Heat Transfer Performance in Double-Pass Flat-Plate Heat Exchangers with External Recycle

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Abstract

The effect of external recycle on the performance in double-pass rectangular heat exchangers was investigated. Theoretical equations for heat transfer rate were derived based on the effectiveness-effect analysis. In contrast to a single-pass device without recycle, improvement in heat transfer of more than 20% is achievable if parallel-flow heat exchange is operated in a double-pass device of the same size with external recycle which provides the increase of fluid velocity, resulting in reduction of heat-transfer resistance. It is found that the performances obtained in the double-pass devices with external recycle operating under concurrent and countercurrent flows are exactly the same.

Key Words: Heat Transfer, External Recycle, Effectiveness-Factor Analysis, Double Pass, Heat Exchanger, Performance Improved

1. Introduction

Heat and mass transfer in a duct has long been a problem of interest in engineering. The complete analysis of recuperator processes without recycle was already given in literature [1]. The heat-transfer problems for uniform wall temperature, or uniform wall heat-flux, with recycle at the ends were investigated [2–7].

The application of recycle operation indeed has much influence on heat transfer [8] and mass transfer [9–11], which in turn plays a significant role on the design, calculation, and operation of the equipment. Actually, the application of recycle operation to heat and mass exchangers has two conflicting effects: one is the effect of increase in fluid velocity, which is good for increasing the heat- or mass-transfer efficiency while the other is the effect of decrease in driving force (temperature or concentration difference) due to remixing, which is bad for heat or mass transfer. It is the purpose of present study to investigate the effect of external recycle on the performance in double-pass parallel-plate heat exchangers with the expectation that the desirable effect of increase in fluid velocity can overcome the undesirable effect of decrease in temperature in double-pass parallel-plate heat exchangers.

2. Theory

Heat and mass exchangers may be very different in design and construction, but in principle, the only difference is in the relative direction of each of the two fluids, a and b. Accordingly, a distinction is made between parallel flow and cross flow. Two fluids in parallel flow may be concurrent or countercurrent.

2.1 Overall Energy Balance

The assumptions made in this analysis are as follows: steady state, uniform temperature and velocity over the cross-section of flow, constant rates of flow, a constant overall heat transfer coefficient, and negligible heat loss by well insulation.

A energy balance for fluid over a single-pass or a double-pass heat exchanger operated with the flow rates of \(q_a\) and \(q_b\) and the temperature of \(t_{ai}\) and \(t_{bi}\) at the inlet
and those of $t_{a,e}$ and $t_{b,e}$ at the outlet, as shown in Figures 1 and 2, gives the total heat transfer rate $Q$ as

$$Q = m_a(t_{a,i} - t_{a,e}) = m_b(t_{b,i} - t_{b,e}) = m_a(1 + R)(t_{a,i} - t_{a,e}),$$  \hspace{1cm} (1)

\text{if } t_{a,i} > t_{b,e}

or

$$Q = m_a(t_{a,i} - t_{a,e}) = m_b(t_{b,i} - t_{b,e}) = m_a(1 + R)(t_{a,e} - t_{a,e}),$$  \hspace{1cm} (2)

\text{if } t_{a,i} < t_{b,e}

where the time rate of heat capacities are defined by

$$m_a = \rho_a C_p a$$  \hspace{1cm} (3)

$$m_b = \rho_b C_p b$$  \hspace{1cm} (4)

It should be mentioned that the effect of recycle operation with reflux ratio $R$ on the performance, which was not consider in Underwood’s work [12], will be thoroughly discussed in present study. The mixed inlet temperature $t_{a,i}$ arose due to the recycle operation will be determined by one more energy balance, in terms of the known inlet temperature $t_{a,i}$ and reflux ratio $R$, as well as the unknown outlet temperature $t_{a,e}$.

