Optimization of the Investment Effect for the Solar Hot-Water Heater in Taiwan

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Abstract

Due to the shortage of fossil energy and the serious environmental pollution problems, the heavy issue must be faced for searching the new source of energy to substitute the fossil energy. The solar power will be very important energy in the future. Because it is not only used to generate electrical power, but also is used to heat water. The maximal electrical power is obtained for the photovoltaic system. The least auxiliary heat energy and the shortest time of the capital recovery are searched for the solar hot-water heater system, respectively. In this study, Taipei, Taichung, Tinan and Hengchung are selected to analyze. First, the sun’s position at any time and in any city is predicted with the mathematical procedure of Julian dating. Then, we adopt a step by step method to calculate the optimal size of the solar panel which has maximal investment effect for the solar hot-water heater on each site. The results show that the optimal size of solar panel are not only affected by the location of the solar panel, but also are affected by the water storage. The economy of energy consumption is obvious while both the water storage and size of solar panel are increased. The investment effect of the solar hot-water heater is installed in south is larger than that is installed in north.

Key Words: Solar Power, Tilt Angle, Azimuth Angle, Heating Energy, Capital Recovery Time

1. Introduction

The storage of fossil energy is decreased hastily and the fossil fuel based power generators are also blamed for being a major contributor to air pollution and greenhouse effect. The solar energy is treasured and is used gradually due to the cost down. The solar energy is majorly applied to hot-water heater systems and small-power photovoltaic products, including the lights, the monitors and traffic signals on the road. The solar energy will be expanded to the power system of residential and business district in the future.

The power output of the photovoltaic (PV) module is affected by both the temperature and the angle of photovoltaic modules installed. The optimal angle of the photovoltaic module installed is developed by various methods [1–4]. There are three modes of PV modules installed, including track-mode, fix-mode and combine-mode. The effect of irradiation absorbed by the PV modules is the worst, but the cost of equipment and maintain is the lowest for the fix-mode. The electric output of the PV modules installed at different tilt angle approaches near to that of the PV modules installed in track-mode [1], this method analyzes the electric power output of the PV module which is located at five tilt angles and azimuth angles. Two azimuth angles are applied to evaluate the irradiation of the PV module under clear sky, and the electric power output of the PV module are compared with the connection of the modules in parallel and series [2]. The sun’s position at any time and any region is predicted by Julian dating to evaluate the radiation, and then the electric power output is simulated under at any tilt angle and azimuth angle [3]. The Genetic Algorithm (GA) and the Simulated Annealing (SA) are both
applied to obtain the optimal tilt angle of the PV module installed in fix-mode, where the relationship among incidence of solar radiation, tilt angle of the PV module and the irradiation on the PV module will be analyzed [4]. The hybrid ant direction and differential evolution method is developed to solve the solution by Chang al. [5], this algorithm applies the ant colony search for finding the proper mutation operator, uses both the acceleration procedure and the migration procedure to shorten the simulation time and obtain the optimal solution. Most of the mentioned algorithms search the optimal tilt angle of the PV module and the azimuth angle is faced due south.

When the latitude and climatic condition of a region are changed, the irradiation on the region is changed too. In Taiwan, the latitude of Hengchung, 22 degrees at south, is lower than that of Taipei, 25 degrees at north. Therefore, the temperature and irradiation in the south are higher than those in the north. Generally speaking, the characters of the photovoltaic (PV) systems are not similar to that of the solar hot-water heater system. The maximal output of electrical power must be obtained for the photovoltaic system, and the least auxiliary heat energy and the shortest time of the capital recovery are searched for the solar hot-water heater system, respectively. In this paper, Taipei, Taichung, Tinan and Hengchung are selected to analyze the optimal investment effect for the solar hot-water heater system. The sun’s position at any time and any region is evaluated by a way of mathematical procedure [3,5,6]. Then, a step by step method is adopted to calculate the optimal size of the solar panel with maximal investment effect.

