Heat Transfer Effects on Rotating MHD Couette Flow in a Channel Partially Filled by a Porous Medium with Hall Current

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Abstract

A viscous incompressible electrically conducting fluid flow is considered in a parallel plate horizontal channel in the presence of an inclined magnetic field. The channel is rotating with uniform angular velocity about an axis normal to the plates. A porous material of finite thickness is attached to the lower impermeable plate which is stationary and kept at a constant temperature $T_0$, while the upper impermeable plate is moving with a uniform velocity and kept at a constant temperature $T_1$. For the strong applied magnetic field, the Hall current effects are considered. In the energy equation viscous dissipation and Ohmic dissipation effects are also taken into account. Exact solutions are obtained for the flow, magnetic field and temperature distributions. Effects of the pertinent parameters on the velocity distribution, temperature distribution, induced magnetic field and rate of heat transfer are depicted graphically and discussed.

Key Words: Rotating MHD Couette Flow, Porous Medium, Permeability, Hall Current, Heat Transfer

1. Introduction

The study of fluid flow through porous media and heat transfer is fundamental in nature. It is of great practical importance in view of several physical problems such as seepage of water in river beds, porous heat exchangers, cooling of nuclear reactors, filtration and purification processes. Because of its industrial importance, problem of flow and heat transfer in porous medium in the presence of magnetic field has been the subject of many experimental and analytical studies. McWhirter et al. [1], and Geindreau and Auriault [2] discussed in detail magnetohydrodynamic flow through porous medium. The investigations considering rotational effects are also very important, and the reason for studying flow in a rotating porous medium or rotating flow of a fluid overlying a porous medium in the presence of a magnetic field is fundamental because of its numerous applications in industrial, astrophysical and geophysical problems.

The problem of MHD Couette flow and heat transfer between parallel plates is a classical one that has several applications in MHD accelerators, MHD pumps and power generators, and in many other industrial engineering designs. Thus such problems have been much investigated by researchers such as, Seth et al. [3], Singh et al. [4], Chauhan and Vyas [5], Attia [6,7], Attia and Ewis [8], Seth et al. [9], and Attia et al. [10].

In most of the investigations, as above, we notice that, the Hall term is neglected for small or moderate values of the magnetic field in applying Ohm’s law in the analysis. When a strong magnetic field is applied, the influence of electromagnetic force is noticeable, and the strong magnetic field induces many complex phenomenon in an electrically conducting flow regime including Hall currents, Joule’s heating etc. as stated by Cramer and Pai, [11]. Infact, in an ionized gas under strong magnetic field when the density is low, the Hall
current is induced which is mutually perpendicular to both electrical and magnetic fields. It has significant effect on the current density and hence on the electromag-
netic force. Sato [12] and Sutton and Sherman [13] in-
vestigated the hydromagnetic flow of a viscous ionized
gas between two parallel plates taking Hall effects into
account. It was the first significant study to include Hall
effect in the analysis and indicated that the fluid flow in
the parallel plate channel becomes secondary in nature.
Hall currents can have strong influence on the fluid flow
distributions in MHD flow systems, e.g. in MHD power

generators, electrically conducting aerodynamics and at-
mospheric science. Hall effects on MHD flow in a rotat-
ing channel have been investigated by Ghosh and
Bhattacharjee [14] in the presence of inclined magnetic
field. Further studies of Hall effects on MHD flow in par-
allel plate channel with perfectly conducting walls with
heat transfer characteristics have been presented by
Ghosh et al. [15] who analyzed the asymptotic behavior
of the solution.

Hall currents in MHD Couette flow and heat transfer
effects have been investigated in parallel plate channels
with or without ion-slip effects by Soundalgekar et al.
[16], Soundalgekar and Uplekar [17], and Attia [18].
Hall effects on MHD couette flow between arbitrarily
conducting parallel plates have been investigated in a
rotating system by Mandal and Mandal [19]. The same
problem of MHD couette flow rotating flow in a rotating
system with Hall current was examined by Ghosh [20] in
the presence of an arbitrary magnetic field.

