Uncertainties in the Calibration System for Invar Leveling Rods

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Abstract

Bar code line and barcode invar leveling staffs must be calibrated for precise leveling. According to first-order leveling specifications instituted by the Ministry of Internal Affairs, the fixed standard for rod calibration uncertainties is $\pm 50 \mu m$. At the National Measurement Laboratory (NML), no equipment meets this requirement. The purpose of this study of the existing calibration system at the National Measurement Laboratory is to reduce the calibration uncertainties to an allowable level. An automated calibration system for invar leveling staffs was established in this study, and the uncertainty of the calibration data was evaluated. The National Measurement Laboratory, supervised by the Development Center for Measuring Technology of the Industrial Technology Research Institute, employs a stabilized-frequency laser interferometer, a charge-coupled device (CCD) microscopic monitor and a precise movement mechanism for the calibration of invar leveling staffs. As part of the process, the error factors and the amount of uncertainty associated with line scale and barcode leveling staffs are assessed. At the 95% confidence level, the expanded uncertainties of the graduations were 22 $\mu m$ and 21 $\mu m$ for line scale and bar coded staffs, respectively, which meet user requirements.

Key Words: Invar Leveling Staffs, Calibration System, Estimate of Uncertainty, Measuring Technology

1. Introduction

During precise leveling, a digital level with a barcode invar leveling staff or an optical level with a linescale invar leveling staff is frequently used for elevation measurements. After extended periods of field work, barcode invar leveling staffs can deform slightly, which affects measurement accuracy. To obtain precise results, levels must be calibrated regularly. For normal use, a calibration value $\pm 50 \mu m$ is the invar standard instituted by the Ministry of Internal Affairs [1]. Unfortunately, there is no calibration system in the National Measurement Laboratory (NML) in Taiwan. The standards associated with reeling staffs, which are dependent on the equipment used, are different in different countries. For example, the Korea Research Institute of Standards and Science (KRISS) standard is 13 $\mu m$, the U.S. National Institute of Standards and Technology (U.S. NIST) standard is 20–50 $\mu m$, the Swiss Federal Office of Metrology and Accreditation (SFOM) standard is 16 $\mu m$, and the standard in Canada is 10 $\mu m$. The calibration ability at the 95% confidence level is shown in Table 1.

This study evaluates the sources of measurement errors and the uncertainties associated with the calibration
system for reeling staffs that is currently used in the National Measurement Laboratory. The experimental result is within the standard using the proposed calibration system approach. Furthermore, this study solves the problem of the supports of the staffs during the measurement process.

2. System Configuration

2.1 Introduction and Retrace of the Calibration System

The calibration system examined in this study is designed to measure data directly. The components of the system are a stabilized-frequency laser interferometer, a CCD microscopic monitor and a precise movement mechanism. Figure 1 illustrates the system configuration [2, 3]. The arrows indicate data output directions.

The measurement theory for the invar leveling calibration system requires setting a critical value to acquire and calculate the strip edge on the marked invar leveling rods. This study uses a movement mechanism to move the edge of the invar leveling rods to the CCD image center. This process is repeated until the edges of the invar leveling rods are moved to the CCD image center. The reading value of the laser interferometer and the circumstance parameters are then recorded. Using these reading values, the optical wave length and the thermal expansion coefficients can be modified [4]. Figure 2 illustrates this process.

In the field of the measurement of quantities, the measurement retrace can be connected to a special reference system using a continuous comparison chain with uncertainty. In the long-rule calibration system, the standard-stabilized-frequency laser interferometer can be retraced to the stabilized-frequency laser calibration system. The laser wavelength and frequency calibrations are used to obtain the definition of a meter; that is, a meter is defined as 1/299792458th of the movement of light in one second in a vacuum. The retracing process is illustrated in Fig. 3.

2.2 Preparation before Calibration:

The following tasks must be completed before measuring.

