EQWEAP Analysis and Its Applications to Seismic Performance Based Design for Pile Foundations

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Abstract

One-dimensional wave equation can be used to model the seismic responses of piles. This paper intends to discuss the corresponding analysis EQWEAP and its application to seismic performance based design of the pile foundations. Probability analysis suggested by PEER is adopted to analyze the seismic performance of a typical bridge pile foundation in Taipei. With simplified procedures, the single piles subjected to seismic ground motions can be investigated with various design concerns. In the analysis, the time-dependent displacements and bending moments of the pile are closely monitored during the earthquake shaking, and the nonlinear moment-curvature relationship is used for damage measure. It is found that the EQWEAP analysis can provide fast and effective solution to pile foundation design where the seismic ground motion is extremely important.

Key Words: Seismic Response, Wave Equation, Pile Foundation, Performance Based Earthquake Engineering

1. Introduction

Performance Based Design (PBD) has received tremendous attentions from geotech societies in recent years. GeoCode-21, and Eurocode-7 and -8 were both developed for PBD concerns. Honjo [1] has pointed out that the PBD design can be done by three methods, i.e., LRFD, probability and reliability analyses. The subjects of PBD on foundations could be also divided into static and seismic aspects as well as bearing capacity and deformation (settlement) problems. To analyze the seismic performance of the structures (typically on deformations), the so called Framing equation was suggested by US PEER. It is called the Performance Based Earthquake Engineering (PBEE) analysis. In such analysis, the annual rate of exceedance for the seismic Intensity Measure (IM), the Engineering Demand Parameter (EDP) and the Damage Measure (DM) of the structure as well as the Decision Variable (DV) can be evaluated using the step-by-step discrete procedures. One can estimate the probabilities of structural parameters and compare them to the limited values for design purpose. For a pile foundation located at a site where ground information is known, the displacements and internal moments of the piles are the major concerns to the earthquake excitations. One can estimate the probabilities of these quantities following the PBEE procedures where the structural performance can be affected by many influence factors. With proper controls of the factors, the analysis is also applicable to evaluate the seismic performance of an earth structures. In this paper, a wave equation analysis called EQWEAP is suggested to simulate the pile responses under the earthquake. The solutions from such analysis on a typical pile foundation in Taipei were analyzed using the PBEE procedures. Discussions are made and details of the studies are presented next.
2. EQWEAP Analysis

Seismic responses of the piles could be analysed using the time-dependent Winkler type foundation model, whereas a simplified two-step procedure EQWEAP has been suggested by the author [2,3]. In such analysis, the free-field ground motions are obtained first and then applied to the pile for the solutions, and the discrete wave equations are used to solve for the pile displacements. Figure 1 illustrates the layout of the procedure. This modelling was reported in good agreements with the FE solutions. To model the soil liquefaction and/or liquefaction induced lateral spreading, a number of alternative models have also been suggested [4–9].

The 1st step in EQWEAP procedure simply adopts lumped mass model to solve for the free-field ground responses. It is rather convenient and simple analysis, nevertheless one must be cautioned to conduct the analysis using the bedrock accelerations and the baseline corrections of the displacements. In the 2nd step, the resolved ground motions should be applied to the springs and dashpots along the pile in order to calculate the seismic forces applied to the pile. The effects of pile-to-pile interactions and the soil-cap-pile interactions can be further included. If the seismic earth pressures were prescribed, then the 1st step analysis can be omitted. On the other hand, if the seismic ground motions were known already and the subgrade reaction modulus of the soils was available, the corresponding earth pressures could be computed and applied to obtain the solutions. All these modelling are feasible solutions to model the seismic pile responses.

For solution of the liquefied soil, the soil parameter reduction coefficients suggested by the Japan Road Association [10] could be considered. The reduction coefficients are applied to both the free-field ground response analysis and the wave equation analysis to reduce the soil stiffness due to liquefaction. This approach is simple but rational enough to reproduce the degraded modulus of liquefied soil.

