Three Dimensional Compensation Spherical Coils for Compact Atomic Magnetometer

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Abstract

To avoid the broadening of Zeeman resonances of the vector atomic magnetometer working in an unshielded environment, a real-time magnetic compensation is necessary. A three dimensional mini spherical compensation system is presented, which can be used for a compact atomic magnetometer to realize an ultra-high precision field measurement. Based on the field gradient method, parameters are optimized to obtain a uniformity of $10^{-3}$ over the region of one half radius with a good tolerance on dimensional variations. A prototype has been built and the measurement by traditional fluxgate magnetometer and the integration into the atomic system both demonstrate the validity of the design.

Key Words: Spherical Compensation Coils, Field Gradient Method, Unshielded Atomic Magnetometer, Vector Field Measurement

1. Introduction

Magnetic field characterization is of great importance in a wide variety of areas of modern science. Traditionally, superconducting quantum interference devices (SQUIDs) with a sensitivity reaching down to sub-fT/Hz$^{1/2}$ have been dominant in these demanding applications. Recent development of atomic magnetometers enables them to become competitive with or even better than SQUID detectors in sensitivity, reaching an ultra-high level of 0.16 fT/Hz$^{1/2}$ [1]. They have the intrinsic advantage of not requiring cryogenic cooling and more potential on miniaturization by the advent of inexpensive semiconductor lasers and micro-fabricated vapor cells.

Atomic magnetometry relies on the accurate measurement of Lamor precession frequency of alkali metal atoms polarized by a pumping laser. The basic principles have been illustrated in Refs [2] and [3]. Compared to SQUIDs which only measure the field component along one particular direction, vector atomic magnetometers can provide all three axes information [4]. And these compact magnetometers are more preferred to be configured in a single-beam arrangement [5].

The fundamental sensitivity limitation due to shot noise is associated with the number density of atoms and the spin-coherence time $T_2$ [6]. $T_2$ related to the resonance linewidth $\Delta\omega = 1/T_2$ can be lengthened when spin relaxation due to spin-exchange collisions are completely eliminated by operating in the near zero magnetic field ($< 10$ nT), i.e. by realizing SERF (spin exchange relaxation free) regime. The former constraint is more challenging for the unshielded magnetometer which is extremely attractive for potential applications in geophysical surveying, anti-submarine ordnance and space exploration [7] when the magnetic sources are outside of the magnetometer. It uses active magnetic compensating systems instead of multi-layer shields, avoiding the imperfection of continuous variation and inaccurate mea-
surement of the shielding factor. Given the high field attenuation ratio and the fluctuation of earth field, a rapid and accurate compensation is indispensable and must be able to produce a highly uniform field throughout the cell center.

Helmholtz coils are commonly-used compensating actuators in these atomic magnetometer setups [4,8]. The main disadvantage of the Helmholtz coils is their bulky volume with the problem of miniaturization. This also makes it hard to isolate disturbances from the environment, especially the earth field gradient which decreases the sensitivity of the magnetometer [9]. Moreover, the uniformity can hardly satisfy the requirements of many applications. Since it was demonstrated that a uniform spherical current shell would produce a complete uniform field throughout the inner volume [10], there have been many investigations on spherical coils to improve the uniformity. Everett and Osemeikhian used closely wound coils to approximate the spherical current shell [11]. And a four circular system was described by Gottari with a uniformity of 1% over a region half the diameter [12]. However, these arrangements mostly rely on complex mathematical models and the impact of dimensional variations is not deeply considered.

In this paper, a design of a mini spherical coil system producing highly uniform field with a tolerance on dimensional variations is modeled, simulated and fabricated. This work uses a more empirical approach instead of complicated field potential expansion. Simple numerical calculation considering the field points at the center and away from the center is introduced to optimize the coil parameters. Finally two experiments are carried out to prove the feasibility of the design.

2. Coil Design

The compensation system consists of three sets of spherical coils orthogonal to each other and with slightly different diameters. A fundamental theorem demonstrates that a spherical shell with constant winding density can produce a complete uniform field. After experiment, a simple four couples’ coil system can well satisfy the requirement of uniformity and is also easy to be established. As shown in Figure 1, the left part gives a cross-sectional view of the real system where grooves are curved to accommodate the square windings. And in the right, each individual coil can be approximated by four couples symmetrically arranged loops satisfying a spherical constraint. By definition, Z-axis is the axis of rotational symmetry. For simplicity, we set the number of turns, current through each coil and the radius as one. Here, \( r_0 \times h \) is the dimension of a single square winding. And \( d_1 \) to \( d_4 \) are distances between the coils and the center, which are waiting to be optimized.