2.2 Temperature Distributions in Double-Pass Module with Recycle

The double-pass parallel-plate heat-exchanger (Figure 2) of fixed configuration is similar to the single-pass one (Figure 1), such as having the heat-transfer medium of same size (with length L and width W) inserted in parallel between two parallel plates with same distance from them to divide the conduit into two channels (channels a and b) of same height $h$, except that an insulated plate with negligible thickness is placed in vertical to the upper plate and the heat-transfer medium at the center line of channel a (fluid a), to divide the channel into two subchannels (subchannels $a_1$ and $a_2$) of same width ($W/2$), as shown in Figure 2. Being a device of double flow in channel a, there are two different flow patterns for operation. The system with concurrent flow first and then followed by countercurrent flow, as shown in Figure 2(a) called A-B type. On the other hand, Figure 2(b) illustrates the system with countercurrent flow first and then followed by concurrent flow, called B-A type.

2.2.1 A-B type Module

The schematic diagram in Figure 2(a) may serve to explain the nomenclature to be employed for the recycle-type double-pass system with concurrent flow first and then followed by countercurrent flow. Energy balance over the right-hand section of the heat exchanger is

$$m_b(t_{b,e} - t_b) = m_a(1 + R)(t_{a,i} - t_{a,e})$$  \hspace{1cm} (5)

Energy balances over a differential length $dx$ are

Figure 1. Single-pass parallel-plate heat exchanger.

Figure 2. Double-pass parallel-plate heat exchanger with external recycle.
where \( t_{a,1} \) and \( t_{a,2} \) are the fluid temperatures in sub-channels \( a_1 \) and \( a_2 \), respectively, \( t_b \) denotes the fluid temperature in channel \( b \), and the overall heat-transfer coefficient \( U \) for laminar flow in the all channels with the neglect of heat-transfer resistance through the heat-transfer medium may be defined by [13].

\[
\frac{1}{U} = \frac{1}{h_{a,1}} + \frac{1}{h_{a,2}} + \frac{1}{h_b}
\] (9)

where

\[
h_{a,1} = h_{a,2} = 1.75(k_a / D_{eq,a}) \left[ \frac{m_a(1 + R)}{k_a L} \right]^{1/2}
\] (10)

\[
h_b = 1.75(k_b / D_{eq,b}) \left[ \frac{m_b}{k_b L} \right]^{1/2}
\] (11)

\[D_{eq,a} = D_{eq,a} = D_{eq,b} \approx 2h_a \text{ if } w >> h
\] (12)

Substitution of Eqs. (7) and (8) into Eq. (6) yields

\[
\frac{dt_b}{dx} = -\left( \frac{U W}{m_b} \right) t_b + \left( \frac{U W}{2m_b} \right) (t_{a,1} + t_{a,2})
\] (13)

Differentiating Eq. (13) with respect to \( x \) gives:

\[
\frac{d^2t_b}{dx^2} = -\left( \frac{U W}{m_b} \right) \frac{dt_b}{dx} + \left( \frac{U W}{2m_b} \right) \left[ \frac{dt_{a,1}}{dx} + \frac{dt_{a,2}}{dx} \right]
\] (14)

Substitution of Eqs. (7) and (8) into Eq. (14) results in

\[
\frac{d^2t_b}{dx^2} = -\left( \frac{U W}{m_b} \right) \frac{dt_b}{dx} + \left[ \frac{U^2 W^2}{4m_b m_b(1 + R)} \right] (t_{a,1} - t_{a,2})
\] (15)

Substituting from Eq. (5) and using the following dimensionless groups:

\[
\xi = \frac{x}{L}
\] (16)
Applying Eq. (23) and the following boundary conditions:

\[ \text{at } x = 0 \ (\xi = 0), \ t_{a,1} = t_{a,j}^0 \text{ and } t_{a,2} = t_{a,e} \]  
\[ (29), (30) \]

Eq. (28) becomes:

\[ A\alpha + B\beta = -mt \left( t_{a,j}^0 + t_{a,e} \right) / 2 \]  
\[ (31) \]