2. Evaluation of the Effective Irradiation on the Solar Panels

The effective irradiation on the solar panels can be both measured directly and evaluated by a mathematical procedure [5,6]. Before evaluating the effective irradiation on the solar panels, the elevation and azimuth of the sun, Greenwich mean sidereal time (Gmst) and local mean sidereal time (Lmst) must be decided firstly.

2.1 Evaluation of the Gmst and Lmst

Both the Gmst and Lmst are expressed as below

\[
Gmst = 6.697357 + 0.065709 \times JDn \tag{1}
\]

\[
Lmst = Gmst + \gamma + (\text{local time} - tl) \times 1.002738 \tag{2}
\]

where

- \(JDn\): Julian dating
- \(\gamma\): longitude at the site
- \(tl\): the time lag, it is 8 hours in Taiwan

2.2 The Elevation and Azimuth of the Sun

After the Gmst and Lmst are obtained, the elevation and azimuth of the sun can be calculated as below

\[
ad = \tan^{-1}\left[\frac{\sin(Lmst - \alpha)}{\sin\phi \cos(Lmst - \alpha) - \cos\phi \tan\delta}\right] \tag{3}
\]

\[
ah = \sin^{-1}\left[\sin\phi \sin\delta + \cos\phi \cos\delta \cos(Lmst - \alpha)\right] \tag{4}
\]

where

- \(\alpha\): solar longitude
- \(\delta\): solar declination
- \(\phi\): latitude at the site

As shown in Figure 1, \(\eta\) is the tilt angle of the solar panels, and \(\theta\) is the contain angle between the normal of solar panel and the incidence of solar radiation.

The effective component of sunshine on the solar panel is found as the following

\[
\cos\theta = \sin(\alpha)\cos(\eta) + \cos(\alpha)\sin(\eta)\cos(ad - \tau) \tag{5}
\]

\(\tau\): the azimuth of the solar panel

The irradiation absorbed by the solar panel is obtained as below [7]
\[ S_{1} = S_{\text{max}} \left[ 1 + 0.034 \cos \left( \frac{2 \pi n}{365} \right) \right] \cos \theta \]

\[ = S_{\text{max}} \times \omega \times \cos \theta \]  

(6)

where
- \( S_{\text{max}} \): the sun constant, it equals to 1367 W/m²
- \( n \): day number

Because the part of solar radiation can only reach to the ground from the outside of the atmosphere, the transparency of the atmosphere must be considered. Then the sun radiation on the ground can be expressed as below

\[ S_{t} = S_{1} \times p_{n} \]  

(7)

where
- \( p_{n} \): the transparency of the atmosphere

When the climatic condition and the size of solar panel are also considered, the irradiation absorbed by the solar panel is found as the following

\[ S_{h}(h) = S_{t} \times I_{R} \times h \]  

(8)

h: the superficies of the solar panel (m²).
- \( S_{h}(h) \): the irradiation on the solar panels for the size of the solar panel (h m²)
- \( I_{R} \): the insolation ratio, it is the ratio between the practical time and the theoretic time of the solar radiation.

### 3. Problem Statement

When the solar energy is applied to a hot-water heater system, the solar panel is used to absorb the solar radiation to heat the water. The size of solar panel will affect the temperature of the water heated, and it also affects the equipment cost. This study is to evaluate the optimal size of the solar panel and make the capital recovery time be the shortest for the hot-water heater. The heat energy which the water heated at an expected temperature is expressed as the following equation

\[ W_{h}(i) = w \times (T_{\text{set}} - T_{\text{ini}}(i)) \times 4.2/3600/1000 \]  

(9)

where
- \( W_{h}(i) \): the heat energy that the water is heated at the expected temperature for the w liter of the water storage at i-th day (kWH)
- \( w \): the water storage (liter)
- \( T_{\text{set}} \): the expected temperature after the water heated (Celsius)
- \( T_{\text{ini}}(i) \): the initial temperature before the water heated at i-th day.