2.1 Parameter Optimization

The field distribution can be well described by Biot-Savart law in a cylindrical coordinate system. Since the coil loops are symmetric about the equatorial plane, when the \( +i \)-th (\( i = 1, 2, 3, 4 \)) turn on the positive hemisphere is located at \( z_i = +d_i \), there will be a counterpart at \( z_i = -d_i \). The magnetic field at an arbitrary point \( P(z, \rho) \) is a function of \( Z \) and \( R \) only. The field projection parallel to the axis which is contributed by the \( +i \)-th turn is written by:

\[
H_{\rho i}(z, \rho) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\sqrt{1 - d_i^2} - \rho \cos \phi}{\left[ \rho^2 + (1 - d_i^2) + (z - d_i)^2 - 2\rho \sqrt{1 - d_i^2} \cos \phi \right]^{\frac{3}{2}}} d\phi
\]

while the radial projection is written by:

![Figure 1. Cross section of one set of the spherical system. Grooves are curved on the surface to accommodate windings and distances \( d_1 \) to \( d_4 \) are parameters waiting to be optimized.](image)
where $\varphi$ is the angle between the projection of the selected point $P$ and a fixed radius on the coil plane. The total fields are:

$$H_{tot}(z, \rho) = \frac{1}{2\pi} \int_0^\pi \frac{(z - d_i) \cos \varphi}{\rho^2 + (1 - d_i^2) + (z - d_i)^2 - 2\rho \sqrt{1 - d_i^2} \cos \varphi} \rho^2 \, d\varphi \, dz$$  \hspace{1cm} (2)

When $\rho = 0$, the field on the $z$ axis which is used to compensate the earth field is written by:

$$H_z(z, 0) = H(z, 0) = \sum_{i=1}^{N} (1 - d_i^2)$$

$$H_z(z, 0) = \frac{1}{\left[1 - \left(1 - d_i^2 + (z - d_i)^2\right)^{1/2}\right]^{1/2}} + \frac{1}{\left[1 - \left(1 - d_i^2 + (z + d_i)^2\right)^{1/2}\right]^{1/2}}$$  \hspace{1cm} (6)

The field uniformity $n_z$ within the coils can be briefly described as:

$$n_z(z, \rho) = \frac{|H(z, \rho) - H(0, 0)|}{H(0, 0)}$$  \hspace{1cm} (7)

The uniform half length $L_z$ on $z$ axis is defined as:

$$L_z = \max \left[ |z| : \frac{|H_z(z, 0) - H_z(0, 0)|}{H_z(0, 0)} < n_z \right]$$  \hspace{1cm} (8)

Here, $n_z$ is the selected level of uniformity. Because a coil system can not simultaneously gain the largest uniform volume at different levels of uniformity [13], we set $n_z$ one part in $10^3$. This absolutely guarantees the fluctuation of the field is within 10 nT in the center zone where the alkali vapor cell is placed.

First consider an ordinary spherical system in equidistance arrangement. That means, the $i$th turn lies on $z_i = d_i = (2i - 1)b/2$ and $b = 2d_i$. Also, $b$ satisfies the constraint $1 - [b/2 + (N - 1)b] < b$ and $b/2 + (N - 1)b < 1$. $N = 4$ is the number of loops. Clearly seen from Figure 2, there are two extremum values of the uniform length on $z$ axis when $b$ varies from 2/9 to 2/7. We choose the value easier for manufacture: $b = 0.238$ and $L_z = 0.496$.

The uniform region can be further enlarged by slightly changing distances from locations of the equidistance mode. By numerical calculation, the optimal values are rewritten as: $d_1 = 0.136, d_2 = 0.414, d_3 = 0.551, d_4 = 0.874$ and $L_z = 0.649$.

In order to reduce parameter errors on the uniform length produced by fabrication, installation or nonmagnetic heating, it is worthy studying the parameter robustness. That means to find the parameters which can produce a large uniform region and at the same time the uniformity will not drop sharply when there is a small change in them. Thus, the field gradient is introduced. Because $d_3$ and $d_4$ are larger than $d_1$ and $d_2$, we vary $d_1, d_2$ discretely by 15% around their optimal values and $d_3, d_4$ by 10%. Write these discrete parameters as a set of vectors $\mathbf{d}_{i,j,k,l} = (d_1, d_2, d_3, d_4)$. The corresponding uniform half length is $L_z(\mathbf{d}_{i,j,k,l})$. Then, we define the field gradient as:

$$G(\mathbf{d}_{i,j,k,l}) = \frac{L_z(\mathbf{d}_{i+1,j,k,l}) - L_z(\mathbf{d}_{i,j,k,l})}{\Delta d_i}, \frac{L_z(\mathbf{d}_{i,j+1,k,l}) - L_z(\mathbf{d}_{i,j,k,l})}{\Delta d_j}, \frac{L_z(\mathbf{d}_{i,j,k+1,l}) - L_z(\mathbf{d}_{i,j,k,l})}{\Delta d_k}, \frac{L_z(\mathbf{d}_{i,j,k,l+1}) - L_z(\mathbf{d}_{i,j,k,l})}{\Delta d_l}$$  \hspace{1cm} (9)