1. Invar the source of electricity in the laser stable-pressure instrument.
2. Activate the thermometer.
3. Activate the electricity source for the movement system.
4. Check that the multimeter and the IEEE488 port-connected line of the switch is OK, and activate the sensor.
5. Place the unit to be tested on the granite platform, tune the unit until the level vial is at the space in the center joint of the platform. Move the movement

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Table 1. Calibration standards for invar leveling rods in various countries

<table>
<thead>
<tr>
<th></th>
<th>Taiwan CMS</th>
<th>Korea KRISS</th>
<th>America NIST</th>
<th>Canada NRCC</th>
<th>Switzerland SFOM</th>
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<tr>
<td>Measurement range</td>
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<td>3 m</td>
<td>3 m</td>
<td>3 m</td>
<td>4 m</td>
</tr>
<tr>
<td>Uncertainty (μm)</td>
<td>Q [47 L]</td>
<td>Q [10,3 L]</td>
<td>20–50</td>
<td>&gt;10</td>
<td>Q [11,1.4 L]</td>
</tr>
</tbody>
</table>

The unit of L is m.
system to the center of the unit (which needs to be calibrated), and activate the light, CCD, and fluorescence source screen. Then the cushion is placed below the standard unit.

(6) The microscope object lens is set at 5 when the unit under test is a strip invar leveling rod, and it is set at 3 when the unit under test is a mark invar leveling rod. A cushion is necessary in both cases.

(7) The alignment adjustment of the unit is based on the lower line of the strip line of the unit. The lower line is tuned to the two parallel lines on the monitor.

(8) Move the movement system to the initial position. In calibrating strip invar leveling rods, move the moving system to 16 cm in front of the strip manually. A strip appears every 200 nm. In calibrating mark invar leveling rods, move the moving system to the trapezoidal sculpture of zero, and be careful; the reflector lens must not bump the spiral cap of the unit.

(9) Activate the air-conditioning system in the lab.

(10) Maintain a stable lab temperature for 8 hours.

2.3 Calibration steps

(1) Turn on the power for the movement system; activate the reset key for the remote control; and turn on the red bulb in the control box.

(2) The calibrated file for the strip invar leveling rods is saved as C:\tape\Miron.exe, whereas the calibrated file for the mark invar leveling rods is saved as C:\tape\Mlevel.exe. Establish the input file and then execute the automatic measuring program. The output file is the uncalibrated results file; see Table 2.

(3) To measure the next set of data, the power for the remote control must be turned off. Move the movement system to its original position manually. Turn on the power for the remote control again, press the reset key, and execute the next measurement. Turn off the power to the remote control before moving the movement platform manually.

2.4 Data Analysis

After the calibration procedure, the raw data file is obtained automatically. For example, the raw data file for the strip invar leveling rod is Routput.dat. With the strip reading, the laser interferometer, the temperature of the lab, the relative humidity, and the atmospheric pressure, Excel software can be used to correct the coefficient of thermal expansion and the refraction.

3. Discussion of the Position of the Support

Because the graduated invar level must be used in an aluminum frame, the invar bar code does not lie smoothly on the support surface when the staff is placed on a granite bench. Thus, a pad must be inserted under the staff to facilitate the calibration and reduce straightness errors associated with the leveling staff. The pad size and location were selected on the basis of previous experimental results. The experiments were conducted in the following order. Four steel pads of the same size (100 mm long, 24 mm wide, and 8 mm high) were prepared. Then, according to the three following rules, the pads were placed and the reading value was measured. Measurements were taken ten times for each support case. The results are shown in Tables 1 and 2.
(A) Two-point support: pads are placed at 1 m and 2 m along the 3-m-long invar leveling staff.
(B) Three-point support: pads are placed at 0.75 m, 1.5 m, and 2.8 m along the 3-m-long staff.
(C) Four-point support: pads are placed at 0.6 m, 1.2 m, 1.8 m, and 2.4 m along the 3-m-long invar leveling staff.

The average reading values and the standard deviations are shown in Tables 3 and 4. We conclude that the three-point and four-point support systems are superior to the two-point support system. The difference in the standard deviation between the results for the three-point and four-point support systems is only 1 μm. The three-point support system, however, is the better of the two in terms of time and cost.

4. Discussion of Uncertainty

The uncertainty of the invar leveling staff is evaluated according to the “Guide to the Expression of Uncertainty in Measurement” recommended by the International Organization for Standardization (ISO) [5].

4.1 Mathematical model

The uncertainty is evaluated using the following equation:

\[ L = \left(\frac{\lambda_0}{128 n_{ref}}\right)N[1 - \frac{\theta^2}{2}] + d(\sin \beta) \]  

(1)

where \( L \) is the distance the platform moves, \( \lambda_0 \) is the vacuum laser wavelength, \( d \) is the distance between the laser optical axis and the measuring axis, \( \beta \) is the straight-line error of the moving platform, \( \theta \) is the angle between the laser optical axis and the moving axis, and \( n_{ref} \) is the refractive index of air for temperature \( t \), pressure \( p \), and humidity \( f \). By considering additional factors such as the temperature, thermal expansion coefficient, modification of air refraction, and positioning...