By Substituting from Eqs. (25) and (26) into Eq. (31) with the use of the following dimensionless groups:

\[ \xi_{b,e} = \frac{t_{b,e} - t_{b,j}}{t_{a,e} - t_{b,j}} \]  
\[ (32) \]

\[ \xi_{a,e} = \frac{t_{a,e} - t_{a,j}^0}{t_{a,e} - t_{b,j}} \]  
\[ (33) \]

\[ = - \left( \frac{\xi_{b,e}}{\ell} \right) \]  
\[ (34) \]

one has

\[ - \frac{a\epsilon^\beta - \beta e^\alpha}{e^\beta - e^\alpha} \xi_{b,e} = -(mt) \left( \frac{1}{2\ell} \right) - \xi_{b,e} - 1 \]  
\[ (35) \]

where Eq. (34) is obtained from Eq. (1). Finally, the dimensionless outlet temperature of fluid b may be rewritten from Eq. (35) as

\[ \xi_{b,e} = \left( \frac{1}{2\ell} \right) - \frac{a\epsilon^\beta - \beta e^\alpha}{mt(e^\beta - e^\alpha)} \]  
\[ (36) \]

### 2.2.2 B-A Type Module

Figure 2(b) shows a schematic diagram of the system with countercurrent flow first and then followed by concurrent flow, which is exactly the same as Figure 2(a) except that in this figure the flow directions of fluid b are in opposite. The energy balance over the right-hand section of the heat exchanger is

\[ m_a(t_b - t_{b,j}) = m_a(1 + R)(t_{a,j}^0 - t_{a,2}) \]  
\[ (37) \]

Energy balances over a differential length \( dx \) are

\[ m_a dt_b = m_a(1 + R)(dt_{a,j}^0 - dt_{a,2}) \]  
\[ (38) \]

\[ m_a(1 + R)dt_{a,j} = -U \left( \frac{W}{2} \right) (t_{a,j} - t_b) dx \]  
\[ (39) \]

\[ m_a(1 + R)dt_{a,2} = U \left( \frac{W}{2} \right) (t_{a,2} - t_b) dx \]  
\[ (40) \]

Eqs. (37)–(40) were solved by following the same procedure from Eq. (9) through Eq. (36) with the use of the following boundary conditions:

\[ \text{at } x = 0, \ t_{a,1} = t_{a,j}^0 \]  
\[ (41) \]

\[ t_{a,2} = t_{a,e} \]  
\[ (42) \]

\[ t_b = t_{b,j} \]  
\[ (43) \]

\[ \text{at } x = L, \ t_b = t_{b,e} \]  
\[ (44) \]

The result is

\[ \xi_{b,e} = \left[ 1 + \left( \frac{1}{2\ell} \right) + \frac{a\epsilon^\beta - \beta e^\alpha}{mt(e^\beta - e^\alpha)} \right]^{-1} \]  
\[ (45) \]

It is easy to know from Eq. (21) and (22) that

\[ mt = -(a + \beta) \]  
\[ (46) \]

With this relation Eq. (45) can be rearranged turning to Eq. (36). Thus, for double-pass parallel-flow heat exchanger, both flow types, A-B type and B-A type, have same result of performance.

### 2.3 Heat-Transfer Rate in Double-Pass Modules with Recycle

The total heat-transfer rates for both flow patterns, A-B system and B-A system, can be calculated from Eq. (1), in which the outlet temperatures, \( t_{a,e} \) and \( t_{b,e} \), can be calculated from Eqs. (32)–(34) once \( \xi_{b,e} \) is known from Eq. (36), i.e.,

\[ t_{a,e} = t_{a,j}^0 + \xi_{a,e}(t_{a,j}^0 - t_{b,j}) \]  
\[ (47) \]

\[ = t_{a,j}^0 - \left( \frac{\xi_{b,e}}{\ell} \right) \left( t_{a,j}^0 - t_{b,j} \right) \]  
\[ (48) \]
It is mentioned in the previous section that the outlet temperatures of A-B and B-A systems are the same as shown by Eqs. (36) and (45), with the use of Eq. (46). Therefore, the equations for calculating the total heat-transfer rates in such two double-pass parallel-flow modules with external recycles can be derived from Eqs. (1) and (49), and the results are the same as

\[ Q_z = Q_{z,A-B} = Q_{z,B-A} = (m_b \xi_b \epsilon) [t_{e,j} - t_{b,j}] \]  

(50)