Figure 2. The extremum values of the uniform half length on $z$ axis when $b$ varies. The smaller value of $b$ (triangle point) should be chosen so that more space can be saved for heating and pumping laser holes at the end of the shell.
which can reflect the varying extend of the uniform region. $\Delta d_i$ is the discrete step length of each loop ($i = 1, 2, 3, 4$).

By setting thresholds, parameters are selected while the uniform region is large enough and the gradient is in a reasonable range:

$$\begin{align*}
\begin{cases}
L_T(d_{i,j,k,l}) > L_T \\
|G(d_{i,j,k,l})| < G_T
\end{cases}
\end{align*}$$

$L_T$ and $G_T$ are thresholds on the uniform half length and the field gradient, respectively. They can be chosen according to different experimental requirements. Here we use $L_T = 0.5$ and $G_T = 35$. The step length is 0.01 for $d_1$ and $d_4$, and 0.005 for $d_2$ and $d_3$, since the system is more sensitive to the variation of latter distances. The final parameters are determined either by:

$$\{(d_1, d_2, d_3, d_4): \min_{i,j,k,l} \left| G(d_{i,j,k,l}) \right| \}$$

(12)

to gain the best robustness or:

$$\{(d_1, d_2, d_3, d_4): \max_{i,j,k,l} L_T(d_{i,j,k,l}) \}$$

(13)

to gain the best uniformity.

Based on the first criterion, the solution gives a pretty uniform field generating system:

- $d_1 = 0.125$
- $d_2 = 0.372$
- $d_3 = 0.581$
- $d_4 = 0.856$
- $L_T = 0.506$

### 2.2 Simulation of the Magnetic Field

In the method above, we only use the uniform half length on $z$ axis as a benchmark to evaluate the field uniformity. If we want to ensure the magnetic field inside the center cell is as uniform as possible, the field behavior of the whole system should be analyzed. Using (3), (4) and (5), the magnetic computation of the first quadrant is performed in Figure 3. Figure 3(a) shows the uniformity mapping. Inside the region of about $0.35R \times 0.4Z$, the uniformity is below $10^{-3}$ which is sufficiently large to place the cell. The central field derivation is below $10^{-4}$. This fully guarantees the field within the effective volume of the magnetometer is under 10 nT, even considering the diameter of the laser. Figure 3(b) displays $H_r(z, \rho)/H(0, 0)$, investigating effects of the radial field component. The ratio of $H_r(z, \rho)$ to $H(0, 0)$ inside the region of about $0.3R \times 0.5Z$ is less than $1 \times 10^{-3}$. This means when the field along $z$ axis is compensated, the system produces negligible disturbances in its perpendicular direction.

The advantage of the parameters obtained by using the field gradient is obvious in Figure 4. It plots the uniform half length when critical parameters $d_2$ and $d_3$ vary around their optimal values (white spots). Compared to

![Figure 3. Magnetic field distribution. (a) Uniformity mapping; (b) $H_r/H$. The radial component has negligible effect when the dominant field is along $z$ axis.](image)
the equidistance structure, the gradient method has a larger uniform region (Figure 4(a)). Also, clearly seen from Figure 4(b) where optimal values of the equidistance is located at the color fast-changing ‘edge’ (the uniform length drops quickly from about 0.5 to 0.3), the gradient method is robust against parameter variations, since it stands on the summit of two gentle ‘slopes’.

2.3 Further Discussion

The model discussed above neglects the cross sections of the windings. By taking them into consideration, a more accurate model can be constructed. Hypothesize the cross section is \( r_0 h \), shown in Figure 1. Then the current density through each winding is \( 1/(r_0 h) \). The field component on \( z \) axis contributed by the \( +i \)th turn is written as:

\[
H_{z}(z,0) = \frac{1}{r_0 h} \left[ z - d_i + \frac{h}{2} \right] \ln \frac{\sqrt{1-d_i^2 + \frac{r_0}{2}} + \sqrt{1-d_i^2 + \frac{r_0}{2}}^2 + \left( z - d_i + \frac{h}{2} \right)}{\sqrt{1-d_i^2 - \frac{r_0}{2}} + \sqrt{1-d_i^2 - \frac{r_0}{2}}^2 + \left( z - d_i + \frac{h}{2} \right)}
\]

Assume \( r_0 \) and \( h \) are small enough and use Taylor expansion, the field component on \( z \) axis contributed by the \( +i \)th turn can be simplified as:

\[
\frac{1-d_i^2}{\left[ 1-d_i^2 + (z-d_i)^2 \right]^{3/2}} + o(3)
\]

(15)

\( o(3) \) is the 3rd order infinitesimal of \( r_0 \) and \( h \). The expression on the left side of (15) is exactly the one of (6) which means the simplified model is accurate enough to describe the system.

