The Consistent Performance in Dispersion Analysis for Geotechnical Surface Wave Investigations

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Abstract

This paper presents improved dispersion analysis of the surface wave method for estimations of shear wave velocity profile at near surface soils. The conventional surface wave analysis has limited capability to reduce the irregularities and other noises in the dispersion curve. The frequency-wavenumber (f-k) domains analysis with fitting criteria and different mode analysis is presented to obtain reliable dispersion curve of surface wave velocities. The irregularities and noises presented in absolute dispersion curve can be reduced using updated technique with the f-k analysis. The significance of the paper is to show how reliable apparent dispersion curve is obtained through f-k transformation with fitting criteria implemented in the MASW method.

Key Words: Dispersion Curve, f-k Transform, Fitting Criteria, Surface Wave Method, Multichannel Analysis

1. Introduction

Scientific interest is growing in the application of seismic signal analysis to demonstrate the surface wave profiling of near-surface soil. The main advantage of surface wave method is essentially related to its non-destructive and non-invasive nature that allows the characterization of near surface soils. Thus, the need of boreholes for subsurface seismic methods (such as downhole and cross-hole methods) which are expensive and time consuming can be minimized. The spectral analysis of surface waves (SASW) method which is based on dispersive characteristics of surface waves is one of the familiar seismic methods for determining the shear wave velocity. The SASW method is widely established as a subsurface investigative tool, and is implemented in a wide variety of geotechnical investigations, including pavements, solid waste landfills, and sea beds profile.

The multichannel analysis of surface waves (MASW) method \cite{1-8} represents an improvement over SASW, overcoming the few but significant weaknesses of the SASW method. MASW is also a fast method of evaluating near-surface shear wave velocity, $v$, profile due to the coverage of the entire range of investigation depth by one, or a few generation of ground roll without changing receiver configuration. The main advantage of the MASW method is its ability to take a full account of the complicated nature of seismic waves that always contain higher modes of surface waves, body waves, scattered waves, traffic waves. Incorporating multi-station receivers and 2-D wavefield transformation \cite{9,10} improves inherent difficulties in evaluating signal from noise with only a pair of receivers.

The MASW technique has proven to reduce the interfering of unwanted seismic signal in the shot records and filter noisy surface-wave modes. Thus, this method significantly improves the range and resolution of multimodal dispersion curves in the phase-velocity-frequency domain. Observation and detection of higher mode surface wave is successfully obtained in the MASW me-
MASW method is convenient to apply in a shallow marine environment and to evaluate stiffness of water-bottom sediments. This method is also impressive to determine a sinkhole impact area as well as collapse and subsidence feature, to represent near surface anomalies, to delineate dissolution features and to map bedrock surface [11–23]. The MASW method is manipulated in three stages: data acquisition with multiple receivers, dispersion analysis and inversion analysis. The dispersion analysis is one of the significant considerations to reveal consistent shear wave velocity profile to evaluate near surface soil properties.

The dispersion analysis through frequency-wave-number (f-k) domain is gaining popularity for surface wave method due to the consistence in performance. Conventionally, the dispersion analysis using f-k domain for MASW method is advanced to demonstrate the different mode in dispersion curve. Usually, only fundamental mode is considered as higher energy disclosure and other higher modes are neglected in final estimation of the shear wave velocity. Moreover, there is no estimator to find the deviation in dispersive outcomes. In this study, different types of modes are taken into account to obtain more accurate dispersion curve through f-k analysis with fitting criteria. Data fitting is one of the significant considerations in research analysis to reduce the presence of error in final outcomes. Thus, the irregularities and noises presented in absolute dispersion curve can be reduced using the updated technique with the f-k analysis. The aim of the paper shows how reliable apparent dispersion curve is obtained through f-k transformation with fitting criteria implemented in the MASW method.

2. Improved Dispersion Analysis

The dispersion analysis is performed through f-k transformation through the fitting criteria to obtain reliable dispersion curve. The recorded signal by acquisition device, RAS-24 is stored in time domain signal. Figure 1 shows time domain output for 24 receivers through a single shot which is recorded in data acquisition. The time domain signals are then converted to frequency domain using Fourier analysis by MATLAB Programming. The data acquisition and analysis are performed since January, 2009 to June, 2010 at University Kebangsaan Malaysia (UKM) in Bangi, Selangor, Malaysia.

Fourier analysis is a mathematical technique which breaks down a signal into constituent sinusoids of different frequencies and transforms our view of the signal from time-based to frequency-based. The time domain signals shown in Figure 2(a) is the 1st receiver trace of Figure 1, and composed of 1200 samples. The time domain signal shown in Figure 2(a) is converted to frequency domain revealed in Figure 2(b) with the help of Discrete Fourier Transformation (DFT) analysis.