When the temperature of the water cannot be heated at the expected value by the heat energy on the solar panel, the auxiliary heater (electric, gas or oil) must be used, or the size of the solar panel must be enlarged. But, the investment cost will be increased for both conditions. The aim of this paper is to search the optimal superficies of the solar panel and make the capital recovery time be the shortest. The shortest capital recovery time corresponding to the specified capacity of the water storage and the superficies of the solar panel is expressed as follows

\[ mCRT(w, h) = \min \left( \frac{T_{m}}{P_{tw} \times \alpha 1} \right) \]

\[ = \min \left( \frac{T_{m}}{(P_{ty} - \sum_{i=1}^{n} P_{wd}(i)) \times \alpha 1} \right) \]

\[ = \min \left( \frac{T_{m}}{\sum_{j=1}^{n} W_{h}(j) - \sum_{i=1}^{n} \max(W_{h}(i) - S_{h}(i), 0)) \times \alpha 1} \right) \]  

(10)

where
- \( Y_{n} \): the total days in a year
- \( P_{wd}(i) \): the needed auxiliary heat power at i-th day. When the temperature of the water can be heated at the expected value by the heat energy on the solar panel, the \( P_{wd}(i) \) is zero.
- \( P_{ty} \): the total hot energy of the hot-water heater without the solar energy a whole year (kWH)
- \( P_{tw} \): the total stinted energy of the hot-water heater with the solar energy a whole year (kWH)
- \( P_{wy} \): the total auxiliary energy of the hot-water heater with the solar energy a whole year (kWH)
- \( T_{m} \): total equipment cost
- \( \alpha 1 \): the electricity charge (NTS/kWH)
- \( CRT(w, h) \): the capital recovery time corresponding to the specified capacity of the water storage (w liters) and the superficies of the solar panel (h m²)
mCRT(w, h): the shortest capital recovery time corresponding to the specified capacity of the water storage (w liters) and the superficies of the solar panel (h m²)

Hereafter, the SCRT corresponding to the specified water storage can be computed as below

\[
SCRT(w) = \min(mCRT(w, h)); \quad h \in \Omega
\]  

SCRT(w): the shortest capital recovery time corresponding to the specified w liters of water storage.

Ω: the set of the all sizes of the solar panel which are simulated in this paper

4. Formulation

In order to avoid the optimal operation point loss, a step by step method is applied to obtain the shortest capital recovery time corresponding to the specified capacity of the water storage and the specified superficies of the solar panel, mCRT. This method increases one degree in the tilt angle or the azimuth at each iteration, the searched procedure of the method is shown as Figure 2. In the figure, \( \eta_0 \) is the initial tilt angle of the solar panel installed, \( \tau_0 \) is the initial azimuth of the solar panel installed, \( \eta_{opt} \) is the optimal tilt angle and \( \tau_{opt} \) is the optimal azimuth, respectively.

When the all sizes of the solar panel considered are analyzed, the shortest capital recovery time corresponding to the specified capacity of water storage, SCRT, can be obtained.

5. Numerical Results

This study will analyze the optimal size of the solar panel which with the shortest capital recovery time corresponding to the specified capacity of the water storage for the solar hot-water heater in Taipei, Taichung, Tinan and Hengchung. Here, the superficies of one piece of the solar panel is set as 1.92 (m²) and the equipment cost with 100 liters of water storage and one piece of solar panel is NT$40,000. The incremental cost is NT$10,000 for increasing 100 liters of water storage or increasing one piece of solar panel. The longitude and latitude of the four selected regions are listed in Table 1 [8].

<table>
<thead>
<tr>
<th>Position</th>
<th>Northern Latitude</th>
<th>East Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taipei</td>
<td>25°02'</td>
<td>121°30'</td>
</tr>
<tr>
<td>Taichung</td>
<td>24°09'</td>
<td>120°41'</td>
</tr>
<tr>
<td>Tinan</td>
<td>23°</td>
<td>120°14'</td>
</tr>
<tr>
<td>Hengchung</td>
<td>22°</td>
<td>120°44'</td>
</tr>
</tbody>
</table>

There are several conditions assumed, including the expected temperature of the water after heated is 55 degrees Celsius, the initial temperature of the water before heated is the surrounding temperature, the effective pe-
period of the irradiation absorbed by the solar panel is given [8], the transparency of the atmosphere is 0.75 and the electric heater is used to provide the auxiliary heat power. The monthly-averaged surrounding temperature and monthly-averaged insolation ratio in each region are given in Table 2 [6,9], this table shows that the temperature in south is hotter than that in north.

