Hydromagnetic Flow of a Conducting Micropolar Fluid over a Plane Wall with Heat Transfer

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Abstract

The steady hydromagnetic laminar flow of an incompressible non-Newtonian micropolar fluid impinging on a plane wall with heat transfer is investigated. A uniform magnetic field is applied normal to the plate which is maintained at a constant temperature. Numerical solution for the governing nonlinear momentum and energy equations is obtained. The effect of the uniform magnetic field and the characteristics of the non-Newtonian fluid on both the flow and heat transfer is presented and discussed.

Key Words: Stagnation Point Flow, Non-Newtonian Fluid, Magnetic Field, Numerical Solution, Heat Transfer

1. Introduction

The two-dimensional flow of a fluid near a stagnation point is a classical problem in fluid mechanics. It was first examined by Hiemenz [1] who demonstrated that the Navier-Stokes equations governing the flow can be reduced to an ordinary differential equation of third order using similarity transformation. Owing to the nonlinearities in the reduced differential equation, no analytical solution is available and the nonlinear equation is usually solved numerically subject to two-point boundary conditions, one of which is prescribed at infinity.

Later the problem of stagnation point flow was extended in numerous ways to include various physical effects. The axisymmetric three-dimensional stagnation point flow was studied by Homann [2]. The results of these studies are of great technical importance, for example in the prediction of skin-friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery. Either in the two or three-dimensional case Navier-Stokes equations governing the flow are reduced to an ordinary differential equation of third order using a similarity transformation. Later the problem of stagnation point flow has been extended in numerous ways to include various physical effects. The effect of suction on Hiemenz problem has been considered in the literature. Schlichting and Bussman [3] gave the numerical results first. More detailed solutions were later presented by Preston [4]. An approximate solution to the problem of uniform suction is given by Ariel [5]. The effect of uniform suction on Homann problem where the flat plate is oscillating in its own plane is considered by Weidman and Mahalingam [6]. In hydromagnetics, the problem of Hiemenz flow was chosen by Na [7] to illustrate the solution of a third-order boundary value problem using the technique of finite differences. An approximate solution of the same problem has been provided by Ariel [8]. The effect of an externally applied uniform magnetic field on the two or three-dimensional stagnation point flow was given, respectively, by Attia in [9] and [10] in the presence of uniform suction or injection.

The study of heat transfer in boundary layer flows is of importance in many engineering applications such as the design of thrust bearings and radial diffusers, transpiration cooling, drag reduction, thermal recovery of oil, etc. [11]. Massoudi and Ramezan [11] used a perturba-

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tion technique to solve for the stagnation point flow and heat transfer of a non-Newtonian fluid of second grade. Their analysis is valid only for small values of the parameter that determines the behavior of the non-Newtonian fluid. Later Massoudi and Ramezan [12] extended the problem to nonisothermal surface. Garg [13] improved the solution obtained by Massoudi [12] by computing numerically the flow characteristics for any value of the non-Newtonian parameter using a pseudo-similarity solution.

Non-Newtonian fluids were considered by many researchers. Thus, among the non-Newtonian fluids, the solution of the stagnation point flow, for viscoelastic fluids, has been given by Rajeshwari and Rathna [14], Beard and Walters [15], Teipel [16], Arial [17], and others; for power-law fluid by Djukic [18]; and for second grade fluids by Teipel [19] and Ariel [20] in the hydrodynamic case and by Attia [21] in the hydromagnetic case. Stagnation point flow of a non-Newtonian micropolar fluid was studied by Nath [22] and Nazar et al. [23] in the hydrodynamic case.

The purpose of the present paper is to study the effect of uniform magnetic field applied normal to the wall on the steady laminar flow of an incompressible non-Newtonian micropolar fluid at a two-dimensional stagnation point with heat transfer. The wall and stream temperatures are assumed to be constants. The induced magnetic field is neglected by assuming small magnetic Reynolds number [24]. A numerical solution is obtained for the governing momentum and energy equations using finite difference approximations which takes into account the asymptotic boundary conditions. The numerical solution computes the flow and heat characteristics for the whole range of the non-Newtonian fluid characteristics, the magnetic field parameter and the Prandtl number.

