
the intermediate good of the foreign firm, which is vital for the production of the
domestic final good. We expect that in addition to the traditional profit-shifting effect,
there will be an extra symbiotic effect, where a rise in the subsidy to the import of the
intermediate good would decrease the price of the domestic firm’s final good by
reducing its marginal cost. This will in turn raise the profit of the domestic firm and
the welfare level of the domestic country. Thus, the symbiotic effect resulting from
the existence of symbiotic production tends to impose a subsidy on the imports of the
intermediate good under Cournot competition. On the other hand, since the final
products are strategic complements in prices under Bertrand competition, a rise in the
tariff on the imports of the intermediate good can raise the prices of both domestic
and foreign final products by increasing the marginal cost of the domestic firm. The
profit of the domestic firm and welfare level of the domestic country will then
increase. Thus, the optimal import policy under Bertrand competition should be to
impose a positive tariff.
Appendix A

The profit function of each firm can be rewritten as follows:

(A.1) \[ \pi^d = \{(m^d - c^d - v)y^f + \{(p - m^f - c^f + s - t)y^d \} \}, \]

(A.2) \[ \pi^f = \{(m^f - c^f - \tau)y^d \} + \{(p - m^d - c^f)y^f \}. \]

Following the same procedures as in section II and assuming a linear demand function for the final product, \( p = \alpha - \beta(y^d + y^f) \), we can derive the following comparative statics while ignoring the detailed procedures in order to save space:

(A.3) \[
\begin{align*}
\frac{dy^d}{dm^d} &= \frac{1}{3\beta}, \quad \frac{dy^d}{dm^f} = \frac{-2}{3\beta}, \quad \frac{dy^f}{dm^d} = \frac{-2}{3\beta}, \quad \frac{dy^f}{dm^f} = \frac{1}{3\beta}, \\
\frac{dy^d}{ds} &= \frac{2}{3\beta}, \quad \frac{dy^d}{dt} = \frac{-2}{3\beta}, \quad \frac{dy^f}{ds} = \frac{-1}{3\beta}, \quad \frac{dy^f}{dt} = \frac{1}{3\beta}.
\end{align*}
\]

(A.4) \[
\begin{align*}
\frac{dm^d}{ds} &= \frac{2}{33}, \quad \frac{dm^d}{dv} = \frac{20}{33}, \quad \frac{dm^d}{dt} = \frac{-2}{33}, \quad \frac{dm^d}{d\tau} = \frac{-2}{33}, \\
\frac{dm^f}{ds} &= \frac{13}{33}, \quad \frac{dm^f}{dv} = \frac{-2}{33}, \quad \frac{dm^f}{dt} = \frac{-13}{33}, \quad \frac{dm^f}{d\tau} = \frac{20}{33}.
\end{align*}
\]

In stage 1, the welfare function of the foreign country, \( W^f \), is defined as follows:

(A.5) \[ W^f = \pi^f + (t + \tau)y^d. \]

Totally differentiating (12) with respect to \( s \) and \( v \), (A.5) with respect to \( t \) and \( \tau \), and then applying the comparative static results, we can obtain:

(A.6) \[
\begin{align*}
[-2/(9\beta)]s + [-4/(99\beta)]v + [-20/99\beta]t \\
= [20/99\beta]M^d(m^d, m^f, 0) + (13/33)(d\pi^d / dm^f),
\end{align*}
\]

(A.7) \[
\begin{align*}
[-2/(9\beta)]s + [-40/(99\beta)]v + [-2/99\beta]t \\
= [2/99\beta]M^d(m^d, m^f, 0) + (2/33)(d\pi^d / dm^f),
\end{align*}
\]
\[(A.8) \quad \left[\frac{26}{99}\beta\right]r + \left[\frac{-42}{99}\beta\right]t = \left[\frac{68}{99}\beta\right]M^f(m^d, m^f, 0) + \left(\frac{2}{33}\beta\right)\left(\frac{d\pi^d}{dm^f}\right) - y^d,\]

\[(A.9) \quad \left[\frac{-40}{99}\beta\right]r + \left[\frac{-42}{99}\beta\right]t = \left[\frac{2}{99}\beta\right]M^f(m^d, m^f, 0) + \left(\frac{2}{33}\beta\right)\left(\frac{d\pi^d}{dm^f}\right) - y^d.\]