One can also use the excess pore-water pressures (PWP) model to simulate the liquefaction. For example, Finn’s model [11] has been adopted by the authors to simulate the liquefaction influences on pile. The volumetric strains of the soils were computed and accumulated during the seismic excitations to obtain the excess pore water pressures. Soil liquefaction is modeled through the ground response analysis. Shear modulus of the soil could be calibrated using the suggestion of Seed and Idriss [12] or any other similar ones with the dependence of shear strains. Iterative procedures were performed to ensure the convergence and equilibrium of the structural system. This approach waives the conduction of liquefaction potential analysis. For solutions adopting the direct earth pressures and the indirect ones from the empirical ground displacement profiles, a more recent study of the author [13] can be referred. It has been reported that the EQWEAP can provide good estimations for seismic pile responses to an extent that the pile damages may occur.

For large earthquake excitations, nonlinear pile behaviors based on the moment-curvature relationships were considered. It can be obtained from both experiments and rigorous computations. For simplicity, one can approximate the nonlinear curves with the bi-linear (steel pipe piles) or tri-linear (concrete piles) relations. For given values of the cutting-point moments and their corresponding curvatures, one can find the approximate model constants for each line. With the EI values adjusted iteratively through the wave equation analysis, one can obtain approximately the nonlinear pile responses. The details of EQWEAP can be found in a recent summary made by the authors [13]. Assuming fixed head and long pile conditions, the basic forms of the solutions of EQWEAP can be derived as follows.

\[
\begin{align*}
\mathbf{u}_p(i, j + 1) &= \frac{1}{C_1 + C_2} \begin{bmatrix}
-u_p(i + 2, j) +
(4 - C_2)u_p(i + 1, j) \\
-(6 - 2C_1 - 2C_2 + C_4)u_p(i, j) +
(4 - C_2)u_p(i - 1, j) \\
-u_p(i - 2, j) -
(C_1 - C_2)u_p(i, j - 1) +
C_2[u_p(i, j + 1) - u_p(i, j - 1)] +
C_3u(i, j)
\end{bmatrix}
\end{align*}
\] (1)
Note that $C_1 = A \Delta z^4 / V_c^2 \Delta t^2$, $C_2 = P_x \Delta z^2 / EI$, $C_3 = C_1 \Delta z^2 / 2 \Delta t EI$, $C_4 = K_s \Delta z^4 / EI$. In above equation, $i$ is the $i$th nodal point, $j$ is the $j$th time step, $V_c$ is the compressive wave velocity of the pile, and is equal to $(E/\rho)^{1/2}$, $\Delta z$ and $\Delta t$ are the thickness of the pile segment and time increment respectively, $E$ = Young’s modulus of pile, $\rho$ = mass density of pile, $A$ = cross-section area of pile, $P_x$ = vertical load, $u_p$ = absolute pile displacement, $u_s$ = absolute soil displacement, $u = u_p - u_s$ = relative pile displacement, $C_s$ and $K_s$ = damping coefficient and spring constant of the soils along the pile. For concerns of different boundary conditions, one can find more detailed information from Chang et al. [13].

Pile nonlinearity can be approximated using a simplified tr-linear moment-curvature model for concrete pile. The equation of such relationship was suggested by Kunnath and Reinhorn [14] as

$$M = \alpha(EL)\phi + (1 - \alpha)M_Y \phi$$

where $M = \text{moment}$, $\phi = \text{curvature}$, $M_Y = \text{moment when the bar to yield}$, $\alpha$ and $Z$ are parameters evaluated knowing the intersection points of the linear lines, where the corresponding moments of the intersections are known to be $M_{cr}$ (moment where the cracks start to occur), $M_U$ and $M_{ult}$ (ultimate moment where the plastic hinge of the pile start to occur). Eq. (2) was conveniently adopted to model the nonlinear pile analyses. The ratios of moments and curvatures, standing for $EI$ values, were computed for each node and then averaged for next time step. This process is carried out through the whole analysis unless the ultimate moment is reached. In that case, the pile is broken. Of course, the time increment and the length of the pile segment need to be cautioned in order to remain the stability of the solutions.

Figure 2 depicts the tri-linear model of moment and curvature for concrete pile. Table 1 summarizes the model parameters $a$ and $Z$ for the tri-linear relationships, where the experimental data were collected and analyzed for the suggestions.