The study of hydromagnetic couette flow in a porous
channel has become important in the applications of
fluid engineering and geophysics. Krishna et al. [21] in-
vestigated convection flow in a rotating porous medium
channel. Bég et al. [22] investigated unsteady magneto-
hydrodynamic couette flow in a porous medium channel
with Hall current and heat transfer. When the viscous
fluid flows adjacent to porous medium, Ochoa-Tapia
[23,24] suggested stress jump conditions at the fluid
porous interface when porous medium is modeled by
Brinkman equation. Using these jump conditions, Kuznetsov [25] analytically investigated the couette flow
in a composite channel partially filled with a porous
medium and partially with a clear fluid. Chauhan and
Rastogi [26], heat transfer effects on MHD conducting
flow with Hall current in a rotating channel partially
filled with a porous material using jump conditions at
the fluid porous interface. Chauhan and Agrawal [27] in-
vestigated Hall current effects in a rotating channel partially
filled with a porous medium using continuity of velocity
components and stresses at the porous interface. Chauhan
and Agrawal [28] further studied effects of Hall current
on couette flow in similar geometry and matching con-
ditions at the fluid porous interface.

The current work deals with the rotating magne-
tohydrodynamic Couette flow of electrically conducting
fluid and heat transfer in a channel partially filled by
a porous medium, with Hall effects. The jump condition is
applied at the fluid porous interface suggested by Ochoa
Tapia and Whitaker [23,24].

2. Formulation of the Problem

The problem under investigation comprises the la-
minar, steady, viscous, incompressible, electrically-con-
ducting fluid flow between two parallel electrically non-
conducting infinitely long plates, rotating with constant
angular velocity $\Omega$ about $y$ axis, in an $x$-$z$ plane. A strong
magnetic field, $B_0$, is applied at an angle $\theta$ to the positive
direction of the axis of rotation. The Couette flow geo-
metry is shown in Figure 1. The bottom wall is stationary
where a layer of porous material of thickness ‘$a$’ is at-
tached and the upper wall translates at a constant speed $U$
relative to rotating frame of reference. The upper plate is
fixed at a distance ‘$d$’ from the porous medium interface.
Consider that the origin of Cartesian coordinate system
lies on the porous-clear fluid interface, the lower and

![Figure 1. Schematic diagram.](image_url)
upper plates are therefore located at \( y = -a \) and \( y = d \) which are maintained at constant temperatures \( T_0 \) and \( T_1 \) respectively. For the strong applied magnetic field, the Hall Effect is taken into consideration and, consequently, a \( z \)-component for the velocity is expected to arise. Since plates are of infinite dimensions in the \( x-z \) directions, we may assume that all flow variables are functions of \( y \) only.

The flow field in the channel is divided into two regions:
- Region-I, \( (0 \leq y \leq d) \) clear fluid region;
- Region-II, \( (-a \leq y \leq 0) \), porous medium region.

Following Sutton and Sherman [13], the following assumptions are made which are compatible with the fundamental equations of magnetohydrodynamics:

\[
q = (u, 0, w), \quad \vec{q} = (\vec{u}, 0, \vec{w}), \\
H = (H_x + H_0 \sin \theta, H_0 \cos \theta, H_z), \\
\vec{H} = (H_x + H_0 \sin \theta, H_0 \cos \theta, \vec{H}_z), \\
E = (E_x, E_y, E_z), \quad \vec{E} = (\vec{E}_x, \vec{E}_y, \vec{E}_z), \\
J = (J_x, 0, J_z), \quad \vec{J} = (\vec{J}_x, 0, \vec{J}_z)
\] (1)

where \( q, H, E, J \) are the velocity vector, magnetic field, electric field, and current density vector in region-I respectively and \( \vec{q}, \vec{H}, \vec{E}, \) and \( \vec{J} \) are the corresponding vectors in region-II.