<table>
<thead>
<tr>
<th>Times\Methods</th>
<th>Two-point supporting (A)</th>
<th>Three-point supporting (B)</th>
<th>Four-point supporting (C)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>999.977 mm</td>
<td>999.991 mm</td>
<td>999.986 mm</td>
</tr>
<tr>
<td>2</td>
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<tr>
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<td>999.979 mm</td>
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<td>999.983 mm</td>
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<td>9</td>
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<td>999.987 mm</td>
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<tr>
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<td>999.990 mm</td>
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<tr>
<td><strong>Average</strong></td>
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<td><strong>999.987 mm</strong></td>
<td><strong>999.988 mm</strong></td>
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<tr>
<td><strong>St. dev.</strong></td>
<td><strong>0.003 mm</strong></td>
<td><strong>0.003 mm</strong></td>
<td><strong>0.002 mm</strong></td>
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<td>1999.978 mm</td>
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<td>4</td>
<td>1999.952 mm</td>
<td>1999.978 mm</td>
<td>1999.976 mm</td>
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<td><strong>Average</strong></td>
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<td><strong>1999.977 mm</strong></td>
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<td><strong>0.002 mm</strong></td>
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</table>
error of a graduation on the invar staff, Eq. (1) can be re-
written as Eq. (2):

\[ L' = \left[ 1 + \alpha_s (20 - t_s) \right] \left[ \frac{\lambda_0}{128 n_{gp}} \right] N \left( 1 - \frac{\theta^2}{2} + \delta (\sin \beta) \right) + E \]

(2)

where \( \alpha_s \) is the thermal expansion coefficient of the in-
var leveling staff, \( t_s \) is the temperature of the leveling
staff, \( N \) is the reading of the laser interferometer
\( (\lambda_{gp}, \alpha_s, t_s, b, 0, d, N, E) \)

where \( \lambda_0 \) is the wavelength in air, and \( E \) is the
positioning error of the graduation line on the ribbon. A
change in the refractive index of air can generate laser
wavelength errors. Therefore, the laser wavelength
must be modified by measuring the refractive index of
air. It is generally known that the refractive index of air
is affected by the air temperature, relative humidity and
vapor pressure. Values for these parameters are ob-
tained from sensors. These parameters are then inserted
into the experimental formula developed by Edlen [6],
the modified equation proposed by Birch and Downs
[7], and the modified equation for vapor pressure pro-
posed by Wexler to compute the air refraction index and
modify the laser wavelength using Eq. (3):

\[ \lambda_{npf} = \frac{\lambda_o}{n_{npf}} = \frac{\lambda_o}{1 + A_1 \times B_2 - C_2} \]

(3)

where,

\[ A_1 = \frac{[7843.05 + 2406294(130 - \sigma^2)^{-1} + 15999(38.9 - \sigma^2)^{-1}10^{-8}]}{96095.43}, \]

(3a)

\[ B_2 = \left[ 1 + p(0.601 - 0.00972t_{90})10^{-8} \right] \]

(3b)

\[ C_2 = f(3.7345 - 0.0401 \sigma^2)10^{-10}, \]

(3c)

\[ f = RH \times e_{sw} (T) / 100, \]

(3d)

\[ e_{sw} (T) = \exp \left[ \sum_{j=0}^{6} A_j \times T^{j-2} + A_7 \times 1_n (T) \right], \]

(3e)

where \( P \) is the air pressure (Pascals), \( \lambda_0 \) is the air tem-
perature (°C), \( \sigma \) is \( \frac{1}{\lambda_0} \) (\( \mu \)m⁻¹), \( f \) is the partial water vapor
pressure (Pascals), \( RH \) is the relative humidity, \( e_{sw} (T) \) is
the saturated water vapor pressure (Pascals) and \( T \) is the
absolute temperature (K), where \( T = 273.15 + t \).

The investigation of the various factors that affect
the uncertainty, i.e., the laser wavelength, the environ-
ment, the invar leveling staffs, human negligence, and
instrument errors, is described below.