### 3. PBEE Analysis

Comprehensive overview of the PBEE analysis can be found in Kramer [15]. The ground motions, structural responses, physical damages and loss should be carefully analyzed considering the occurrence of the influence factors and the reliability of the design factors of interest. The IM, EDP, DM and DV values are to be analyzed accordingly. The Framing equation proposed by PEER is written as follows.

$$\lambda_{DV}(DV) = \int \int \int \int \lambda_{IM}^c(DV | DM) \, dG(DM | EDP) \, dG(EDP | IM) \, d\lambda_{IM}(IM)$$

In Eq. (3), $G(a|b)$ denotes a complementary cumulative distribution function (CCDF) for $a$ conditioned upon $b$ (the absolute value of the derivative of which is the probability density function for a continuous random variable). The three CCDFs result from the loss, damage, and response models; the final term, $d\lambda_{IM}(IM)$ is from the seismic hazard curve. This triple integral can be solved numerically for most practical problems as follows.

$$\lambda_{IM}^c(DV) = \sum_{k=1}^{N_{DM}} \sum_{j=1}^{N_{EDP}} \sum_{i=1}^{N_{IM}} \left[ P(DV > dv | DM = dm_j) \right] \left[ P(DM > dm_j | EDP = edp_i) \right] \left[ P(EDP > edp_i | IM = im_k) \lambda_{IM}^c(im_k) \right]$$

The numerical integration can be accomplished as where $P[a|b]$ describes the probability of $a$ given $b$, and where $N_{DM}$, $N_{EDP}$ and $N_{IM}$ are the number of increments. According to Kramer [15], the discrete form of Eq. (4) can be broken down into a series of components. The individual conditional probability terms can be expressed in the form of fragility curves. With some simplifying assumptions, the Framing equation can be solved in a closed form with the use of a power law relationship between mean annual rate of exceedance and $IM$.

$$\lambda_{IM}^c(IM) = k_0 (IM)^{\beta}$$

In Eq. (5), $k_0$ is the value of $\lambda_{IM}^c$ when $IM = 1$ and $k$ is the slope of the seismic hazard curve. If the response model is also assumed to be of power law form, then

$$EDP = a(IM)^k$$

Based on lognormal dispersion that has statistically independent aleatory and epistemic components of uncertainty $\beta$, the analytical form of $EDP$ hazard curve can be expressed as

$$\lambda_{EDP}(edp) = k_0 \left[ \frac{edp}{a} \right]^\beta \exp \left[ \frac{k^2}{2b^2} (\beta^2) \right]$$
Eq. (7) describes the mean annual rate of exceeding some level of response, $EDP = edp$, given the seismic hazard curve and a probabilistic response model. One could find detailed explanations regarding the use of this equation and corresponding ones when the damage and loss models were involved in Kramer [15]. Again, if Power laws could also be used for DM and EDP relations, e.g., $DM = c(EDP)^d$, then the analytical solution of the annual rate of exceedance, $\lambda$ for a certain level of $dm$ can be expressed as:

$$
\lambda_{dm}(dm) = k_0 \left\{ \frac{1}{a} \left[ \frac{dm}{c} \right]^{\gamma_0} \right\}^{1/\gamma} \exp \left[ \frac{k^2}{2b^2d^2} (d^2 \beta_{\phi}^2 + \beta_{\varphi}^2) \right] \tag{8}
$$

$\beta_{\phi}$ and $\beta_{\varphi}$ are the coefficients of variations associated with the EDP and DM data, respectively. If one would like to omit the hazard rate increments used for the integrations, simplified procedures to obtain the “stripes” data and the “cloud” data can be used to find out the simple relationships of EDP and IM, and the results shall then become much easier to obtain [15]. In using this procedure to analyze the bridge pier foundation, Shin [16] found that the uncertainty of the earthquake is mostly significant to the analysis. More than 80% uncertainties will resolved from this variable. Sometimes, the effects of the soil parameters and the geological profiles were studied too. It is necessary to point out that any proper structural analysis can be incorporated with the PBEE procedures for the estimations.