For a constant property Newtonian fluid, the complete set of MHD equations includes the Navier-Stokes equations of motion, the continuity equation, the Maxwell equations, and the Ohm’s law. The MHD equations governing the steady flow in a rotating frame of reference for clear fluid region-I can be expressed as follows:

Mass Conservation
\[
\nabla \cdot q = 0
\] (2)

Momentum Conservation
\[
(q \cdot \nabla)q + 2(\Omega \times q) = -\frac{1}{\rho} \nabla p + \nu \nabla^2 q + \frac{\mu_s}{\rho} J \times H
\] (3)

where the MHD body force and Coriolis terms are included in the Navier-Stokes equations to model the momentum equation.

Maxwell Equations
\[
\nabla \cdot B = 0
\] (4)

\[
\nabla \times E = 0
\] (5)

\[
\nabla \times H = J
\] (6)

\[
\nabla \cdot J = 0
\] (7)

Following Cowling [29], the modified Ohm’s law taking Hall current into account for strong magnetic field is given by

\[
J + \frac{\tau_s \omega_e}{H_0} (J \times H) = \sigma \left( E + \mu_s q \times H + \frac{1}{\eta_s} \nabla p_e \right)
\] (8)

where \( B = \mu_s H \) is the magnetic induction vector and \( \Omega = (0, \Omega, 0) \) is the angular velocity vector. Here \( \rho, \mu_s, \nu, \sigma, \omega_e, \tau_s, e, \eta_s, p_e \) are the fluid density, magnetic permeability, kinematic viscosity, electrical conductivity, cyclotron frequency, electron collision time, electric charge, number density of electron and electron pressure respectively. It is assumed that the \( \tau_s \omega_e \approx o(1) \) and \( \tau_s \omega_i < 1 \), where \( \omega_e \) is the cyclotron frequency of ions and \( \tau_i \) is the collision time of ions. In writing the induction equation (8), the ion slip and the thermo electric effects are neglected. Also the term involving electron pressure gradient in equation (8) is negligible for ionized gas, and dropped in the further analysis.

All physical quantities are functions of \( y \) only, under assumption (1). Together with equation (2) and (4)–(7), the equation of momentum (2) and magnetic field equation (8) in rotating frame of reference reduce to:

\[
\nu \frac{d^2 u}{dy^2} + \frac{\mu_s H_e \cos \theta}{\rho} \frac{dH_z}{dy} - 2\Omega w = 0
\] (9)

\[
\nu \frac{d^2 w}{dy^2} + \frac{\mu_s H_e \cos \theta}{\rho} \frac{dH_z}{dy} + 2\Omega u = 0
\] (10)

\[
\frac{d^2 H_z}{dy^2} + m \cos \theta \frac{d^2 H_z}{dy^2} + \mu_s \sigma H_e \cos \theta \frac{dw}{dy} = 0
\] (11)

\[
\frac{d^2 H_x}{dy^2} - m \cos \theta \frac{d^2 H_x}{dy^2} + \mu_s \sigma H_e \cos \theta \frac{du}{dy} = 0
\] (12)

where, \( m = \tau_i \omega_e \), is the Hall current parameter.

For region-II \((-a \leq y \leq 0)\) following Nield and Bejan [30], the governing equation of motion is given by
Equation of continuity, Maxwell equations, and generalized Ohm’s law neglecting ion slip and thermoelectric effect are given by

\begin{align}
\nabla \cdot \vec{q} &= 0 \\

\nabla \cdot \vec{B} &= 0 \\

\nabla \times \vec{E} &= 0 \\

\nabla \times \vec{H} &= \vec{J} \\

\nabla \cdot \vec{J} &= 0
\end{align}

(13)

where \( K_0 \) is the permeability of the porous medium and \( \bar{\nu} \) is the effective kinematic viscosity of the fluid in porous medium.

Since all physical quantities are functions of \( y \) only, using equation (1), equations (14), (15) and (18) are satisfied identically; using equations (16) and (17), equations (13) and (19) reduce to:

\begin{align}
\nabla \cdot \vec{q} &= 0 \\

\nabla \cdot \vec{B} &= 0 \\

\nabla \times \vec{E} &= 0 \\

\nabla \times \vec{H} &= \vec{J} \\

\nabla \cdot \vec{J} &= 0
\end{align}

(19)

The corresponding boundary conditions are

- at \( y = -a, \ \bar{u} = 0, \ \bar{w} = 0, \ \bar{H}_x = 0, \ \bar{H}_z = 0 \) (24)

where, \( \beta \) is a constant.