3. Coil Construction and Experimental Test

The shell body is made of aluminum to attenuate 50 Hz magnetic noise. In Figure 5, three shells about 3 mm thickness are in a nesting structure with axes perpendi-

Figure 4. Comparison of the uniform half length. (a) Method using gradient has better uniformity and a good tolerance on dimensional variations; (b) Uniform length drops sharply in the equidistance structure.

Figure 5. Construction of the spherical coils. A is holes for single laser beam and nonmagnetic heaters, B is the center chamber and C is three orthogonal coil shells.
cular to each other to compensate the three dimensional fields. The orthogonality is fixed by dowels and the diameter of the largest coil set is 7 cm. Four couples of grooves are curved on the surface to accommodate windings and there are $5 \times 5$ turns in each groove. Holes for the nonmagnetic heating and the pumping laser are drilled to satisfy the requirement of the single beam scheme. The alkali metal cell is placed in the center chamber to sense outer field variation.

The field uniformity is measured by fluxgate magnetometer CTM-6W with about 40 mA current through the coils. Figure 6 shows the theoretical prediction together with the measurement results, which verifies the validity of the design. The fluctuation inside the area within 0.1% uniformity is generated by stray earth field, which does not influence the performance of the final atomic magnetometer. Given the averaging effect due to the length of the magnetometer probe [14], the experiment results achieve even better uniform region than the theoretical analysis.

This compensation structure is integrated into our unshielded vector atomic magnetometer system. We use the single beam arrangement illustrated in Figure 7. A vapor cell containing one drop of Cs atoms and 30 Torr N$_2$ for quenching is heated up to 52 °C by nonmagnetic heaters with 20 kHz alternating current. A multimode DFB laser with 45 mA current pumps the atoms in the cell. The precession of the atomic spin carrying three axes magnetic information is detected by the pumping laser tuned to the center of Cs D1 line and then received by the photodetector (PD). The output of PD is proportional to $P_z$, the polarization of atoms along $z$ axis, whose evolution is well described by the Block equation. The steady-state solution is:

$$P_z = P_0 \frac{B_z^2 + (R_{rot}/\gamma)^2}{B_x^2 + B_y^2 + B_z^2 + (R_{rot}/\gamma)^2}$$

where $P_0$ is the equilibrium polarization along the pumping beam direction; $B_x$, $B_y$, and $B_z$ are the vectors of the magnetic field; $R_{rot}$ is the sum of the pumping rate and the relaxation rate; $\gamma$ is the gyromagnetic ratio of electron. From the equation, we can see that when the field in $x$ or $y$ reaches zero, the signal received by PD is maximized, but in $z$, the signal minimized. Based on such a principle, a feedback algorithm is introduced to make the spherical coils produce reverse fields to null earth fields in all three directions [15]. By symmetrically adding a weak sweeping field across zero point in $y$ axis, the resonance curve is shown in Figure 8. It is fitted by a Lorentzian curve. And the half linewidth is 469 nT. When compensation process is completed, currents through each coil in $x$, $y$ and $z$ are 10.9 mA, 26.9 mA and 18.0 mA, respectively. By multiplying the coil constants, we can deduce that the earth fields in three axes are $16.7 \times 10^{-9}$ T, $34.3 \times 10^{-9}$ T and $27.7 \times 10^{-9}$ T, which is consistent with the result of the fluxgate magnetometer.

![Figure 6](image6.png) **Figure 6.** Agreement is very close between the theoretical analysis and the field output of the intermediate coil set on $z$ axis. The data has been averaged for five times.

![Figure 7](image7.png) **Figure 7.** Experimental setup of Cs atom magnetometer. Polarization is detected by the pumping laser. MCU produces controlled current on the coils to null the earth field.
4. Conclusions

The present work has demonstrated a compact compensation coils used for atomic magnetometers. Mathematical models are constructed and the gradient method is introduced to optimize the parameters. The length within $10^{-3}$ uniform level is over one half radius and it is robust against parameter variations. The measurement of the prototype by fluxgate magnetometer is in good agreement with the calculation. And the integration into the unshielded vector atomic magnetometer further demonstrates its feasibility.

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