In the frequency domain analysis, the cross spectral density between each pair of receivers is determined using a combination formula. Each term of the cross spectral density is represented by

$$R_{i,j}(f) = \frac{1}{B} \sum_{n=1}^{N} S_{i,n}(\omega) S_{j,n}(\omega^*)$$

in which * indicates complex conjugation, $R_{i,j}(f)$ is the cross spectral density between each pair of receivers, $i$ and $j$ indicates the position of $n$ number of receivers, $B$ is the block of signal, $\omega$ is the circular frequency and $n$ is the total number of receivers. $S_{i,n}(\omega)$ represents the

![Figure 1. Seismic signal reception with 24 receivers at UKM, Malaysia.](image)
frequency response of \( i^{th} \) receivers used in mathematical expression of the matrix, \( R \). The Fourier transformed response of each sensor, \( S(\omega) \) can be written as

\[
S(\omega) = \sum_{i=0}^{M-1} s(t) \exp \left( -\frac{\omega t}{M} \right)
\]  

(2)

where, \( s(t) \) is the time domain signal and \( M \) is the samples of signal. If no noise was present, the transformed representation of the perturbation, obtained at the fixed frequency, would show only one peak at about the frequency of excitation depicted in Figure 3.

The next step concerns the Fourier transformation from the space domain to the wave number domain. This task is accomplished by means of the Frequency Domain Beamformer (FDBF) analysis. The FDBF method consists of transforming the measured wave field from the time-space domain of acquisition to the frequency-wave number domain. This double transformation can be performed by means of a 2-D Fourier transformation, one from time to frequency and the other from space to wave-number.

The 2-D Fourier transform is a linear operator that when applied to time-offset data, gives a frequency-wave number (\( f-k \)) spectrum represented by

\[
U(f, k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, t) e^{-i2\pi(f \cdot t - k \cdot x)} dx dt
\]

(3)

where, \( U(f, k) \) is the \( f-k \) spectrum and \( u(x, t) \) is the time-space domain signal. This transformation is used in seismic processing because it allows separation and filtering at events having different frequencies, wave-numbers and apparent velocities.

Using \( f-k \) analysis, the local maxima can be associated to modes of propagation and from their location in
the \((f, k)\) plane. The displacements, \(s(x, t)\) that are caused by an impulsive point source acting on the ground surface can be evaluated by mode superposition as (4).

\[
s(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{m=1}^{\infty} S_m(\omega, x) e^{i(\omega x - k_m \omega t)} d\omega
\]

(4)

where, \(m\) is the mode number. The factor is determined as,

\[
S_m(\omega, x) = I(\omega) P_m(\omega) R_m(\omega) \frac{e^{-\alpha_m(\omega)x}}{\sqrt{x}}
\]

(5)

where, \(I(\omega)\) is the instrument response, \(P_m(\omega)\) is the source spectrum, \(R_m(\omega)\) is the path response and \(\alpha_m(\omega)\) is the attenuation coefficient. The modal wavenumber, \(k_m\) generated by (6) is inversely proportional to phase velocity, \(V_R\) or equivalently proportional to its inverse, i.e. the slowness \(p_m(\omega)\), and is given by

\[
k_m(\omega) = \frac{\omega}{V_R(\omega)} = \omega p_m(\omega)
\]

(6)

If there are \(N\) aligned geophones with spacing \(\Delta x\), the phase difference for the mode \(m\) can be written as \(k_m(\omega)\Delta x\).

As the material attenuation is concerned, its contribution can be taken out from the expression of \(S_m\) so that this last quantity becomes only a function of frequency. Finally, applying the Fourier transform over the time and recalling the relationship between \(k\) and \(p\) as shown in (7), the \(f-k\) spectrum can be written as,

\[
F(f, k) = \sum_{m} S_m(f) \left[ \sum_{n=1}^{N} e^{-\alpha_n(f)\Delta x} e^{i(k_n(f)\Delta x)} \right]
\]

(7)

Thus, using the FDBF analysis \(f-k\) curve is obtained using MATLAB programming shown in Figure 4. The maximum wavenumber, \(K_{peak}\) is located as 0.1 rad/ft at the determined frequency of 10 Hz. With the execution of (6), the phase velocity, \(V_R\) regarding different mode is generated corresponding to frequency. \(F-K\) method is the representation of the frequency and wavenumber versus maximum power spectrum. Figure 5 shows the \(f-k\) representation which is capable to deliver the experimental apparent dispersion curve.

The 2-D dispersion analysis of surface wave method using \(f-k\) transformation is demonstrated in the study. The results of dispersion analysis are evaluated based on the matching with the standard theoretical dispersion curve. The conception is also supportive to the Levenberg-Marquardt Algorithm to minimize the problems arising for nonlinear programming outcomes.