Subtracting (A.9) from (A.8) gives:

\[(A.10) \quad \tau^* = M^f(m^d, m^f, 0),\]

where \( M^f(m^d, m^f, 0) = \frac{d\pi^f}{dy^f} = \{m^f - c^f\} - \{p - m^d - c^f\}.\]

Substituting (A.10) into (A.9), we have:

\[(A.11) \quad \tau^* = -M^f(m^d, m^f, 0) + \left(\frac{-1}{7}\beta\right)\left(\frac{d\pi^f}{dm^d}\right) + \left(\frac{33\beta}{14}\right)y^d.\]

As the difference in profit margins must be greater than zero to ensure the existence of the interior solutions for the equilibrium price of the intermediate products, i.e., \( M^f > 0 \), it follows from (A.10) that the optimal policy for the foreign country is to impose a tax on the exports of the intermediate goods. Intuitively, the effects of the optimal tax on the exports of foreign intermediate goods can be decomposed into three effects: the strategic, the symbiotic and the tax-revenue effects. Similar to the intuition stated before, the strategic effect is negative while the symbiotic effect is positive. Moreover, the tax-revenue effect improves foreign welfare via increasing its tax revenue. As a result, we can figure out from (A.10) that the positive symbiotic and tax-revenue effects outweigh the negative strategic effect.

Recall that \( M^f > 0 \) and \( \left(\frac{d\pi^f}{dm^d}\right) = -y^f < 0 \). We see from (A.11) that the effects
of the optimal tax on the imports of domestic final goods consist of three effects: the strategic, the symbiotic and the tax-revenue effects, which are represented by the terms on the right-hand side of (A.11) in that order. Similarly, the strategic effect is negative while the symbiotic effect is positive. Moreover, the tax-revenue effect is also positive. As a result, we find from (A.11) that the optimal policy on the imports of final product is ambiguous.

Solving (A.6) and (A.7), we obtain:

\[(A.12) \quad s^* = -t - M^d(m^d, m^f, 0) + 2\beta(\frac{d\pi^d}{dm^f}), \]

\[(A.13) \quad \nu^* = (1/2)t + (1/2)M^d(m^d, m^f, 0) - (5\beta/4)(\frac{d\pi^d}{dm^f}), \]

where \( M^d(m^d, m^f, 0) = \frac{d\pi^d}{dm^f} = \{m^d - c^d\} - \{p - m^f - c^f\}. \)

Recall that \( M^d > 0 \) and \( (d\pi^d / dm^f) = -y^d < 0. \) We find from (A.12) and (A.13) that in addition to the strategic and symbiotic effects, there is an extra tariff effect denoted by the first term in the right-hand of the equations, as the optimal trade policies of foreign country are taken into consideration. A rise in foreign tariff increases the marginal cost of producing the domestic final product. This will worsen the profitability of the final good attracting domestic government to reduce the subsidy on the exports of the final good for switching resources to more profitable intermediate good. However, as the optimal tariff is ambiguous, this tariff effect is also ambiguous. Substituting (A.11) into (A.12) and (A.13), these two equations can be rewritten as
follows:

\[(A.12)' \quad s^* = -[M^d(m^d, m^f, 0) - M^f(m^d, m^f, 0)] + (1/7)(d\pi^f / dm^d) + 2\beta(d\pi^d / dm^f) - (33\beta / 14)y^d,\]

\[(A.13)' \quad \nu^* = (1/2)[M^d(m^d, m^f, 0) - M^f(m^d, m^f, 0)] - (1/14)(d\pi^f / dm^d) - (5\beta / 4)(d\pi^d / dm^f) + (33\beta / 28)y^d.\]

Note that \([M^d(m^d, m^f, 0) - M^f(m^d, m^f, 0)] = 0, (d\pi^f / dm^d) = -y^f < 0\) and \((d\pi^d / dm^d) = -\nu^f < 0\). It follows from \((A.12)'\) and \((A.13)'\) that the optimal subsidy on the final good is negative while the optimal tax on the intermediate good is positive. This result occurs because the extra tariff effect is too weak to dominate the strategic and the symbiotic effects.