2. Formulation of the Problem

Consider the two-dimensional stagnation point flow of an incompressible non-Newtonian micropolar fluid impinging perpendicular on a permeable wall and flows away along the x-axis. This is an example of a plane potential flow that arrives from the y-axis and impinges on a flat wall placed at y = 0, divides into two streams on the wall and leaves in both directions. The viscous flow must adhere to the wall, whereas the potential flow slides along it. (u, v) are the components for the potential flow of velocity at any point (x, y) for the viscous flow whereas (U, V) are the velocity components for the potential flow. A uniform magnetic field B₀ is applied perpendicular to the wall and the magnetic Reynolds number is assumed very small so that the applied magnetic field is undisturbed [24]. The velocity distribution in the frictionless flow in the neighborhood of the stagnation point is given by

\[ U(x) = ax, \quad V(y) = -ay \]

where the constant a(> 0) is proportional to the free stream velocity far away from the surface. The simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar and incompressible micropolar fluid are [22,23]

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \quad (1) \\
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= U \frac{dU}{dx} + (\mu + h) \left( \frac{\partial^2 u}{\partial y^2} \right) \\
&+ h \frac{\partial N}{\partial y} + \sigma B_0^2 (U(x) - u) \quad (2) \\
\rho \left( u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) &= \gamma \frac{\partial^2 N}{j \partial y^2} - k \left( \frac{2N + \partial u}{j \partial y} \right) \quad (3)
\end{align*}
\]

where \( N \) is the micro-rotation or angular velocity whose direction of rotation is in the x-y plane, \( \mu \) is the viscosity of the fluid, \( \rho \) is the density of the fluid, \( \sigma \) is the electrical conductivity of the fluid and \( j \), \( \gamma \) and \( h \) are the micro-inertia per unit mass, spin gradient viscosity and vortex viscosity, respectively, which are assumed to be assumed to be constant. The last term in the right-hand side represents the electromagnetic Lorentz force [24]. Here \( \gamma \) is assumed to be given by [22,23]

\[ \gamma = (\mu + h/2) j \quad (4) \]

and we take \( j = v / a \) as a reference length and \( v \) is the kinematic viscosity. Relation (4) is invoked to allow Eqs. (1)–(3) to predict the correct behaviour in the limiting case when microstructure effects become negligible, and the microrotation, \( N \), reduces to the angular velocity [23].
The appropriate physical boundary conditions of Eqs. (1)–(3) are

\[ u(x, 0) = 0, v(x, 0) = 0, \quad N(x, 0) = -n \frac{\partial u}{\partial y} \]  
\[ y \to \infty : u(x, y) \to U(x), \quad v(x, y) \to 0, \quad N(x, y) \to 0 \]  
\[ \text{(5a)} \]
\[ \text{(5b)} \]

where \( n \) is a constant and \( 0 \leq n \leq 1 \). The case \( n = 1/2 \) indicates the vanishing of anti-symmetric part of the stress tensor and denotes weak concentration [23] of microelements which will be considered here. The governing equations (1)–(4) subject to the boundary conditions (5) can be expressed in a simpler form by introducing the following transformation

\[ \eta = \sqrt{\frac{a}{v}} y, \quad u = ax f'(|\eta|), \quad v = -\sqrt{av} f(|\eta|), \quad N = ax \sqrt{av} g(|\eta|), \]
\[ g(|\eta|) = -\frac{1}{2} f''(|\eta|) \]  

so that Eqs. (2) and (3) reduce to the single equation

\[ \left( 1 + \frac{K}{2} \right) f'' + f' f'' - f'^2 + 1 + Ha^2 (1 - f') = 0 \]  
\[ \text{(7)} \]

subject to the boundary conditions

\[ f(0) = 0, f'(0) = 0, f'(\infty) = 1 \]  
\[ \text{(8)} \]

where \( K = \frac{h}{\mu} > 0 \) is the material parameter, \( A = \frac{v_o}{ \sqrt{av}} \) is the suction parameter, \( Ha^2 = \sigma B^2 / \alpha p \) is the modified Hartmann number squared, and primes denote differentiation with respect to \( \eta \). For micropolar boundary layer flow, the wall skin friction \( \tau_w \) is given by

\[ \tau_w = \left[ \left( \mu + h \right) \frac{\partial u}{\partial y} + hN \right]_{\eta=0} \]  
\[ \text{(9)} \]

Using \( U(x) = ax \) as a characteristic velocity, the skin friction coefficient \( C_f \) can be defined as

\[ C_f = \frac{\tau_w}{\rho U^2} \]  
\[ \text{(10)} \]

Substituting (6) and (9) into (10), we get

\[ C_f \frac{\text{Re}^{1/2}}{x} = (1 + K/2) f''(0) \]  
\[ \text{(11)} \]

where \( \text{Re}^{1/2} = xU / v \) is the local Reynolds number.