3. Solution of the Problem

3.1 Flow and Magnetic Induction

Let us introduce the following new variables

\begin{align}
\nF &= u + iw, \ \ h = H_z + iH_x, \ \ \bar{F} = \bar{u} + i\bar{w}, \ \ \bar{h} = \bar{H}_z + i\bar{H}_x
\end{align}

(25)

In order to obtain the dimensionless form of governing equations, dimensionless variables are defined as follows:

\begin{align}
\eta &= \frac{y}{d}, \ \ F^* = \frac{F}{U}, \ \ \bar{F}^* = \frac{\bar{F}}{U}, \ \ K_0^* = \frac{K_0}{d^2}, \\
\h^* &= \frac{h}{\sigma \mu UdH_0}, \ \ \bar{h}^* = \frac{\bar{h}}{\sigma \mu UdH_0}
\end{align}

(26)

Using equations (25) and (26), equations (9)–(12), (20)–(23) takes the following non-dimensional form:

\begin{align}
\frac{d^2 F}{d\eta^2} + M^2 \cos \theta \frac{d h}{d\eta} + 2iR F = 0 \\
(1 + im \cos \theta) \frac{d^2 \bar{F}}{d\eta^2} + \cos \theta \frac{d\bar{F}}{d\eta} = 0
\end{align}

(27)

(28)

where \( \phi_1 = \frac{\bar{F}}{U} \), \( \mu \), the clear fluid viscosity; \( \bar{\mu} \), the effective viscosity of the clear fluid in porous medium; \( M = \mu / \mu_0 d^2 (\sigma / \rho_0) \), the Hartmann number, and \( R = \Omega d^2 / \mu \) is the rotation parameter.

The corresponding non-dimensional boundary conditions that follow from eq. (24) are:

- at \( \eta = 1, \ F = 1, \ h = 0 \)
- at \( \eta = 0, \ F = \bar{F}, \ h = \bar{h}, \)
- \( \phi_1 \frac{d\bar{F}}{d\eta} - \frac{d\bar{F}}{d\eta} = -\frac{\beta}{\sqrt{K_0}} \bar{F}, \ \ \phi_1 \frac{d\bar{h}}{d\eta} - \frac{d\bar{h}}{d\eta} = -\frac{\beta}{\sqrt{K_0}} \bar{h} \)
- at \( \eta = -\alpha, \ \bar{F} = 0, \ \ h = 0 \)

where \( \alpha = a/d \).
Here, asterisks have been dropped for convenience in eqs. (27)–(31).

On solving equations (27)–(30) under the boundary conditions (31), we obtain the velocity distributions and the magnetic field distributions in both regions as follows:

\[ F = B_i \frac{M^2 \cos \theta}{A^2} + B_2 e^{\alpha y} + B_3 e^{-\alpha y} \]  
(32)

\[ \overline{F} = B_k \frac{M^2 \cos \theta}{\phi_1 C^2} + B_6 e^{\alpha y} + B_7 e^{-\alpha y} \]  
(33)

\[ h = \left( 1 - \frac{M^2 \cos^2 \theta}{A^2 (1 + \text{im cos} \theta)} \right) B_i y - \frac{\cos \theta}{1 + \text{im cos} \theta} \left( \frac{B_2 e^{\alpha y} - B_3 e^{-\alpha y}}{A} \right) + B_4 \]  
(34)

\[ \tilde{h} = \left( 1 - \frac{M^2 \cos^2 \theta}{\phi_1 C^2 (1 + \text{im cos} \theta)} \right) B_i y - \frac{\cos \theta}{1 + \text{im cos} \theta} \left( \frac{B_6 e^{\alpha y} - B_7 e^{-\alpha y}}{C} \right) + B_8 \]  
(35)

where, \( A^2 = \frac{M^2 \cos^2 \theta}{1 + \text{im cos} \theta} - 2iR \)

and \( C^2 = \frac{1}{\phi_1} \left( \frac{M^2 \cos^2 \theta}{1 + \text{im cos} \theta} + \frac{1}{K_0} - 2iR \right) \).