Inspection of Eq. (50) shows that the mixed inlet temperature \( t_{e,j} \) is not specified prior. Mathematically, one more relation is needed for the determination of this value. For this purpose, an energy balance for fluids at the inlet of phase a is readily obtained as

\[ t_{a,j} + R t_{a,x} = (1 + R)t_{e,j} \]  

(51)

Substituting Eq. (47) into Eq. (51), one has

\[ t_{a,x} = \frac{t_{a,j} - R \xi_b t_{b,j}}{1 - R \xi_b} \]  

(52)

and the outlet temperature \( t_{a,e} \) is ready obtainable from Eq. (47) with the use of Eq. (52). Finally, the heat-transfer rate in double-pass modules with external recycle is derived by substituting Eq. (52) into Eq. (50), as

\[ Q_z = \frac{m_b \xi_b \epsilon}{1 + (R \xi_b \epsilon)} [t_{e,j} - t_{b,j}] \]  

(53)

2.4 Heat-Exchanger Efficiency in Double-Pass Modules with Recycle

The heat-exchanger efficiency may be defined by

\[ \eta = \frac{\text{actual heat-transfer rate}}{\text{maximum heat-transfer rate}} \]  

(54)

For the case of cooling \( (t_{a,j} > t_{b,j}) \)

\[ \eta_c = \frac{Q}{U(W/\xi_b)(t_{a,j} - t_{b,j})} = \frac{m_b(t_{a,j} - t_{a,x})}{US(t_{b,j} - t_{b,i})} = \frac{m_b[(t_{a,j} / t_{b,j}) - (t_{a,x} / t_{b,x})]}{US[t_{b,j} / t_{b,i} - 1]} \]  

(55)

For the case of heating \( (t_{a,j} < t_{b,j}) \)

\[ \eta_h = \frac{m_b(t_{a,x} - t_{a,j})}{US(t_{b,j} - t_{b,i})} = \eta_c = \eta \]  

(56)

where the term \( (t_{a,x} / t_{b,i}) \) in Eq. (55) can be obtained from Eqs. (48) by substituting Eq. (52), as

\[ \frac{t_{a,x}}{t_{b,i}} = \frac{(t_{a,j} / t_{b,i})[1 - (\xi_b \epsilon / \ell)] + (\xi_b \epsilon / \ell)(1 + R)}{1 + (R \xi_b \epsilon / \ell)} \]  

(57)

Some of the graphical representations for the effectiveness factor \( \eta (US/m_a) \) vs. reflux ratio \( R \) with \( (m_a/m_b) \) as parameter in the double-pass devices with external recycle are given in Figures 3–5. Once \( \eta \) is known from these figures, the heat-transfer rates in these devices are

\[ \frac{t_{a,j}}{t_{b,i}} = 2 \text{ or } 1/2 \]

\[ m_a = 0.1664 \text{ kJ/K} \cdot \text{s} \]

\[ m_a = 0.3328 \text{ kJ/K} \cdot \text{s} \]

Figure 3. \( \eta (US/m_a) \) vs. \( R \) with \( (m_a/m_b) \) as parameter for \( (t_{a,j}/t_{b,i}) = 2 \text{ or } 1/2 \).
3. Numerical Calculation

3.1 Heat-Transfer Rate for Single-Pass Heat-Exchangers

In order to show the benefit of recycle operation, the performance in a single-pass device without recycle should be briefly described. Figure 1 illustrates the flows ready obtained by

\[ Q = \eta \frac{US(t_{a,j} - t_{b,j})}{R} \quad \text{for} \quad t_{a,j} > t_{b,j} \]  

(58)

and

\[ Q = \eta \frac{US(t_{a,j} - t_{b,j})}{R} \quad \text{for} \quad t_{a,j} < t_{b,j} \]  

(59)

in single-pass parallel-plate heat-exchangers. The total heat-transfer rates in single-pass devices for concurrent \((Q_{c,i})\) and countercurrent \((Q_{c,ii})\) flows, which were already derived by Underwood [12] without recycle operation, are

\[ Q_{c,i} = US(\Delta m)_{c,i} \]  