The flow-chart of matching technique to be implemented on surface wave analysis is revealed in Figure 6. The apparent dispersion curve is compared with a theoretical approach of a standard curve to reduce the irregularities during analysis. The irregularities observed in dispersion analysis are adjusted through the fitting criteria. The subsequent data is stored corresponding to fre-
quency and final dispersion curve is then obtained. To obtain improved performance in dispersion analysis, the data fitting criteria through Levenberg-Marquardt method is implemented in this work. Using the fitting Algorithm, the perfect modal velocity is taken corresponding to the frequency in surface wave analysis. The system is repeated with the increment of frequency until getting the highest frequency shown in the flow-chart of Figure 6. In each repetition of the system, experimental data is compared with standard theoretical data to reduce the observed irregularities in dispersion analysis.

3. Result and Discussion

Dispersion analysis is significant step to show the shear wave velocity profile in surface wave analysis. The dispersion analysis is conducted through $f$-$k$ transformation which provides the clear information of different modes in soil characterizations. Indeed, events having different dips in the $(x, t)$ plane can be separated in the $(f, k)$ plane by their dips. A selective filtering process using an appropriate window can eliminate most of the undesired energy related to ground roll (mainly surface waves), guided waves and side-scattered waves. The result is a much clear representation in time, and space of arrivals of reflected and refracted waves, on which geophysical seismic methods are based. Moreover, the collaboration of fitting criteria with $f$-$k$ transformation reveals the robust performance in dispersion analysis of surface wave method.

The $f$-$k$ domain analysis is important to produce better results in near surface soil characterizations. It is possible to get separate dispersion curves for different modes, instead of only one cumulated curve as in MASW test. Moreover, the resulting dispersion curves are smoother since they represent the average over a given spatial extent. These factors give the opportunity of applying more stable and robust inversion processes, partially mitigating the non-uniqueness problem. Indeed, using $f$-$k$ simulation, it has been shown that the undesired distortions produced by body waves on the computed dispersion curve are minimized.

In general, the $f$-$k$ analysis consists of matching at different mode to demonstrate the subsequent dispersion curve. The procedure to individuate the maxima that are related to different mode of propagation was based on a process of successive muting in the $f$-$k$ domain. Using such algorithm, after the dispersion curve for a mode is determined, the corresponding zone in the $(x, t)$ space is muted and data are again reported in the $f$-$k$ domain to reveal the maxima that are related to next higher mode.

In addition, the $f$-$k$ analysis allows for a better signal to noise ratio, hence resulting a more accurate evaluation of the experimental apparent dispersion curve. In evaluating the experimental dispersion curves shown in Fig-

![Flow-chart for the dispersion curve with fitting criteria.](image-url)
ure 7 only the absolute maximum in the spectrum has been selected for each frequency. More information could be obtained looking for all the relative maxima that exist at each frequency.

This study involves the cooperation of fitting criteria with $f-k$ transformation. This approach started from an initial guess, the theoretical dispersion curve is then compared to experimental dispersion curve and the profile is modified until a good fitting is obtained. Automated iterative technique has been used with the least square criterion by many authors, with different updating criteria, damping and weighting which can be recognized as updated technique. This technique is also implemented in this research analysis to get better results in dispersion analysis.

Figure 8 shows the linearity of frequency versus wave-number relationship from which the theoretical dispersion can be depicted. The adjustment of irregularities of apparent experimental dispersion is conducted through the comparison between theoretical dispersion curve. This procedure would be repeated iteratively until the vector of the predicted error is small enough and hence a good fitting is established.

The performance of fitting criteria for modal phase velocity in apparent dispersion curve is shown in the Figure 9. The irregularities are clearly shown in the figure. For example, before the fitting is exercised, at the frequency of 10 Hz, the phase velocity is induced as 700 ft/sec whereas at 5 Hz and 15 Hz the phase velocities are 2300 ft/sec and 1400 ft/sec, respectively. After fitting criteria, the shear wave velocities are well adjusted at their respective frequencies.

4. Conclusion

This paper demonstrates the modified multichannel analysis of surface wave method to generate a better pro-

![Figure 7](image.png)

**Figure 7.** Representation of the modal analysis in the frequency-phase velocity plane.

![Figure 8](image.png)

**Figure 8.** Standard theoretical dispersion curve from wave-number versus frequency linearity.

![Figure 9](image.png)

**Figure 9.** Dispersion curve with and without fitting criteria.
procedure to construct the experimental dispersion curve from field measurements at surface wave including data acquisition technique, test configuration and data analysis. The $f$-$k$ transform is implemented to produce the better dispersion curve for estimating shear wave velocity profile of near surface soil. In this study, dispersion analysis is implemented incorporating fitting criteria with $f$-$k$ transformation to generate more reliable velocity profiles. The effects of multiple modes on multi-station measurements are investigated and the criteria of mode separation are established. The higher modes are considered to form the apparent experimental dispersion curve which is essential to obtain reliable dispersion curves.

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