4.2 Uncertainty of the laser wavelength (\( \lambda_0 \))

The uncertainty of the laser wavelength (\( \lambda_0 \)) is attrib-
utable to the following three factors.

(a) The standard laser head is the red Zeeman
frequency-stabilized HeNe laser, which is related to the
iodine frequency-stabilized red HeNe laser, with a
central wavelength of 0.632991376 \( \mu \)m and a report-
ing code B910771 of a relative expanded \( (k = 2) \) un-
certainty of \( 1.52 \times 10^{-9} \). The estimated relative uncer-
tainty is 10% and the number of degrees of freedom
is 50. The standard uncertainty \( u_1(\lambda_0) \) resulting from
tracing back the laser wavelength is \( 0.48 \times 10^{-9} \) m.

(b) The temperature changes the laser cavity length and
the effective index refraction when the laser is acti-
vating. An increase in the semiconductor tempera-
ture produces a dilatation of the laser cavity shifting
the emission wavelength. In order to maintain stable
length of the laser cavity, the material of high ri-
gidity and small expansion coefficient is usually
used, and then the laser cavity deformation is mea-
sured by sensor, and compensated by reflecting mir-
ror. However, no matter how good the sealing tech-
ology is, the helium in the laser cavity still dissis-
pates little by little, so the laser wavelength has
long-term drift. The long-term standard uncertainty

\[ A_0 = -2.9912729 \times 10^3 \]
\[ A_1 = -6.0170128 \times 10^3 \]
\[ A_2 = 1.88764354 \times 10^4 \]
\[ A_3 = -2.8354721 \times 10^2 \]
\[ A_4 = 1.7838301 \times 10^5 \]
\[ A_5 = -8.41504171 \times 10^{-10} \]
\[ A_6 = 4.4412543 \times 10^{-13} \]
\[ A_7 = 2.858487 \]
(k = 1), which must be obtained from the value of the laser wavelength in the calibration report, is valid for three years. The value must be less than 2 × 10^{-10} \text{ m}.

If the distribution is rectangular, the relative uncertainty is estimated as 10%, with 50 degrees of freedom. The standard (k = 1) uncertainty \( u(\lambda_0) \) resulting from long-term drift of the laser wavelength is 0.577 × 10^{-10} \text{ m}.

The non-linear error of the Zeeman frequency-stabilized HeNe laser is 2 nm, with a rectangular distribution. The standard uncertainty \( u_3(\lambda_0) \) is 1.15 nm.

Combining the analysis of (a)–(c), the standard uncertainty (k = 1) of the laser wavelength is

\[
u(\lambda_0) = \left[ u_1^2(\lambda_0) + u_2^2(\lambda_0) + u_3^2(\lambda_0) \right]^{1/2} \approx 1.15 \text{ nm} \quad (4)
\]

### 4.3 Refraction index of air

Changes in the air refraction index will change the laser wavelength. The change in the laser wavelength associated with changes in the refraction index is a function of the temperature (t), the pressure (p), and the water vapor pressure (f). Thus, air temperature, air pressure, and relative humidity along the laser light path must be considered and input into the Edlen formula to modify the laser wavelength.

**Air refraction index–Equation for the air refraction index**

The modified Edlen equation for the air refraction index developed at the National Physical Laboratory (NPL) in England [1] was adopted in this study. The error associated with this equation is ± 3 × 10^{-8}. If its distribution is rectangular, the standard uncertainty \( u(\text{ntrp}) \) is 1.732 × 10^{-8}.

**Air refraction index–Air temperature (t)**

The effect of a change in the air temperature can be modeled using the Edlen equation. The range of temperatures in the laboratory was (20 ± 0.5) °C. Three thermometers are placed at 1m, 5m and 10m, and the temperature data are substituted in the equation to modify the laser wavelength when the laser distance is measured each time. This range is characterized by a U-shaped distribution. The standard uncertainty \( u(t) \) is 0.289 °C.

**Air refraction index–Air pressure (p)**

From the Edlen equation, an error of 1 Pa in the air pressure is found to cause an error of -2.68 × 10^{-9} L Pa^{-1} in the length measurement. Its standard uncertainty \( u(p) \) is 151.84 Pa.

**Air refraction index–Relative humidity (f)**

Using the Edlen equation, we can assess the influence of the relative humidity. The standard uncertainty \( u(f) \) is 5.77% R.H., with the relative humidity maintained at less than 10% and assumed to have a rectangular distribution.

