MHD Couette Flow and Heat Transfer of a Dusty Fluid with Exponential Decaying Pressure Gradient

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Abstract

In the present study, the unsteady Couette flow with heat transfer of a viscous incompressible electrically conducting fluid under the influence of an exponentially decreasing pressure gradient is studied. The parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below while the fluid is acted upon by an external uniform magnetic field applied perpendicular to the plates. The equations of motion are solved analytically to yield the velocity distributions for both the fluid and dust particles. The energy equations for both the fluid and dust particles including the viscous and Joule dissipation terms, are solved numerically using finite differences to get the temperature distributions.

Key Words: Couette Flow, Magnetohydrodynamics, Heat Transfer, Dusty Fluid, Numerical Solution

1. Introduction

The importance and application of solid/fluid flows and heat transfer in petroleum transport, wastewater treatment, combustion, power plant piping, corrosive particles in engine oil flow, and many others are well known in the literature [1–7]. Particularly, the flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters and has possible applications in nuclear reactors, filtration, geothermal systems, and others. The possible presence of solid particles such as ash or soot in combustion MHD generators and plasma MHD accelerators and their effect on the performance of such devices led to studies of particulate suspensions in conducting fluids in the presence of magnetic fields. For example, in an MHD generator, coal mixed with seed is fed into a combustor. The coal and seed mixture is burned in oxygen and the combustion gas expands through a nozzle before it enters the generator section. The gas mixture flowing through the MHD channel consists of a condensable vapor (slag) and a non-condensable gas mixed with seeded coal combustion products. Both the slag and the non-condensable gas are electrically conducting [1,2]. The presence of the slag and the seeded particles significantly influences the flow and heat transfer characteristics in the MHD channel. Ignoring the effect of the slag, and considering the MHD generator start-up condition, the problem reduces to unsteady two-phase flow in an MHD channel [8–12].

In the present work, the transient Couette flow with heat transfer of an electrically conducting, viscous, incompressible, dusty fluid is studied. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The fluid is acted upon by an exponentially decaying with time pressure gradient. The fluid is assumed to be incompressible and electrically conducting and the particle phase is assumed to incompressible, electrically non-conducting dusty and pressureless. The fluid is flowing between two infinite electrically insulating porous plates maintained at two constant but different temperatures while the particle phase is assumed to be electrically non-conducting. The fluid is subjected to

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a uniform suction from above and a uniform injection from below and mass conservation is assumed. An external uniform magnetic field is applied perpendicular to the plates while no electric field is applied and the induced magnetic field is neglected by assuming a very small magnetic Reynolds number. The governing equations for both fluid and dust particles are solved numerically taking the Joule and the viscous dissipations into consideration in the energy equations. The effect of the magnetic field, the Hall current, the ion slip, and the suction velocity on both the velocity and temperature fields are reported.

2. Description of the Problem

The dusty fluid is assumed to be flowing between two infinite horizontal porous plates located at the \( y = \pm h \) planes. The upper plate is moving with a constant velocity \( U_0 \) while the lower plate is kept stationary. The plates are subjected to a uniform suction from above and a uniform injection from below. Thus the \( y \)-component of the velocity of the fluid is constant and denoted by \( v_0 \). The dust particles are assumed to be electrically non-conducting spherical in shape and uniformly distributed throughout the fluid and to be big enough, so that they are not pumped out through the porous plates and have no \( y \)-component of velocity. The two plates are assumed to be electrically non-conducting and kept at two constant temperatures \( T_1 \) for the lower plate and \( T_2 \) for the upper plate with \( T_2 > T_1 \). A uniform pressure gradient, which is taken to be exponentially decaying with time, is applied in the \( x \)-direction. A uniform magnetic field \( B_y \) is applied in the positive \( y \)-direction. This is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number \([11]\). The fluid motion starts from rest at \( t = 0 \), and the no-slip condition at the plates implies that the fluid and dust particles velocities have neither a \( z \) nor an \( x \)-component at \( y = \pm h \). The initial temperatures of the fluid and dust particles are assumed to be equal to \( T_1 \). It is required to obtain the time varying velocity and temperature distributions for both fluid and dust particles. Due to the inclusion of the Hall current term, a \( z \)-component of the velocities of the fluid and of dust particles is expected to arise. Since the plates are infinite in the \( x \) and \( z \)-directions, the physical quantities do not change in these directions that is \( \partial / \partial x = \partial / \partial z = 0 \) and the problem is essentially one-dimensional. The governing equations for this study are based on the conservation laws of mass, linear momentum and energy of both phases. In this work, it is assumed that both phases are treated as two interacting continua. The interaction between the phases is restricted to the interphase drag force which is modeled by Stokes linear drag theory and the interphase heat transfer \([1,2]\).