The constants of integration \( B_1, B_2, B_3, B_4, B_5, B_6, B_7 \) and \( B_8 \) are obtained by the corresponding boundary conditions and are not reported for the sake of brevity.

### 3.2 Heat Transfer

The governing MHD equations in a rotating frame of reference for temperature distribution in region I \((0 \leq y \leq d)\) and II \((-a \leq y \leq 0)\) are given by

\[ 0 = k \frac{d^2 T}{d \eta^2} + \mu \left( \frac{d u}{d \eta} \right)^2 + \left( \frac{d w}{d \eta} \right)^2 \]  
(36)

\[ + \frac{1}{\alpha \rho C_p} \left[ \left( \frac{d H_s}{d \eta} \right)^2 + \left( \frac{d H_s}{d \eta} \right)^2 \right] \]

\[ 0 = \bar{k} \frac{d^2 \overline{F}}{d \eta^2} + \frac{1}{\alpha \rho C_p} \left[ \left( \frac{d H_s}{d \eta} \right)^2 + \left( \frac{d H_s}{d \eta} \right)^2 \right] \]  
(37)

\[ + \frac{\mu}{\rho C_p K_0} \left( \overline{u} + \overline{w} \right) + \frac{\mu}{\rho C_p} \left( \frac{d \overline{u}}{d \eta} \right)^2 + \left( \frac{d \overline{w}}{d \eta} \right)^2 \]

with the corresponding boundary conditions:

\[ \text{at } \eta = 1, \ T = 1 \]

\[ \text{at } \eta = 0, \ T = \overline{T}, \ \frac{d T}{d \eta} = \phi_2 \frac{d \overline{T}}{d \eta} \]  
(42)

where \( \phi_2 = \bar{K} / k \) is the thermal conductivities ratio; \( Pr = \mu C_p / k \) is the Prandtl number; and \( Ec = U^2 C_i (T_1 - T_0) / \) is the Eckert number.

Substituting the values of \( F, \overline{F}, h \) and \( \tilde{h} \) from equations (32)–(35) and solving the resulting equations sub-
ject to the boundary conditions (42), we obtain:

\[ T = E_1 \eta + E_2 - PrEc \left[ \frac{F_1 e^{(A+\bar{A})\eta} + F_2 e^{-(A-\bar{A})\eta}}{(A+\bar{A})^2} \right. \\
\left. + \frac{F_1 e^{(A-\bar{A})\eta} + F_2 e^{-(A+\bar{A})\eta}}{(A-\bar{A})^2} \right. \\
\left. + \frac{F_4 e^{\eta} + F_5 e^{-\eta}}{(A)^2} \right] \]

(43)

where, \(A\) and \(\bar{C}\) are the complex conjugates of \(A\) and \(C\) respectively.

The constants of integration \(E_1, E_2, E_3, E_4\) are determined by the corresponding boundary conditions and are not reported for the sake of brevity.

4. Discussion

Magnetohydrodynamic Couette plasma flow in a rotating system, subjected to Hall effects is investigated in a channel partially filled by a porous medium. The heat transfer characteristics are also analyzed by taking viscous and Ohmic dissipation effects in the energy equation. Such study has applications in designing cooling and MHD power generation systems.

Figures 2a–c demonstrate the velocity distribution for different values of the pertinent parameters, such as, Rotation parameter \(R\), magnetic field parameter \(M^2\), non-dimensional permeability parameter \(K_0\), angle of inclination of magnetic field \(\theta\), Hall current parameter \(m\), viscosity ratio parameter \(\phi_1\), and jump parameter \(\beta\). The velocity profiles for the primary flow are plotted vs. \(\eta\) in Figure 2a. It is observed that the primary flow decreases with the increase in the value of \(R\) and even back flow occurs in the channel for high rotation. This flow enhances in the channel by increasing the permeability \(K_0\) of the porous substrate attached to the lower stationary plate, as a result of decreasing the damping force (Darcy’s resistance) on \(u\). However the effect of the Hall parameter \(m\) is to reduce the primary flow. Infact, this parameter \(m\) induces a secondary flow in the channel which affects the primary flow as well. By increasing the magnetic field parameter \(M^2\), it is seen that the primary flow velocity decreases near the upper moving plate.