(60)

\[ Q_{c,ii} = US(\Delta m)_{c,ii} \]  

(61)

where

\[ \Delta m_{c,i} = \frac{(t_{a,i} - t_{b,i}) - (t_{a,j} - t_{b,j})}{\ln[(t_{a,i} - t_{b,i})/(t_{a,j} - t_{b,j})]} \]  

(62)
Combination of Eqs. (60) and (62) with the use of Eq. (1), in which \( Q \) is replaced by \( Q_{1,a} \) to eliminate \( t_{a,e} \) and \( t_{b,e} \) yields:

\[
Q_{1,a} = \frac{(t_{a,e} - t_{b,e}) \left\{ 1 - e^{-\left(\frac{3}{(1/m_a) + (1/m_b)}\right)} \right\}}{(1/m_a) + (1/m_b)}, \quad t_{a,e} > t_{b,e}
\]  

(64)

Similarly, from Eqs. (61) and (63) as well as Eq. (1) in which \( Q \) is replaced by \( Q_{1,b} \), one has

\[
Q_{1,b} = \frac{(t_{a,e} - t_{b,e}) \left\{ 1 - e^{-\left(\frac{3}{(1/m_a) - (1/m_b)}\right)} \right\}}{(1/m_a) - (1/m_b)e^{-\left(\frac{3}{(1/m_a) - (1/m_b)}\right)}}, \quad t_{a,e} > t_{b,e}
\]  

(65)

### 3.2 Numerical Example

For the purpose of illustration, let us employ some numerical values as follows:

- The parallel-flow heat exchanger (\( L = 1.2 \) m, \( W = 0.2 \) m, \( h = 0.02 \) m) is insulated except the heat-transfer medium between channel a and channel b, and the heat loss is neglected.

- Hot and cold waters are employed as the working fluids flowing in channels a and b (\( t_{a,i} > t_{b,i} \)), or channels b and a (\( t_{a,i} < t_{b,i} \)), respectively, with \( t_{av} = (t_{a,i} + t_{b,i})/2 = 40 \) °C, \( \rho_a = \rho_b = 992 \) kg/m³, \( C_{pa} = C_{pb} = 4.181 \) kJ/kg·K, \( k_a = k_b = 0.629 \) W/m·K, and \( \mu_a = \mu_b = 0.656 \) kg/m·s [14].

With the use of above numerical value given, the outlet temperatures and heat-transfer rates were calculated from Eqs. (47) and (53), respectively, with the specified values of flow rates, \( q_a \) and \( q_b \), inlet temperatures, \( t_{a,i} \) and \( t_{b,i} \), and reflux ratio \( R \). The results are presented in Tables 1–3.

### 4. Results and Discussion

#### 4.1 Effects of Operating Parameters on \( Q \)

As seen in Tables 1–3, the heat-transfer rates, \( Q \) (\( Q_{1,a} \), \( Q_{1,b} \) and \( Q_2 \)), increases when \( q_a \) or \( (q_b/q_a) \), or \( R \) increases. Further, \( Q \) increases with the temperature differences of fluids at the inlet, \( (t_{a,i} - t_{b,i}) \) for cooling operation and \( (t_{b,i} - t_{a,i}) \) for heating operation, and the values are the same for the same inlet temperature-differences in both operations. As indicated in Eqs. (1) and (2), the above tendency of increasing \( Q \) with operating parameters will also lead to the increase in the fall of temperature, \( (t_{a,i} - t_{a,e}) \), for cooling operation, as well as to the increase in the rise of temperature, \( (t_{a,e} - t_{b,i}) \), for heating operation. The heat-transfer rate \( Q_2 \) in the double-pass devices, either with or without recycle, are the same for both A-B type and B-A type operations, as shown by Eqs. (36) and (45) with the use of the relation indicated by Eq. (46) while, under single-pass operation, the heat-transfer rates \( Q_{1,a} \) obtained in a countercurrent-flow device are rather larger than those \( Q_{1,a} \) obtained in a concurrent-flow one. However, both \( Q_{1,a} \) and \( Q_{1,b} \) are unchanged as the inlet volume rates in fluid a and fluid b are interchanged. Taking the case of \( t_{a,i} = 53.3 \) °C and \( t_{b,i} \) 26.7 °C, as well as \( t_{a,i} = 26.7 \) °C and \( t_{b,i} = 53.3 \) °C, in