4. Pile Foundation Design in Taiwan

A number of design codes are available for pile foundation design in Taiwan. All the design codes and specifications require the checks for foundation capacities at ordinary and seismic conditions. The settlements and deformations of the foundation also need inspections. In general, both working stress design (WSD) and limit state design (LSD) are adopted in current design practice. Figure 3 shows the flowchart of a generalized pile design procedures taken in Taiwan. It can be seen that the seismic concerns were mainly focusing on the foundation capacities, whereas the liquefaction effects are considered independently. The flow pressure model for liquefaction-induced lateral spreads and the soil parameter reduction coefficients from liquefaction potential analysis of the site were mainly used. According to the suggestions made by Chen et al. [17], the seismic performances of the pile foundations could be categorized into three levels with the concerns of foundation serviceability, rehabilitation and safety, respectively (see Table 2).

Performance Level I indicates that the structure is mainly governed by the elastic behaviors under small to...
medium earthquakes, where soil liquefaction does not occur or occurs slightly. The major interest of Level I is the serviceability of the structure. Conventional design methods are applicable in this case. Performance Level II is applicable to medium to large earthquakes, nonlinear structural responses can be resulted, in which the ground tends to liquefy. The major concern of Level II is the rehabilitation and safety of the structure, both short term and long term should be evaluated. The engineers need to make sure that any local damage of the structure is not allowed in this case. Performance Level III is amendable to nonlinear responses of the structures that are affected by soil liquefaction and liquefaction induced lateral spread of the ground under very large earthquakes. The fatal collapse of the structure is prohibited in this requirement. Notice that the relationships between theses performance levels and the return periods of 30, 475 and 2500 years are referable in Table 2.

5. Seismic Hazard Curves in Taiwan

The importance of the fault sources and closest distance to fault in developing the ground-motion attenuation relationships is pronounced in PSHA. Cheng [18] has successfully used the logic tree and weightings at branches to discuss the uncertainty of PSHA considering the earthquakes in Taiwan. The characteristics of seismic sources in vicinities by deaggregating hazard contributed from different magnitude and distance were carefully examined. 3-D plate source to model fault planes and subduction zone plates was used besides the regional sources. Truncated-Exponential model developed by mainshock of EQ in MW from 1900 to 1999 was used to describe the magnitude distribution of regional sources. Characteristic-Earthquake model developed by fault slip rate was used to describe the magnitude distribution of active fault and subduction interface sources.

Adopting suitable attenuation relationship for each source in PSHA, especially the crustal source including the Chi-Chi earthquake sequence, the hanging-wall effect and site condition for specific site was revealed. According to the iso-seismic hazard map of PGA, 0.2 sec and 1.0 sec spectral acceleration, the hazard level is strongly dependent of the fault. The hazard was found significant around the centre of the hanging wall. The highest hazard level can be found in the eastern longitudinal valley and western foothills to coast plain, separated by the central mountain range in low hazard level. Furthermore, the hazard level considering faults activity

Table 2. Seismic performances and return periods for transportation structures (after Chen et al., [17])

<table>
<thead>
<tr>
<th>Hazard Level</th>
<th>Embankment</th>
<th>Bridge pile foundation</th>
<th>Underground structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>S30</td>
<td>Level I</td>
<td>Level I</td>
<td>Level I</td>
</tr>
<tr>
<td>S475</td>
<td>Level III</td>
<td>Level III</td>
<td>Level II</td>
</tr>
<tr>
<td>S2500</td>
<td>N/A</td>
<td>N/A</td>
<td>Level III</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ordinary</td>
<td>Important</td>
</tr>
</tbody>
</table>

1. Level I: elastic deformations, no or rare liquefaction, in normal condition. 2. Level II: plastic deformations allowed, slight to medium liquefaction, recoverable damages. 3. Level III: ultimate deformations occurred, severe soil liquefaction, structure not allow to collapse.
divided by regional sources shows that the prominent ratio always distribute on hanging wall. This occurs especially on the low background seismicity region such as Taichung, Hsinchu and Miali. Figure 4 presents the hazard curves for major cities in Taiwan from the deaggregation of PSHA, Cheng was able to show that the hazard is contributed mainly from the distance and magnitude bin by different return periods. The deaggregation process could provide information for hazard mitigation while choosing scenario earthquakes.