Using the boundary layer approximations and neglecting the dissipation, the equation of energy for temperature \( T \) is given by [11,12],

\[ \rho c_p \left( \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} \]  
\[ \text{(12)} \]

where \( c_p \) is the specific heat capacity at constant pressure of the fluid, and \( k \) is the thermal conductivity of the fluid. A similarity solution exists if the wall and stream temperatures, \( T_w \) and \( T_a \) are constants – a realistic approximation in typical stagnation point heat transfer problems [11,12].

The boundary conditions for the temperature field are

\[ y = 0 : T = T_w \]  
\[ \text{(13a)} \]
\[ y \to \infty : T \to T_a \]  
\[ \text{(13b)} \]

Introducing the non-dimensional variable

\[ \theta = \frac{T - T_a}{T_w - T_a} \]  
\[ \text{(14)} \]

and using the similarity transformations given in Eq. (6), we find that Eqs. (12) and (13) reduce to,

\[ \theta'' + Pr \theta' f = 0 \]  
\[ \theta(0) = 1, \theta(\infty) = 0 \]  
\[ \text{(15)} \]

where \( Pr = \mu c_p / k \) is the Prandtl number.

The heat transfer at the wall is computed from Fourier’s law [11,12] as follows;

\[ q_w = -k \left( \frac{\partial T}{\partial y} \right)_{\eta=0} = -k(T_w - T_a) \sqrt{\frac{\pi}{v}} G(Pr) \]

where \( G \) is the dimensionless heat transfer rate which is given by

\[ G^{-1} = \int_0^\infty d\eta \exp(-2Pr \eta) \int_0^\eta f d\zeta \]
The flow Eqs. (7) and (8) are decoupled from the energy Eqs. (14) and (15), and need to be solved before the latter can be solved. The flow Eq. (7) constitutes a non-linear, non-homogeneous boundary value problem (BVP). In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and natural choice. The boundary value problem given by Eqs. (7) and (8) may be viewed as a prototype for numerous other situations which are similarly characterized by a boundary value problem having a third order differential equation with an asymptotic boundary condition at infinity. Therefore, its numerical solution merits attention from a practical point of view. The flow Eqs. (7) and (8) are solved numerically using finite difference approximations. A quasi-linearization technique is first applied to replace the non-linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence. The quasi-linearized form of Eq. (7) is,

\[ (1 + K/2)f''_n + f_n f'_{n+1} + f'_{n} f_{n+1} - f''_{n} f_{n} - 2f'_{n} f'_{n+1} + f_{n}^2 + 1 + Ha^2(1 - f'_{n+1}) = 0 \]

where the subscript \( n \) or \( n+1 \) represents the \( n^{th} \) or \( (n+1)^{th} \) approximation to the solution. Then, Crank-Nicolson method is used to replace the different terms by their second order central difference approximations. An iterative scheme is used to solve the quasi-linearized system of difference equations. The solution for the Newtonian case is chosen as an initial guess and the iterations are continued till convergence within prescribed accuracy. Finally, the resulting block tri-diagonal system was solved using generalized Thomas’ algorithm.

The energy Eq. (14) is a linear second order ordinary differential equation with variable coefficient, \( f(\eta) \), which is known from the solution of the flow Eqs. (7) and (8) and the Prandtl number \( \text{Pr} \) is assumed constant. Equation (14) is solved numerically under the boundary condition (15) using central differences for the derivatives and Thomas’ algorithm for the solution of the set of discretized equations. The resulting system of equations has to be solved in the infinite domain \( 0 < \eta < \eta_{c} \). A finite domain in the \( \eta \)-direction can be used instead with \( \eta \) chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. Grid-independence studies show that the computational domain \( 0 < \eta < \eta_{c} \) can be divided into intervals each of uniform step size which equals 0.02. This reduces the number of points between \( 0 < \eta < \eta_{c} \) without sacrificing accuracy. The value \( \eta_{c} = 10 \) was found to be adequate for all the ranges of parameters studied here. Convergence is assumed when the ratio of every one of \( f, f', f'' \), or \( f''' \) for the last two approximations differed from unity by less than \( 10^{-5} \) at all values of \( \eta \) in \( 0 < \eta < \eta_{c} \).

### 3. Results and Discussion

Figures 1 and 2 present the velocity profiles of \( f \) and \( f' \), respectively, for various values of \( K \) and \( Ha \). The figures show that increasing the parameter \( K \) decreases both \( f \) and \( f' \) and its effect is more pronounced for higher \( Ha \). Increasing \( Ha \) increases them and its effect is more apparent for higher \( K \). Also, increasing \( K \) increases the velocity boundary layer thickness while increasing \( Ha \) decreases it.