### Table 1. Numerical results for: (a) cooling in recycle phase (phase a) with \( t_{a,i} = 53.3 \) °C and \( t_{b,i} = 26.7 \) °C; (b) heating in recycle phase with \( t_{a,i} = 26.7 \) °C and \( t_{b,i} = 53 \) °C

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4.2 Effects of Operating Parameters on $\eta Q$

Figures 3–5 show that the product of heat transfer rate and heat exchanger efficiency, $\eta Q$, increases when $m_a$ or ($m_i/m_o$), or $R$ increases, and the results are the same as the value of $(t_{ia}/t_{bo})$ in cooling operation is equal to the value of $(t_{bo}/t_{ia})$ in heating operation.

4.3 Improvement in Performance

The application of recycle operation to a heat exchanger not only creates the desirable effect of increase in fluid velocity but also the undesirable effect of feed mixing with the recycling stream. The increase of fluid velocity enhances the heat-transfer coefficient while the remixing of inlet fluid lowers the driving force (temperature difference) of heat transfer in the heat exchanger. It is shown in Tables 1–3 that the heat-transfer rates increase with increasing reflux ratio $R$. Therefore, the contribution in increasing fluid-velocity effect compensates for the undesirable remixing effect when the recycle is applied, and the performance in the recycled model developed in present study overcomes that in the Underwood's work without recycle ($R = 0$). The improvement in heat transfer by operating a double-pass device with external recycle is best illustrated by calculating the percent increase in heat-transfer rate based on the operation in a single-pass countercurrent-flow device of same size.
but without recycle,

$$I = \frac{Q_x - Q_y}{Q_x}$$  \hfill (66)

Substitution of Eqs. (53) and (65) into Eq. (66) yields.

$$I = \frac{m_b \tau \left\{ (1/m_b) - (1/m_a) e^{-\frac{1}{\alpha}} \right\}}{[1 + (R \tau \alpha / t)] \left\{ 1 - e^{-\frac{1}{\alpha}} \right\}} = 1$$  \hfill (67)

It is seen in Eq. (67) that the improvement in heat-transfer rate $I$ is independent of the inlet temperature-differences, $(t_{i1} - t_{i2})$ for cooling or $(t_{i2} - t_{i1})$ for heating. The numerical results of $I$ are listed in Table 4, which increase with $m_a$, or $(m_b/m_a)$, or $R$.

### 4.4 Hydraulic-Dissipated Energy

Though considerable improvement in heat transfer can be obtained by double-pass operation coupled with recycle, the hydraulic-dissipated energy due to the friction loss of fluid flow should be discussed. The hydraulic-dissipated power in a parallel-plate channel may be estimated by

$$H = \text{(volume flow rate)} \Delta p$$  \hfill (68)

If laminar flow in the flow channel is assumed, the pressure drop through the flow channel is [13]:

$$\Delta p = \frac{12\mu L}{h^2} \times \text{(cross-section area of channel)} \hfill (69)$$

Accordingly, we have the hydraulic-dissipated powers for single-Pass operation without recycle and double-pass operation with recycle, respectively, as

$$H_1 = \frac{12\mu L q_a^2}{h^2 W} + \frac{12\mu_L L q_b^2}{h^2 W} = \frac{12\mu L}{h^2 W} (q_a^2 + q_b^2)$$  \hfill (70)

$$H_2 = \frac{12\mu_L L q_a (1 + R)^2}{h^2 (W/2)} + \frac{12\mu_L L q_b^2}{h^2 W} = \frac{12\mu L}{h^2 W} (4q_a (1 + R)^2 + q_b^2)$$  \hfill (71)

Here the numerical values presented in section 3.2 will be employed again. Further, the conversion factor, 1 hp = 745.7 Nm/s, for power unit will be used. The enhancement of hydraulic-dissipated energy in the double-pass device based on that in the single-pass one may be calculated by

$$E = \frac{H_2 - H_1}{H_1} = \frac{4q_a^2 (1 + R)^2 + q_b^2}{q_a^2 + q_b^2}$$  \hfill (72)