The data bank and number of sites considered in the seismic hazard study will result different results. One must be cautioned when using the hazard curves to conduct the analyses. For seismic design code used currently in Taiwan, the structures can be designed at three levels of seismic resistances to accommodate the ordinary EQs, the design EQs and the maximum considered EQs. The return periods of these earthquakes for a time of 50 years with the occurrence probability of 80%, 10% and 2% can be found as 30, 475 and 2500 years, respectively. The mean annual rate of exceedance is simply the reciprocal of the return period.

6. Numerical Examples

A numerical example conducted using EQWEAP and PBEE framing equations are discussed next. In Figure 4, the associated PGAs at return period of 30, 475 and 2500 years in Taipei are 0.12, 0.29 and 0.51 g, respectively. Regression analysis shows that the hazard model in Taipei can be expressed as a power function with \( k = 3.071 \) and \( k_0 = 4.917 \times 10^{-5} \) \((r^2 = 0.995)\). The above PGA values can be taken as target PGA values for response analysis of a single pile located at a site in Taipei. A typical bridge pile foundation of the highdeck expressway located in Sinjchuang District of New Taipei City was considered for the example study. Based on significance of the recent seismic records and similarity of the geological conditions, the authors selected 8 accelerogram records from 6 seismic stations in its vicinity. These records include 1999 Chi-Chi earthquake (in-land, active faulting triggered quake, \( M_L = 7.3, \) Taipei Intensity: IV) and 2002 Yi-Lang earthquake (east coast offshore, subduction plate triggered quake, \( M_L = 6.8, \) Taipei Intensity: V). Figure 5 shows the locations of the seismic stations. Table 3 lists the PGA earthquake records in use. Shear wave velocity profiles of the stations are shown in Figure 6. A typical 3 × 3 pile foundation where the piles are of 2 m diameter and 60 m length was considered for geological conditions simplified from the borehole data (see Figure 6). Table 4 summarizes the idealized ground profile and the properties of soils at the foundation site. Vertical design loads applied to the piles were known to be 9000 kN and 18000 kN for ordinary and seismic design cases. The horizontal loads can be 10~15% of the vertical loads. A 15 m height of the horizontal loads was assumed to compute the external moments for the piles. These moments are only required when semi-rigid boun-
The moment condition is assumed at the pile head. The moments of $M_{cr}$, $My$, and $Mult$ used in the study were able to obtain from LPILE analysis, with 2% area percentile of the steel bar and 18000N vertical loads presumed, the $M_{cr}$, $My$, and $Mult$ were calculated as 7347, 22148 and 28679 kN-m, respectively. The value of $EI$ for the piles is $2.4 \times 10^7$ kN-mm$^2$.

The wave equation analyses were conducted using the Finn’s EPWP model. Table 5 summarizes the parameters and empirical equations in use. The dynamic pile responses subjected to the earthquake excitations were obtained for each of the target PGAs. Figure 7 shows the typical results of the maximum pile displacements, the maximum internal moments and the maximum shears for the piles with the target PGA at 0.29 g. Figure 8 presents