### 4.4 Invar Leveling Staff

**Invar leveling staff – Thermal expansion coefficient \((\alpha_s)\)**

No calibration standard exists for the invar leveling staff. However, according to the CNS 3860-Z7048 [8] standard regulation, the measured length for the invar leveling staff must be reduced to the standard temperature of 20 °C. The thermal expansion coefficient for an invar leveling staff is \((1.0 \pm 0.1) \times 10^{-6}/\text{C}\). When the temperature of the invar leveling staff is 20.3 °C, the range of its coefficient of thermal expansion is taken as \(0.1 \times 10^{-6}/\text{C}\), with a rectangular distribution. For an estimated relative uncertainty of 20%, the number of degrees of freedom \( v(\alpha_s) \) is 12.5. The standard (k = 1) uncertainty \( u(\alpha_s) \) is 0.87 × 10^{-6}/°C.

**Invar leveling staff – Measurement of temperature \((t_s)\)**

If the temperature of the invar leveling staff changes during the calibration process, the measurement result is affected. The range of the temperature change is taken as ±0.5 °C. The distribution is rectangular. The estimated relative uncertainty is 10%, and the number of degrees of freedom \( v(t_s) \) is 50. Based on the coefficient of expansion, the standard uncertainty \( u(t_s) \) is 0.29 °C.

### 4.5 Laser interferometer

**Laser interferometer – Error of the degree of straight line (first-order \(\beta\) error) \((\beta)\)**

The first-order \(\beta\) error of the degree of straight line
occurs when the measurement line of the laser axis does not coincide with that of the invar leveling staff. This causes an error in the length measurement. After adjusting the two measurement lines, that of the laser axis and that of the invar leveling staff, the gap \( d \) between the two axes is within 40 mm. The angular misalignment of the two axes is set to within 15 seconds of arc. The distribution is designed as rectangular. The estimated relative uncertainty is 20%. The number of degrees of freedom \( v(\beta) \) is 12.5. The standard uncertainty \( u(\beta) \) is 4.33".

**Laser interferometer – Second-order \( \beta \) alignment error** 

When the laser axis is not parallel to the moving platform, the measured value will be smaller than the actual value; the measurement error is \( -L \theta^2/2 \), where \( \theta \) is the angle between the laser axis and the moving platform axis. After adjustment, the angle \( \theta \) is within \( \pm 20" \). The distribution is taken as rectangular. With an estimated relative uncertainty of 10% and 50 degrees of freedom, the standard uncertainty \( u(\theta) \) becomes 11.55". The value \( d \) of the manual focusing error is within \( \pm 2 \) mm. The distribution is assumed to be rectangular. The estimated relative uncertainty is 20%, the number of degrees of freedom \( v(d) \) is 12.5, and the standard uncertainty \( u(d) \) is 1.15 mm.

**Laser interferometer – Manual focusing error** 

The \( d \) value of manual focusing error is controlled to within \( \pm 2 \) mm. The distribution is designed as rectangular. The estimated relative uncertainty is 20%, the number of degrees of freedom \( v(d) \) is 12.5, and the standard uncertainty \( u(d) \) is 1.15 mm.

**4.6 Edge Detection (E)**

The invar leveling staff is calibrated through edge detection, using equipment such as a microscope, CCD sensor, image processing card, laser interferometer, server control card, driving platform, and direct-circuit server motor. The server mechanism is positioned at the graduation line in the center of the microscope. Therefore, the error in the edge detection is caused by the resolution of the microscope, the CCD and the motor. The value is obtained by moving the driving platform to different positions of the graduation lines in the viewing window and using computer processing to determine and check the edge shown on the CCD. The positioning accuracy is 1 pixel, and the corresponding value of the movement is 5.2 \( \mu \)m. The largest positioning error is thus 1 pixel. The distribution is taken as rectangular. The estimated relative uncertainty is 10%, the number of degrees of freedom is 50, and the standard uncertainty \( u(E) \) is \( 5.2/\sqrt{5} \mu m \approx 1.51 \mu m \).

**4.7 Readout of the laser interferometer (N)**

When the laser interferometer is stationary and resting on the driving platform, with its fluctuation controlled to within three times and with a rectangular distribution, the estimated relative uncertainty is 10%, the number of degrees of freedom \( v(N) \) is 50, and the standard uncertainty \( u(N) = 2 \).