![Figure 2a. Velocity profiles \(u, \bar{u}\) vs. \(\eta\) for \(K_0 = 0.1, m = 1, M^2 = 20, \beta = 0.7, \phi_1 = 1.25, \theta = 45^\circ\).](image)

![Figure 2b. Velocity profiles \(w, \bar{w}\) vs. \(\eta\) for \(K_0 = 0.1, m = 1, M^2 = 20, \beta = 0.7, \phi_1 = 1.25, \theta = 45^\circ\).](image)

![Figure 2c. Velocity profiles \(w, \bar{w}\) vs. \(\eta\) for \(m = 1, M^2 = 20, R = 3, \beta = 0.7, \phi_1 = 1.25, \theta = 45^\circ\).](image)
while it increases in the rest of the channel, however, a reverse effect is observed by increasing the angle of inclination $\theta$ of the applied magnetic field.

The velocity profiles for secondary flow are plotted in Figures 2b and c. Secondary flow is caused by the Hall effect and rotation. It is seen that with increase in $m$ or $R$, the secondary flow velocity increases in the upper part of the channel and decreases in the lower part. However by increasing $R$, it oscillates in the middle part of the channel. It increases by the jump parameter $\phi_1$. It is observed that the secondary flow increases in the channel by increasing permeability ($K_0 < 0.1$) and for higher permeability values it decreases. It also decreases with the increase in the viscosity ratio parameter $\phi_2$. It is noticed that the secondary flow velocity decreases by increasing the magnetic field $M^2$ and a reverse effect is noticed with the increase in $\theta$, because Lorentz force due to applied transverse magnetic field produces resistance to flow.

When flow of a conducting fluid is under the influence of a magnetic field, we know that there is a coupling between the flow field and the magnetic field. From physical considerations it is known that the lines of force representing an applied magnetic field influence the fluid flow, which in turn influences these magnetic lines as well. Thus the parameters which influence the flow field in turn influence similarly the induced magnetic fields in the primary and secondary flow directions. Profiles for induced magnetic field $H_x$ in the primary flow direction and $H_z$ in the secondary flow direction are plotted in Figures 3a and b respectively for various values of the pertinent parameters.

It is observed in Figure 3a that the induced magnetic field $H_x$ in the primary flow direction decreases by the rise of $M^2$, $m$, or $\theta$ in the channel. It decreases by increasing permeability ($K_0 < 0.1$) and increases by high permeability values. It is noticed that by increasing the value of $R$, it increases near the upper moving plate, decreases in the lower part of the channel.

It is noticed in Figure 3b that the induced magnetic field $H_z$ in the secondary flow direction is negative mostly in the upper part of the channel. When $m = 0$, it is negative only in the near region of the upper moving plate and positive in the remaining channel. As $m$ increases it is seen that $H_z$ reduces significantly in the lower part becomes zero at certain $\eta$ and becomes negative in the upper remaining part of the channel. With the increase in the magnetic field $M^2$ it decreases numerically in the upper part and increases in the lower porous part of the channel. It also decreases numerically by increasing the angle $\theta$. Further it is seen that the effects of $K_0$ is to rise $H_z$ when permeability is small ($K_0 < 0.1$) however it decreases numerically for higher permeability values. The rotation parameter $R$ decreases the induced magnetic field $H_z$ in the secondary flow direction flow numerically in the upper part of the channel and enhances it in the lower part.