Table 3. Seismic records used in the study

<table>
<thead>
<tr>
<th>Station</th>
<th>Locations</th>
<th>Earthquake</th>
<th>Longitude latitude</th>
<th>V (g)</th>
<th>EW (g)</th>
<th>SN (g)</th>
<th>Duration (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAP003</td>
<td>Luzhou</td>
<td>Chi-Chi</td>
<td>121.46E/25.09N</td>
<td>0.044</td>
<td>0.129</td>
<td>0.108</td>
<td>175</td>
</tr>
<tr>
<td>TAP010</td>
<td>Sanchong</td>
<td>1999/09/21</td>
<td>121.48E/25.07N</td>
<td>0.028</td>
<td>0.117</td>
<td>0.088</td>
<td>144</td>
</tr>
<tr>
<td>TAP017</td>
<td>Xinzhuang</td>
<td>121.46E/25.05N</td>
<td>0.035</td>
<td>0.113</td>
<td>0.099</td>
<td>151</td>
<td></td>
</tr>
<tr>
<td>TAP087</td>
<td>Wugu</td>
<td>121.42E/25.10N</td>
<td>0.031</td>
<td>0.049</td>
<td>0.077</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>TAP011</td>
<td>Sanchong</td>
<td>Yi-Lan</td>
<td>121.50E/25.06N</td>
<td>0.037</td>
<td>0.074</td>
<td>0.078</td>
<td>105</td>
</tr>
<tr>
<td>TAP037</td>
<td>Xinzhuang</td>
<td>2002/03/31</td>
<td>121.44E/25.03N</td>
<td>0.026</td>
<td>0.071</td>
<td>0.069</td>
<td>90</td>
</tr>
<tr>
<td>TAP017</td>
<td>Xinzhuang</td>
<td>121.46E/25.05N</td>
<td>0.042</td>
<td>0.051</td>
<td>0.082</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>TAP087</td>
<td>Wugu</td>
<td>121.42E/25.10N</td>
<td>0.031</td>
<td>0.049</td>
<td>0.047</td>
<td>71</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Geological conditions and soil properties used in the study

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Thks (m)</th>
<th>Layers</th>
<th>Soil</th>
<th>$\gamma$ (kN/m$^2$)</th>
<th>SPT-N</th>
<th>$\nu$</th>
<th>C (kN/m$^2$)</th>
<th>$S_n$ (kN/m$^2$)</th>
<th>$\phi$ (°)</th>
<th>Vs (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0~4</td>
<td>4</td>
<td>SF</td>
<td>Sand</td>
<td>18</td>
<td>3</td>
<td>0.3</td>
<td>9.8</td>
<td>24.5</td>
<td>30</td>
<td>115</td>
</tr>
<tr>
<td>4~10</td>
<td>6</td>
<td>Sungshan formation VI</td>
<td>CM</td>
<td>19</td>
<td>5</td>
<td>0.48</td>
<td>9.8</td>
<td>44.1</td>
<td>28</td>
<td>171</td>
</tr>
<tr>
<td>10~20</td>
<td>10</td>
<td>Sungshan formation V</td>
<td>SM</td>
<td>20</td>
<td>14</td>
<td>0.47</td>
<td>0</td>
<td>0</td>
<td>33</td>
<td>192</td>
</tr>
<tr>
<td>20~40</td>
<td>20</td>
<td>Sungshan formation IV</td>
<td>CM</td>
<td>20</td>
<td>11</td>
<td>0.49</td>
<td>20</td>
<td>68.7</td>
<td>28</td>
<td>222</td>
</tr>
<tr>
<td>40~50</td>
<td>10</td>
<td>Sungshan formation III</td>
<td>CM</td>
<td>20</td>
<td>21</td>
<td>0.47</td>
<td>0</td>
<td>0</td>
<td>34</td>
<td>221</td>
</tr>
<tr>
<td>50~60</td>
<td>10</td>
<td>Sungshan formation II</td>
<td>CM</td>
<td>20</td>
<td>14</td>
<td>0.49</td>
<td>20</td>
<td>88.3</td>
<td>35</td>
<td>241</td>
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<tr>
<td>60~70</td>
<td>10</td>
<td>Sungshan formation I</td>
<td>SM</td>
<td>20</td>
<td>30</td>
<td>0.47</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>248</td>
</tr>
</tbody>
</table>

Table 5. Soil parameters used in the EQWEAP modelling

<table>
<thead>
<tr>
<th>Model parameters in use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach</td>
</tr>
<tr>
<td>EPWP</td>
</tr>
</tbody>
</table>

Note: $V_s = 80N^{1/3}$ for sand, $V_s = 100N^{1/3}$ for clayey soils.
the demand curves (EDP vs. IM) obtained using the medians of EDP data (maximum pile displacements) at the target PGAs (IM values). If a power law is used, the parameters $a$ and $b$ will be 156.36 and 1.004 ($r^2 = 0.99$). The coefficient of variation, $\beta$ associated with each group of EDP can be computed and substituted into Eq. (7) with the median EDP to compute $\lambda_{EDP}$.