**4.8 Expanded uncertainty**

The errors influencing the calibration of the invar leveling staff are ranked as follows: the error of positioning of the marked graduation line; the error of the straight-line measuring axes of the laser and invar leveling staff, the fluctuation of the coefficient of thermal expansion of the invar leveling staff, and the laser light wavelength.

Typically, the measurement uncertainty is quoted as an expanded uncertainty. The definition of the expanded uncertainty is as follows:

\[
U = k \times u_c
\]

where \( k \) is a coverage factor. Here, the standard uncertainty, the degrees of freedom of various uncertainties and the uncertainty of various error sources are input into the Welch-Satterthwaite formula [6], a valid number of degrees of freedom is obtained, from which—at a 95% confidence level—the value of \( k \) can be obtained from the Student's \( t \) distribution. The value is then multiplied by the combined uncertainty to obtain the expanded uncertainty \( U \).

According to the Welch-Satterthwaite equation, a valid number of degrees of freedom \( v_u \) is:

\[
v_u = \frac{\sum u_i^4(y)}{\sum v_i(y)}
\]
where $u_i$ is the standard uncertainty, $u_i$ is the uncertainty of each component, and $v_i$ is the number of degrees of freedom of each component.

The expanded uncertainty of the invar leveling staff can be obtained by multiplying the standard combined uncertainty by the expanded coefficient $k$. The confidence level is taken as 95% and the number of degrees of freedom is 56. Based on the t distribution, the value of the coverage factor $k$ is 2.01. Thus, the expanded uncertainty ($U$) of a bar code staff is as follows:

$$U = k \times u_c = 2.01 \times [(1.73)^2 + (3.38 L)^2]^{1/2} = [(4)^2 + (7 L)^2]^{1/2} \text{ (µm)} (L \text{ is m}).$$ (6)

The calibration of a traditional line scale invar leveling staff is essentially the same as that of a bar code invar leveling staff. The only difference between the two rods is the error in positioning over a marked line. The shift of 1 pixel on a line scale staff is equivalent to 12.4 µm. The accuracy of the positioning is 1 pixel. The largest range of the positioning error is therefore 1 pixel, with the distribution designed as rectangular.

The relative uncertainty is estimated at 10%, the number of degrees of freedom is 50, and the standard uncertainty $u(E) = 12.4/2\sqrt{3} \text{ µm} = 3.58 \text{ µm}$, with a sensitivity coefficient of 1. The expanded uncertainty of a line scale leveling staff is as follows:

$$U = [(8)^2 + (7 L)^2]^{1/2} \text{ (µm)} (L \text{ is m}).$$ (7)

Inserting $L = 3 \text{ m}$ into Eqs. (6) and (7) yields 21 µm and 22 µm for the expanded uncertainty of the bar code leveling staff and line scale leveling staff, respectively.