**Appendix B**

By the use of modified profit functions \((A.1)\) and \((A.2)\), the equilibrium prices of the final goods solved in stage 3 and those of the intermediate goods solved in stage 2 can be written as follows:

\[(B.1) \quad p^d = \{a(2b + c) + 3bcm^d + (2b^2 + c^2)m^f + 2b(b - c)c^d + c(b - c)c^f - 2b^2s - 2bc\nu + 2b^2t - c^2\tau\} / (4b^2 - c^2),\]

\[(B.2) \quad p^f = \{a(2b + c) + (2b^2 + c^2)m^d + 3bcm^f + c(b - c)c^d + 2b(b - c)c^f - bcs - c^2\nu + bct - 2bct\} / (4b^2 - c^2),\]

\[(B.3) \quad m^{d*} = \{-2bc(b - c)s + 4b^2(b - c)\nu + 2bc(b - c)(t + \tau) + 2b(2b + c)(b - c)c^d - (b - c)(4b^2 - 2bc + c^2)c^f + a(4b^2 - 2bc + c^2)\} / \Delta^m_b,\]
In stage 1, substituting (B.1), (B.3) and (B.4) into (A.1) and then differentiating (12) with respect to $s$ and $v$ respectively, we yield the following welfare-maximizing conditions:

\[
(B.5) \quad \{(4b^2 + c^2)/\Delta_n^m\}(d\pi^d/dm^f) - \{c(8b^4 + 2b^2c^2 - c^4)/[(4b^2 - c^2)\Delta_n^m]\}v \\
-\{b(8b^4 + 2b^2c^2 - c^4)/[(4b^2 - c^2)\Delta_n^m]\}s = 0,
\]

\[
(B.6) \quad \{2bc/\Delta_n^m\}(d\pi^d/dm^f) - \{b(8b^4 + 2b^2c^2 - c^4)/[(4b^2 - c^2)\Delta_n^m]\}v \\
-\{c(8b^4 + 2b^2c^2 - c^4)/[(4b^2 - c^2)\Delta_n^m]\}s = 0.
\]

Solving (B.5) and (B.6) yields:

\[
(B.7) \quad v^{b*} = \left( -\frac{c(4b^2 - c^2)(2b^2 + c^2)}{(b^2 - c^2)(8b^4 + 2b^2c^2 - c^4)} \right) \left( \frac{d\pi^d}{dm} \right),
\]

\[
(B.8) \quad s^{b*} = \left( \frac{2bc(4b^2 - c^2)}{c(8b^4 + 2b^2c^2 - c^4)} \right) \left( \frac{d\pi^d}{dm} \right) - \left( \frac{b}{c} \right) v^{b*}.
\]

Recall that $b > c > 0$ and $(d\pi^d/dm^f = -q^d) < 0$. We see from (B.7) that the optimal policy of the domestic government on the exports of the intermediate good is to impose an export tax. Substituting this result into (B.8), we find that the optimal policy on the exports of the final good is also an export tax.\(^{16}\) Thus, the directions of the optimal trade policies of domestic government remain unchanged under Bertrand

\(^{16}\) Although the variables of foreign tariff $t$ and tax $\tau$ are included in the variable $q^d$ of (26) and (27), we can still figure out the signs of the policy variables of the domestic country $s$ and $v$ because the domestic output $q^d$ must be positive.
competition, even if the optimal trade policies of foreign country are also considered.

Next, substituting (B.1), (B.3) and (B.4) into (A.2) and then differentiating (23) with respect to $t$ and $\tau$, respectively, we yield the following welfare-maximizing conditions:

\[
(B.9) \quad 2c \left( \frac{d\pi}{dm^d} \right) - \left( \frac{8b^4 + 2b^7 c^3 - c^4}{4b^5 - c^2} \right)(t + \tau) = 0,
\]

\[
(B.10) \quad \left[ \frac{2bc}{(8b^2 + c^2)} \left( \frac{d\pi}{dm^d} \right) + q^d + \left[ \frac{8b^3(2c^2 - 3b^2) + bc^2(c^2 - 2b^2)}{(4b^2 - c^2)(8b^2 + c^2)} \right] \right](t + \tau) = 0.
\]

Note that $b > c > 0$ and $\left( \frac{d\pi}{dm^d} = -q^d \right) < 0$. It follows from (B.9) that the sum of the optimal import tariff and export tax has to be less than zero, i.e., $(tB^* + \tau B^*) < 0$. 
References


Economics 56, 205-32.