The Velocity Distribution

The flow of fluid is governed by the momentum equation

\[
\frac{D\rho}{Dt} = -\nabla P + \mu \nabla^2 v + J x B_y - KN(v - v_p)
\]  

(1)

where \( \rho \) is the density of clean fluid, \( \mu \) is the viscosity of clean fluid, \( v \) is the velocity of the fluid, \( v = u(y, t)I + v_0 j \), \( v_p \) is the velocity of dust particles, \( v_p = u_p(y, t)I \), \( J \) is the current density, \( N \) is the number of dust particles per unit volume, \( K \) is the Stokes constant \( = 6 \pi \mu a \), and \( a \) is the average radius of dust particles.

The first three terms in the right-hand side of Eq. (1) are, respectively, the pressure gradient, viscosity, and Lorentz force terms. The last term represents the force due to the relative motion between fluid and dust particles. It is assumed that the Reynolds number of relative velocity is small. In such a case the force between dust and fluid is proportional to the relative velocity \([3]\). The current density \( J \) from the generalized Ohm’s law is given by \([13,14]\)

\[
J = \sigma [E + \nu x B_x]
\]

(2)

where \( \sigma \) is the electric conductivity of the fluid \([13,14]\).

Solving Eq. (2) for \( J \) and substituting the result in Eq. (1), the two components of Eq. (1) read

\[
\rho \frac{\partial u}{\partial t} + \rho v \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu \frac{\partial^2 u}{\partial y^2} - \sigma B_y^2 u - KN(u - u_p)
\]

(3)

The motion of the dust particles is governed by Newton’s second law applied in the \( x \) and \( z \)-directions

\[
m_p \frac{\partial v_p}{\partial t} = KN(u - u_p)
\]

(4)
where \( m_p \) is the average mass of dust particles. It is assumed that the pressure gradient is applied at \( t = 0 \) and the fluid starts its motion from rest. Thus,

\[
\begin{align*}
    t \leq 0 : u &= u_p = 0 \\
    \text{For } t > 0, \text{ the no-slip condition at the plates implies that} \\
    t > 0 : y = -h, u = u_p = 0; y = h, u = U_o, u_p = 0
\end{align*}
\]

\[ (5) \]

\[ (6) \]

**The Temperature Distribution**

Heat transfer takes place from the upper hot plate to the lower cold plate by conduction through the fluid. Since the hot plate is above, there is no natural convection, however, there is a forced convection due to the suction and injection. In addition to the heat transfer, there is a heat generation due to both the Joule and viscous dissipations. The dust particles gain heat from the fluid by conduction through their spherical surface. Since, the problem deals with a two-phase flow, two energy equations are required \([11,15]\). The energy equations describing the temperature distributions for both the fluid and dust particles read

\[
\rho c R \frac{\partial T}{\partial t} + \rho c \gamma_p \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \sigma B_x^2 u^2 + \frac{\rho c}{\gamma_p} (T_p - T)
\]

\[ (7) \]

\[
\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma_p} (T_p - T)
\]

\[ (8) \]

where \( T \) is the temperature of the fluid, \( T_p \) is the temperature of the particles, \( c \) is the specific heat capacity of the fluid at constant volume, \( k \) is the thermal conductivity of the fluid, \( \rho_p \) is the mass of dust particles per unit volume of the fluid, \( \gamma_p \) is the temperature relaxation time, and \( c_p \) is the specific heat capacity of the particles.

The last three terms on the right-hand side of Eq. (7) represent the viscous dissipation, the Joule dissipation \((j^2/\sigma)\), and the heat conduction between the fluid and dust particles respectively. The temperature relaxation time depends, in general, on the geometry, and since the dust particles are assumed to be spherical in shape, the last term in Eq. (7) is equal to \( 4\pi a N(T_p - T) \). Hence

\[
\gamma_p = \frac{3 \Pr \gamma_p c_p}{2c}
\]

where \( \gamma_p \) is the velocity relaxation time = \( 2\rho_0 a^2 / 9\mu \), \( \Pr \) is the Prandtl number = \( \mu c / k \), and \( \rho \) is the material density of dust particles = \( 3\rho_p / 4\pi a^3 N \).

\( T \) and \( T_p \) must satisfy the initial and boundary conditions

\[
\begin{align*}
    t \leq 0 : T &= T_p = 0 \\
    t > 0, y = -h : T &= T_p = T_i \\
    t > 0, y = h : T = T_p = T_j
\end{align*}
\]

\[ (9a) \]

\[ (9b) \]

\[ (9c) \]

The problem is simplified by writing the equations in the non-dimensional form. We define the following non-dimensional quantities

\[
\begin{align*}
    \dot{x} &= \frac{x}{h}, \quad \dot{y} = \frac{y}{h}, \quad \dot{z} = \frac{z}{h}, \quad \dot{u} = \frac{u}{U_o}, \quad \dot{P} = \frac{P}{\rho U_o^2}, \quad \dot{t} = \frac{tU_o}{h}, \\
    \dot{T} &= \frac{T - T_i}{T_j - T_i}
\end{align*}
\]

\[ (10) \]

\[ (11) \]

\[ (12) \]

\[ (13) \]