The effects on the shear stresses $\tau_{yx}$ and $\tau_{yz}$ at the interface of the porous substrate attached to the lower stationary wall are depicted in Figure 4. It is seen that when the permeability of the porous medium is small (i.e. $K_0 < 0.1$) $\tau_{yx}$ at $\eta = 0$, reduces by increasing $K_0$ while it increases by $K_0$ when permeability is high (i.e. $K_0 > 0.1$) for all values of $\theta$ ($0 < \theta < 90$). When $\theta < 45^\circ$, $\tau_{yz}$ enhances by increasing the permeability $K_0$, where as reverse effect is seen for $\theta > 45^\circ$.
For the analysis of heat transfer characteristics in the flow discussed above, energy equation is taken in which all the convective terms become equal to zero because of the assumed temperature boundary conditions. Therefore, the temperature distribution in the rotating channel is due to the heat generation by viscous and Ohmic dissipation, and conduction through the fluid in the transverse direction.

The temperature profiles are plotted in Figure 5 for different values of the various pertinent parameters. It is seen that temperature distribution is linear when \( Br = PrRe \), where \( Br \) is the Brinkman number. However for \( Br > 0 \), heat is generated through viscous and Ohmic dissipation in the fluid and a distribution of parabolic type is obtained. Temperature in the channel increases as we increase the value of \( Br \). The effect of the Coriolis force can be seen by varying the rotation parameter \( R \) on temperature distribution. It is seen that temperature in the channel rises in the upper part by increasing the value of \( R \), while it decreases in the lower porous layer. It is further observed that the effect of the permeability \( K_0 \) is to reduce the temperature in the rotating channel. However with the increase in the magnetic field \( M^2 \), temperature in the channel oscillates. It increases in the upper part, decreases in the middle and again increases in the lower part of the channel.

The rate of heat transfer at the upper moving wall \( T'(1) \) is plotted against \( Br \) in Figure 6 for different values of the parameters \( \theta, m, M^2, \) and \( R \). It is observed that \( T'(1) \) decreases as the value of \( Br \) increases, becomes zero at certain \( Br \) value (critical Brinkman number \( Br^* \)), and then changes sign. In fact, heat transfer will take place from upper moving wall to the fluid if \( T'(1) > 0 \), i.e. when \( Br < Br^* \). On the other hand, when \( T'(1) < 0 \), i.e. \( Br > Br^* \), heat transfer will take place from fluid to the upper wall. There will be no flow of heat either from wall to the fluid or from fluid to the wall when \( Br = Br^* \). This reversal of heat flow occurs, because when there is significant heat generation in the fluid due to viscous and Ohmic dissipation, the temperature of the fluid exceeds the wall temperature which causes heat flow from fluid to the wall. It is observed in Figure 6 that this reversal of heat flow direction takes place at certain Critical Brinkman number (\( Br^* \)), which increases by increasing the value of \( \theta \) and decreases by \( M^2 \). It was expected, because it is known that as we increase the Hartmann number, resistive force increases, while heat transfer decreases. Similar effect is noticed as the value of the rotation parameter \( R \) is increased. Figure 7 is plotted for critical Brinkman number \( Br^* \) vs. \( m \) for different values of the permeability parameter \( K_0 \). It is found that for very small \( m < 0.5 \), \( Br^* \) decreases and for values of \( m > 0.5 \), it increases by rising \( m \). It also increases by increasing the permeability of the porous substrate attached to the lower stationary wall.
5. Conclusion

In this research, it is seen that the Coriolis force is counter-productive for primary flow and for high rotation even back flow occurs in the channel. The permeability of the porous medium causes an increase in this flow. Hall current stifles the primary flow in the channel while the secondary flow responds more positively with it. Secondary flow is due to Hall effect and rotation, hence both effects enhance the secondary flow near the moving plate of the channel. Temperature in the channel increases as we increase the Brinkman number, while permeability of the porous medium reduces it. The rate of heat transfer at the moving plate of the channel decreases as we increase the Brinkman number, becomes zero at certain critical Brinkman number and then changes sign. It is found that critical Brinkman number increases by increasing angle of inclination of the applied magnetic field or permeability of the porous medium, while it decreases by the magnetic field or rotation. The present study concerning magnetohydrodynamics of rotating fluids in the presence of porous medium has immediate relevance to many aspects of fluid engineering, astrophysics and geophysics, and therefore has wide applications in these areas.

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