Figure 9 then presents the regression curve based on the computed annual rate of exceedance for the median EDPs. It can be seen that EDP of 19, 45 and 79 cm are corresponding to the return periods of 30, 475 and 2500 years. Similarly, the damage curves (DM vs. EDP) of these numerical studies can be shown in Figure 10. Again, using a Power Law to curve fitting these data, the parameters of $c$ and $d$ in Eq. (8) are 1368.4 and 0.677 ($r^2 = 0.97$). The corresponding annual rate exceedance of the pile moments can be obtained as shown in Figure 11. The pile moments corresponding to the same return periods would be 10454, 19541 and 25760 kN-m. From Figures 9 and 11, one can easily define the maximum pile displacements and moments allowable for the seismic design particularly considering the ground impacts. Comparing to $Mcr$, $My$ and $Mult$ calculated beforehand, the piles are found to be vulnerable to yield damages around the pile head. For seismic level I where the quakes are relatively small to medium, the cracks will form mainly at the pile head. For design earthquake, the steel bars at the pile head are almost about to yield. For maximum consideration earthquake, the steel bars at the pile head are yielded however the pile will remain elastoplastic and the plastic hinge will not able to find.

![Figure 7](image1.png)
**Figure 7.** Maximum displacement, moment and shear along the pile shaft for PGA = 0.29 g.

![Figure 8](image2.png)
**Figure 8.** Response model of PBEE analysis.

![Figure 9](image3.png)
**Figure 9.** Annual rate of exceedance for EDP in PBEE analysis.

![Figure 10](image4.png)
**Figure 10.** Demand model of PBEE analysis.
7. Discussions

From the example study, it is found that people can always evaluate the seismic performance of the piles based on given information. In common design practice, seismic loads on the foundations are often treated as pseudo static ones. Even for large earthquakes where the sites may liquefy, the ultimate ground displacements and/or the ultimate earth pressures have been suggested to verify the pile resistance. The pseudo static treatments are popularly used in Japan and Taiwan. With the one-dimensional wave equation analysis, one can find that the seismic solution is not that difficult and it is more applicable to help the assessment of seismic performance for pile foundations.

For a given site where the pile foundation is to be built, the geological conditions of the site should be known with reliable bored log data. Soil stiffness parameters must be obtained from proper tests and/or empirical formulations with the correlation from other parameters. The super-structural loads can be analyzed and taken as static loads applying to the foundation. The loads acting on the piles should be calculated using 2D or 3D methods taking the foundation as a beam or a plate. The uncertainties from the seismicity are the major concerns in the design. For better control of curve interpolations, the IMs used for the computation could be increased among the target values.

The uncertainties of the soils are ignored in above study, they can be included as those suggested by Shin [16] based on solving the coefficient of variations, $\beta_s$ from the soils for each group of EEPs. The total uncertainties can be then calculated from both seismicity loadings and soil parameters. With the use of Tornado diagram and First Order Secondary Moment method to compute $\beta_s$, Shin concluded that the influences of the soil uncertainties are far less important than the seismicity of earthquake. For simplicity, one could include both the variations of seismicity and soil parameters in the analyses with the same IMs, the data should be increased with the changing of soil parameters. Same procedures could be followed. In that case, the result can be represented including both influences of the seismicity and the soil.

8. Concerns for Applications

To apply EQWEAP analysis to more realistic cases, the pile group loads distributed to a single pile must be analyzed. For a bridge-pier pile foundation, the super-structural loads transmitting through the pier on the pile cap can be analyzed in the longitudinal and transverse directions. This analysis would simplify the 3D structure to a 2D structure, and the calculations could be performed by simple mechanical analysis. The super-structural loads can be treated statically, as in the conventional designs. By assuming that the cap moves rigidly, the inertia forces of pile cap and the horizontal net earth pressures working beside the cap can be determined. The transmission of typical superstructure loads and the cap loads to the piles is very important. The horizontal soil resistances underneath the cap are ignored. If a 3D calculation is preferred, the suggestions of JRA or those proposed by Chang et al. [19] can be applied to determine the vertical and horizontal loads acting on every single pile. The moments applied to the piles are the same as the total moment. If a 2D calculation is adopted, then simple equations from fundamental mechanics are used to compute the loads.