![Figure 1](image1.png)  
**Figure 1.** Effect of the parameters \( K \) and \( Ha \) on the profile of \( f \).

![Figure 2](image2.png)  
**Figure 2.** Effect of the parameters \( K \) and \( Ha \) on the profile of \( f' \).
Figure 3 presents the profile of temperature $\theta$ for various values of $K$ and $Ha$ and $Pr = 0.5$. It is clear that increasing $K$ increases $\theta$ and the thickness of the thermal boundary layer. Increasing $Ha$ decreases the thermal boundary layer thickness for all $K$. Figure 4 presents the temperature profile for various values of $K$ and $Pr$ and for $Ha = 0.5$. The figure brings out clearly the effect of the Prandtl number on the thermal boundary layer thickness. As shown in Figure 4, increasing $Pr$ decreases the thermal boundary layer thickness for all $K$. It is clear that increasing $K$ decreases $\theta$ and the effect of $K$ on $\theta$ is more pronounced for higher values of $Pr$.

Tables 1 and 2 present the variation of the wall shear stress $C_{f_j} \text{Re}_{j}^{1/2}$ and the heat transfer rate at the wall $G(Pr)$, respectively, for various values of $K$ and $Ha$ and for $Pr = 0.5$. Table 1 shows that, for all $Ha$, increasing $K$ increases $C_{f_j} \text{Re}_{j}^{1/2}$ and then, increasing $K$ more decreases $C_{f_j} \text{Re}_{j}^{1/2}$. Increasing $Ha$ increases $C_{f_j} \text{Re}_{j}^{1/2}$ steadily for all $K$ and its effect is more apparent for smaller $K$. Table 2 shows that increasing $K$ decreases $G(Pr)$ while increasing $Ha$ increases $G(Pr)$ for all $K$.

Table 3 presents the effect of $K$ on $G(Pr)$ for various values of $Pr$ and for $Ha = 0.5$. Increasing $K$ decreases $G(Pr)$ for all $Pr$ and its effect is more for higher $Pr$. Increasing $Pr$ increases $G(Pr)$ for all $K$. Table 4 shows the variation of $G(Pr)$ for various values of $Pr$ and $Ha$ and for $K = 0.5$. Increasing $Ha$ increases $G(Pr)$ and its effect is more apparent for higher $Pr$. Increasing $Pr$ increases $G$ steadily for all $Ha$ and its effect is more apparent for higher $Ha$.

**Table 1.** Variation of the wall shear stress $C_{f_j} \text{Re}_{j}^{1/2}$ with $K$ and $Ha$ ($Ha = 0.5$)

<table>
<thead>
<tr>
<th>$Ha$</th>
<th>$K = 0$</th>
<th>$K = 0.5$</th>
<th>$K = 1$</th>
<th>$K = 1.5$</th>
<th>$K = 2$</th>
</tr>
</thead>
<tbody>
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<td>1.7612</td>
<td>1.7469</td>
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<td>2.2469</td>
<td>2.2419</td>
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<tr>
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<td>2.7421</td>
</tr>
<tr>
<td>2</td>
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<td>3.4107</td>
<td>3.3530</td>
<td>3.3260</td>
<td>3.3186</td>
</tr>
</tbody>
</table>

**Table 2.** Variation of the wall heat transfer $G(Pr)$ with $K$ and $Ha$ ($Pr = 0.5$)

<table>
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<tr>
<th>$Ha$</th>
<th>$K = 0$</th>
<th>$K = 0.5$</th>
<th>$K = 1$</th>
<th>$K = 1.5$</th>
<th>$K = 2$</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>0.4677</td>
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<td>0.4555</td>
<td>0.4504</td>
</tr>
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</table>
## 4. Conclusion

The two-dimensional stagnation point flow of an incompressible non-Newtonian micropolar fluid with heat transfer is studied in the presence of uniform magnetic field. A numerical solution for the governing equations is obtained which allows the computation of the flow and heat transfer characteristics for various values of the non-Newtonian parameter $K$, the magnetic field parameter $H_a$, and the Prandtl number $Pr$. The results indicate that increasing the parameter $K$ increases both the velocity and thermal boundary layer thickness while increasing $H_a$ decreases the thickness of both layers. The effect of the material parameter $K$ on the velocity is more apparent for higher $H_a$. The influence of $K$ or $H_a$ on the temperature is more apparent for higher values of Prandtl number.

### References


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