Table 3 lists the effect of the stinted energy for different sizes of solar panel with the water storages from 100 to 500 liters in Taipei, where the total auxiliary energy is zero when the stinted cost of the solar hot-water heater (SC) equals to the full electricity cost (FEC), and the electricity charge is set as 3 NT$/kWH. The simulated results illustrate that the stinted costs increased when both the water storage is increased and the size of the solar panel is enlarged, but the equipment cost will be also increased. Table 4 lists the comparison of mCRT for different sizes of the solar panel and water storage in Taipei. The table shows that the mCRT for one piece of the solar panel is 10 years, the mCRT for two pieces of the solar panel is 12 years, and the mCRT for three pieces of the solar panel is 14 years when the capacity of water storage is 100 liters, respectively. Therefore, the SCRT is 10 years and the optimal number of the solar panel is one for the 100 liters of water storage. Similarly, the SCRT is 8 years and the optimal number of the solar panel is two pieces for 200 liters of water storage, the SCRT is 7 years and the optimal number of the solar panel is three pieces for 300 liters of water storage.

Table 2. The monthly-averaged surrounding temperature and insolation ratio

<table>
<thead>
<tr>
<th>month</th>
<th>Taipei IR</th>
<th>Tini</th>
<th>Taichung IR</th>
<th>Tini</th>
<th>Tinan IR</th>
<th>Tini</th>
<th>Hengchung IR</th>
<th>Tini</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.264</td>
<td>16.3</td>
<td>0.534</td>
<td>17.7</td>
<td>0.545</td>
<td>18.7</td>
<td>0.521</td>
<td>21.4</td>
</tr>
<tr>
<td>02</td>
<td>0.180</td>
<td>16.5</td>
<td>0.393</td>
<td>18.0</td>
<td>0.490</td>
<td>19.3</td>
<td>0.530</td>
<td>22.0</td>
</tr>
<tr>
<td>03</td>
<td>0.214</td>
<td>19.2</td>
<td>0.380</td>
<td>20.6</td>
<td>0.545</td>
<td>22.3</td>
<td>0.553</td>
<td>24.2</td>
</tr>
<tr>
<td>04</td>
<td>0.239</td>
<td>22.7</td>
<td>0.369</td>
<td>23.8</td>
<td>0.459</td>
<td>25.1</td>
<td>0.528</td>
<td>26.0</td>
</tr>
<tr>
<td>05</td>
<td>0.236</td>
<td>25.6</td>
<td>0.362</td>
<td>26.2</td>
<td>0.545</td>
<td>27.4</td>
<td>0.478</td>
<td>27.5</td>
</tr>
<tr>
<td>06</td>
<td>0.286</td>
<td>28.6</td>
<td>0.408</td>
<td>28.3</td>
<td>0.448</td>
<td>29.1</td>
<td>0.446</td>
<td>28.4</td>
</tr>
<tr>
<td>07</td>
<td>0.411</td>
<td>30.2</td>
<td>0.505</td>
<td>29.1</td>
<td>0.567</td>
<td>29.6</td>
<td>0.585</td>
<td>28.8</td>
</tr>
<tr>
<td>08</td>
<td>0.477</td>
<td>29.6</td>
<td>0.489</td>
<td>28.7</td>
<td>0.500</td>
<td>29.2</td>
<td>0.510</td>
<td>28.5</td>
</tr>
<tr>
<td>09</td>
<td>0.403</td>
<td>24.7</td>
<td>0.510</td>
<td>27.9</td>
<td>0.511</td>
<td>28.7</td>
<td>0.510</td>
<td>27.8</td>
</tr>
<tr>
<td>10</td>
<td>0.366</td>
<td>21.7</td>
<td>0.569</td>
<td>25.7</td>
<td>0.572</td>
<td>26.5</td>
<td>0.574</td>
<td>26.6</td>
</tr>
<tr>
<td>11</td>
<td>0.300</td>
<td>19.2</td>
<td>0.545</td>
<td>22.9</td>
<td>0.541</td>
<td>23.8</td>
<td>0.543</td>
<td>24.8</td>
</tr>
<tr>
<td>12</td>
<td>0.281</td>
<td>16.9</td>
<td>0.552</td>
<td>19.6</td>
<td>0.551</td>
<td>20.4</td>
<td>0.468</td>
<td>22.7</td>
</tr>
</tbody>
</table>