Re = \( \rho h U_o / \mu \), is the Reynolds number, \( S = v_o / U_o \), is the suction parameter, \( Pr = \mu c / k \) is the Prandtl number, \( Ha \) \( = \sigma B_x^2 h^2 / \mu \) where \( Ha \) is the Hartmann number, \( G = m_p \mu / \rho h^2 K \) is the particle mass parameter, \( R = KN h^2 / \mu \) is the particle concentration parameter, \( Ec = U_o^2 / c(T_j - T_i) \) is the Eckert number, \( L_o = \rho h^2 / \gamma_p \) is the temperature relaxation time parameter.

In terms of the above non-dimensional quantities the velocity and energy equations read

\[
\frac{\partial u}{\partial t} + \nabla \cdot (\mu \nabla u) = \frac{1}{Re} \frac{dP}{dx} + \frac{1}{Re} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{R}{Re} (u - u_p)
\]

\[ (14) \]
\[ t \leq 0: T = T_p = 0, t > 0: y = -1, T = T_p = 0, y = 1, T = T_p = 1 \]

where the pressure gradient is assumed in the form \( \frac{dP}{dx} = C e^{-\alpha x} \).

### Numerical Solution of the Governing Equations

Equations (10)–(13) are solved numerically using finite differences [16] under the initial and boundary conditions (14) and (15) to determine the velocity and temperature distributions for different values of the parameters \( Ha \) and \( S \). The Crank-Nicolson implicit method is applied. The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the \( y \)-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. The viscous and Joule dissipation terms are evaluated using the velocity components and their derivatives in the \( y \)-direction which are obtained from the exact solution. Finally, the block tridiagonal system is solved using Thomas’ algorithm. All calculations have been carried out for \( C = -5, \alpha = 1, Re = 1, Pr = 1, R = 0.5, Lo = 0.7, G = 0.8 \) and \( Ec = 0.2 \).

### 3. Results and Discussion

Figure 1 presents, respectively, the profiles of the velocity components and temperature of the fluid \( u \) and \( T \) and particles \( u_p \) and \( T_p \) for various values of time \( t \). The figures are plotted for \( Ha = 1 \) and \( S = 1 \). As shown in Figures 1a and b the profiles of \( u \) and \( u_p \) are asymmetric about the plane \( y = 0 \) because of the suction. It is observed that the velocity component and temperature of the fluid phase reach the steady state faster than that of the particle phase. This is because the fluid velocity is the source for the dust particles’ velocity. It is shown that the velocity components and temperatures of the fluid and dust particles do not reach the steady state monotonically due to the effect of the pressure gradient.

Figure 2 shows the time evolution of the velocity components and temperature at the centre of the channel \( (y = 0) \), respectively, for the fluid and particle phases for various values of the Hartmann number \( Ha \) and \( S = 0 \). Figures 2a and b indicate that increasing \( Ha \) decreases \( u \) and \( u_p \) for all \( t \), as a result of increasing the damping force on \( u \) which decreases \( u \) and consequently decreases \( u_p \). Figures 2c and d indicate that the variation of \( T \) and \( T_p \) with \( Ha \) depends on time. It is clear that for small \( t \), increasing \( Ha \) in-
creases $T$ and $T_p$ due to increasing the Joule dissipation. But, for large $t$, increasing $Ha$ decreases $T$ as a result of decreasing the velocities $u$ and $u_p$, and consequently decreases the viscous and Joule dissipations.

Figure 3 presents the time evolution of the velocity components and temperature at the centre of the channel ($y = 0$), respectively, for the fluid and particle phases for various values of the suction parameter $S$ and $Ha = 0$. It is
clear from Figures 3a and b that increasing the suction parameter decreases both $u$ and $u_p$ due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. Figures 3c and d show that increasing $S$ decreases the temperature at the centre of the channel. This is due to the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel. It is observed from Figures 2 and 3 that the suction has a more pronounced effect on the steady state time of the velocity and temperature of the particles than that of the magnetic field.

4. Conclusion

The unsteady flow with heat transfer of a dusty conducting fluid under the influence of an applied uniform magnetic field has been studied in the presence of uniform suction and injection and an exponential decaying pressure gradient. An analytical solution for the equations of motion has been obtained while the energy equation has been solved numerically using finite differences. The effect of the magnetic field, and the suction and injection velocity on the velocity and temperature distributions for both the fluid and particle phases has been investigated. It is of interest to see that the effect of the magnetic field on the temperatures of the fluid and particles depends on time. Also, it is observed that the suction velocity has a more apparent effect than the magnetic field on the steady state time of the velocity and temperature of the dust particles.

References


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