Possible effects of the pile-to-pile interactions on soil impedance need to be considered. The pile-to-pile interactions could reduce the soil impedance, and they can be calculated through the use of dynamic interaction factors. Based on the study of Chang et al. [19], the interaction effects were found important only for static loads.
and steady-state loads at smaller frequencies. The non-linearities of the soils would also moderate the interaction influences. For large ground excitations where non-linear soil behavior exists, the interaction effects may become trivial. In addition, proper control of the spacing-to-diameter ratio can also eliminate excessive interaction effects. Therefore, the interaction effects on soil impedance and the corresponding earth pressures due to the earthquake could be ignored in the analysis.

With the EQWEAP analysis, the equivalent foundation stiffness $K_{eq}$ and the structural unbalance forces can be calculated easily. For instance, if a bridge-pier pile foundation system were simplified as a 2DOF structure shown in Figure 12, Eq. (9) could be used to compute the displacements of bridge and pier, assuming that the foundation is rigid. The superstructural loads transmitted to the foundation (rigid base) would be $K_s u_s + C_s \ddot{u}_s$. This load can be applied to EQWEAP analysis to solve for the foundation displacements $u_f$. The value of $u_f$ is then used to calculate the unbalanced force $Q$ of the transmitting load as defined in Eq. (10). The unbalanced force $Q$ will then be used to calibrate foundation displacement with its incremental $\Delta u_f$. An iterative process can then be conducted until the force is balanced and the actual foundation displacements are thus obtained.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 + K_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$[K_s u_s + C_s \ddot{u}_s] - [K_s (u_s - u_f) + C_s (\ddot{u}_s - \ddot{u}_f)] = K_s u_f + C_s \ddot{u}_f = Q$$

In the above equations, $C_{11} = \alpha_1 m_1 + \beta_1 K_1$, $C_{22} = \alpha_2 m_2 + \beta_1 K_1 + \beta_2 K_2$, and $C_{12} = -\beta_1 K_1 = C_{21}$, where $\alpha$ and $\beta$ are the parameters that can be simply calculated with the dependence of natural frequency and material damping ratio for the superstructure. By taking a 3DOF structural system in which the 3rd element stands for the foundation, and assuming that the pile cap is massless (see also Figure 12), Eq. (11) can be used to compute the equivalent stiffness for foundation, $K_{eq}$.

$$K_3 = \frac{K_s (u_s - u_f) + \beta_3 K_s (\ddot{u}_s - \ddot{u}_f)}{u_f + \beta_3 \ddot{u}_f}$$

The parameter $\beta_3$ can be calculated with the presumed damping ratio and natural frequency of the foundation. These calculations are also applicable when the foundation stiffness and the corresponding loads are given. The equivalent stiffness of the foundation should be carefully verified to ensure appropriate superstructural loads. This is important and very helpful to the seismic design of pile foundations where the foundation stiffness can be affected by the seismic ground motions.

9. Concluding Remarks

The one-dimensional wave equation analysis can be used to model the seismic response of piles. It provides a fast and effective solution for people to create data bank for performance-based design purpose. Using the PEER framing equation, the authors suggested that the seismic PBD of the piles can be evaluated according to the seismic performance in requirement. For any foundation site of interest, one should have the bored hole data beforehand. The uncertainties of the earthquake and the soils...
could be analyzed accordingly. Numerical examples for piles in Taipei has shown that the rate of exceedance for the maximum pile displacements and the maximum bending moments were achievable from the PBEE analysis. With the suggestions from the authors, these relations should be obtained at a number of design levels. From the design curves established, one could easily find the minimum pile capacities corresponding to different design levels, or simply verify the design pile capacity with respect to its rate of exceedance to see whether the design is acceptable or not.

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