Table 3. The stinted effect for different sizes of solar panel and water storage (Taipei)

<table>
<thead>
<tr>
<th>piece</th>
<th>storage (L)</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 SC</td>
<td>3832</td>
<td>4989</td>
<td>5307</td>
<td>5307</td>
<td>5307</td>
<td></td>
</tr>
<tr>
<td>2 SC</td>
<td>4148</td>
<td>7664</td>
<td>9237</td>
<td>9536</td>
<td>10512</td>
<td></td>
</tr>
<tr>
<td>3 SC</td>
<td>4148</td>
<td>8246</td>
<td>11496</td>
<td>12849</td>
<td>14374</td>
<td></td>
</tr>
<tr>
<td>4 SC</td>
<td>4148</td>
<td>8296</td>
<td>12241</td>
<td>15051</td>
<td>17115</td>
<td></td>
</tr>
<tr>
<td>5 SC</td>
<td>4148</td>
<td>8296</td>
<td>12443</td>
<td>16116</td>
<td>19160</td>
<td></td>
</tr>
<tr>
<td>6 SC</td>
<td>4148</td>
<td>8296</td>
<td>12444</td>
<td>16452</td>
<td>20013</td>
<td></td>
</tr>
<tr>
<td>FEC</td>
<td>4148</td>
<td>8296</td>
<td>12444</td>
<td>16591</td>
<td>20739</td>
<td></td>
</tr>
</tbody>
</table>

Note: (1) The SC denotes the stinted cost of the solar hot-water heater per year that equals to (Pty-Pwy) × α1. (2) The FEC denotes the full electricity cost without solar energy per year that equals to Pty × α1. (3) The L denotes the liter.
Tables 5, 6 and 7 list the comparison of mCRT for the different sizes of solar panel and water storage in Taichung, Tinan and Hengchung, respectively. The tables show that the some results are the same in these regions. For example, the SCRT are 7 years and the optimal number of the solar panel are one for 200 liters of water storage, the SCRT are 6 years and the optimal number of the solar panel are two for 300 liters of water storage, and the SCRT are 5 years and the optimal number of the solar panel are three for 500 liters of water storage in the three regions, respectively.

6. Conclusion

In this paper, the sun’s position at any time and any region is predicted by a mathematical procedure. Then, a step by step method is applied to evaluate the optimal size of solar panels which with the optimal investment effect on each region corresponding to the specified capacity of the water storage for the solar hot-water heater system. The simulated results show that the different SCRT and different optimal number of solar panels for the different region although their water storages are the same. For example, there are 4 members in a family, the required water storage of the solar hot-water heater is about 200 liters, the optimal number of solar panels is two, and the SCRT is 8 years in Taipei from the Table 4. The optimal number of solar panels is one and the SCRT is 7 years in Taichung, Tinan and Hengchung for the same family from the Tables 5, 6 and 7, respectively. The results illustrate that the investment effect of solar hot-water heaters installed in south is better than that installed in north. However, the solar hot-water heaters installed in each region are all helpful for saving the power energy and reducing the CO2. If the solar hot-water heater with 200 liters of water storage and two pieces of solar panels installed in all families (about 950,000 households [7]) in Taipei, the total stinted power energy is 2,382,916,667 kWh per a whole year and it is 4.1% of the total light demand in Taiwan 2007 (58,848,337,029 kWh) [10]. Therefore, a lot of power energy consumption can be stinted when the solar hot-water heaters are used broadly.

References


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