### 4.9 Quality Control for System Measuring

For quality control of the calibration system, the para-

Table 5. Error analysis of invar leveling staff calibration

<table>
<thead>
<tr>
<th>Sources of error ($x_i$)</th>
<th>Estimate error Type</th>
<th>Distribution</th>
<th>Coefficient</th>
<th>Standard uncertainty $u$ ($x_i$)</th>
<th>$\frac{\partial F}{\partial x_i}$</th>
<th>$\frac{\partial F}{\partial y_i} u(y_i)$</th>
<th>$v_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser wave-length ($\lambda_0$)</td>
<td>3 $\times 10^8$</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>1.15 nm</td>
<td>1</td>
<td>1.15 nm</td>
</tr>
<tr>
<td>Refraction of air ($n_{air}$)</td>
<td>0.5 $^\circ$C</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>1.732 $\times 10^{-6}$</td>
<td>-0.9997 L</td>
<td>0.0173 $\times 10^{-6} L$</td>
</tr>
<tr>
<td>Temperature ($t$)</td>
<td>263 Pa</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>0.289 $^\circ$C</td>
<td>9.53 $\times 10^{-7}$</td>
<td>L $^\circ$C$^{-1}$</td>
</tr>
<tr>
<td>Pressure ($p$)</td>
<td>10%</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>5.77%</td>
<td>8.5 $\times 10^{-9}$</td>
<td>L $^\circ$C$^{-1}$</td>
</tr>
<tr>
<td>Humidity ($f$)</td>
<td>1.5 $\times 10^{-6}$</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>0.87 $\times 10^{-6}^\circ$C$^{-1}$</td>
<td>-0.3 L $^\circ$C</td>
<td>0.261 $\times 10^{-6} L$</td>
</tr>
<tr>
<td>Measurement of temperature ($t_o$)</td>
<td>0.5 $^\circ$C</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>0.29 $^\circ$C</td>
<td>-11.5 $\times 10^{-6}$</td>
<td>L $^\circ$C$^{-1}$</td>
</tr>
<tr>
<td>Straight-line degree error ($\beta$)</td>
<td>7.5&quot;</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>4.33&quot; ($= 2.1 * 10^{-5}$)</td>
<td>40 mm</td>
<td>0.84 µm</td>
</tr>
<tr>
<td>Alignment ($\theta$)</td>
<td>20&quot;</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>11.55&quot; ($= 5.6 * 10^{-5}$)</td>
<td>-5.6 $\times 10^{-5}$</td>
<td>L</td>
</tr>
<tr>
<td>Manual focusing error ($d$)</td>
<td>2 mm</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>1.15 mm</td>
<td>6.35 $\times 10^{-7}$</td>
<td></td>
</tr>
<tr>
<td>Edge detection ($E$)</td>
<td>5.2 µm</td>
<td>B</td>
<td>rectangle</td>
<td>$1/2\sqrt{3}$</td>
<td>151 µm</td>
<td>1</td>
<td>1.51 µm</td>
</tr>
<tr>
<td>Numbers of the laser interferometer ($N$)</td>
<td>3</td>
<td>B</td>
<td>rectangle</td>
<td>$1/\sqrt{3}$</td>
<td>2</td>
<td>4.93 nm</td>
<td>9.9 nm</td>
</tr>
</tbody>
</table>
meters and measuring system must be between an upper limit and lower limit using a t-test. The inspection procedure measures the checking unit before measuring the unit under testing. The checking units of this system are a 3-m long strip invar leveling rod made by Zeiss, and a 3-m long mark invar leveling rod made by DDR. The checking parameters selected are 1 m, 2 m and 2.7 m. These checking parameters are utilized to calculate the mean, standard deviation, and upper and lower limits of the calibration system.

The checking parameters for the strip and mark invar leveling rods are 1 m, 2 m, and 2.7 m. Using these checking parameters, the mean $AC$, standard deviation $SC$ and upper control limit $UCL$ and lower control limit $LCL$ can be calculated. After setting up the procedure parameters, inspection of the calibration system is executed before calibration proceeds. The unit tested is measured and a checking parameter $Ci$ is adopted. The t-test is used to determine whether the calibration system is between control limits.

$$t = \frac{|C_i - AC|}{SC} \quad (8)$$

Suppose $t \leq 3$ from Eq. (8), the calibration system is between control limits, whereas $t > 3$, the calibration system is not good. In this case, the calibration system, circumstance, laser interferometer, and thermometer are investigated again.

5. Conclusions

The factors influencing the calibration of invar leveling staffs are the laser wavelength, the refractive index of air, the thermal expansion coefficient of the invar leveling staff, the straight-line error of the laser interferometer and the positions of the graduations and the interferometer. These error sources are independent and affect the calibration of the invar leveling staff in the following order: the error of positioning of the graduation line, the error of the straight line between the laser and the staff of importance axes, the fluctuation of the thermal coefficient of the invar leveling staff; and the laser wavelength. Table 5 shows the results of the error analysis of the recorded data.

Furthermore, as the staff cannot stick smoothly to the platform surface and support pads are needed under the 3-m invar leveling staff, experiments were performed to determine the best method of support. Ten trials each were performed using a two-point support system, a three-point support system, and a four-point support system. Average lengths of 999.975 mm, 999.987 mm, and 999.988 mm, with standard deviations of 0.003 mm, 0.003 mm, and 0.002 mm were obtained for a measuring value of 1000 mm. Average lengths of 1999.958 mm, 1999.977 mm, and 1999.976 mm, respectively, with standard deviations of 0.003 mm, 0.001 mm, and 0.002 mm, respectively, were obtained for a measuring value of 2000 mm. Based on the experimental data, the three-point support system seems to be the best means of support for staffs 0.75 m, 1.5 m, and 2.8 m in length. Based on the experimental data, the three-point support is concluded to be the best method for padding.

References


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