Some of the results are also listed in Table 4. It is shown in this table that $E$ increases when $q_a/q_b$ or $R$ increases. For the two typical cases with $q_a = 16 \times 10^5$ m$^3$/s and $q_b = 4 \times 10^5$ m$^3$/s, $I = 5.89\%$ and $H_2 = (E + 1)H_1 = (14.1 + 1)(2.154 \times 10^4) = 3.25 \times 10^4$ hp for $R = 1$, while for $R = 7$, $I = 13.7\%$ and $H_2 = (240 + 1)(2.154 \times 10^4) = 0.103$ hp. Therefore, the increase of reflux ratio should be suitably controlled to avoid the high hydraulic-dissipated power consumption.

### 4.4 Examination for Application Limit

The Reynolds number in the flow channels $a_1$ and $a_2$ for double-pass operation with recycle may be defined by

| Table 4. Improvement in performance and enhancement in hydraulic-dissipated energy |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $q_a \times 10^4$ (m$^3$/s) | $q_b \times 10^4$ (m$^3$/s) | $H_1 \times 10^4$ (hp) | $I$ (%) | $E$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $R = 0$ | $R = 1$ | $R = 3$ | $R = 5$ | $R = 7$ | $R = 0$ | $R = 1$ | $R = 3$ | $R = 5$ | $R = 7$ |
| 4 | 4 | 0.253 | 0.09 | 2.80 | 5.48 | 7.34 | 8.71 | 1.5 | 7.5 | 31.5 | 71.5 | 127.5 |
| 8 | 0.634 | 1.66 | 4.24 | 7.56 | 9.85 | 11.55 | 15.45 | 0.6 | 3.0 | 12.6 | 28.6 | 51.0 |
| 16 | 2.154 | 2.82 | 5.38 | 9.24 | 11.92 | 13.90 | 18.20 | 0.2 | 0.9 | 3.7 | 8.4 | 15.0 |
| 8 | 4 | 0.634 | 0.92 | 4.99 | 8.71 | 10.94 | 12.50 | 2.4 | 12.0 | 50.4 | 114.4 | 204.0 |
| 16 | 8 | 1.014 | 2.15 | 6.73 | 11.48 | 14.36 | 16.38 | 1.5 | 7.5 | 31.5 | 71.5 | 127.5 |
| 16 | 2.534 | 3.17 | 8.26 | 13.95 | 17.44 | 11.60 | 1.5 | 7.5 | 31.5 | 71.5 | 127.5 |
| 16 | 4 | 2.154 | 1.34 | 5.89 | 9.91 | 12.17 | 13.70 | 15.45 | 1.5 | 7.5 | 31.5 | 71.5 | 127.5 |
| 8 | 2.534 | 2.33 | 7.84 | 13.12 | 16.14 | 18.20 | 15.45 | 2.4 | 12.0 | 50.4 | 114.4 | 204.0 |
| 16 | 4.055 | 3.20 | 9.65 | 16.19 | 19.97 | 22.59 | 1.5 | 7.5 | 31.5 | 71.5 | 127.5 |
Re = \frac{2h(a_\text{in})(1 + R)(W/2)h_{\text{b}}}{\mu_a} = \frac{4q_a(1 + R)\rho W}{\mu_a} \quad (73)

For the typical case that \( q_a = 1.6 \times 10^4 \text{ m}^3/\text{s}, W = 0.2, R = 7, \rho_y = 994 \text{ kg/m}^3 \) and \( \mu_a = 0.656 \text{ kg/m} \cdot \text{s} \), one has Re = 1.55. Therefore, the assumption of laminar flow is confirmed.

5. Conclusions

The heat transfer through a rectangular parallel-flow heat exchanger with external reflux has been analyzed under concurrent-flow and countercurrent-flow operations. The ordinary differential equations for fluid temperature distributions in the flow channels (a_1, a_2, and b) were derived based on energy balances with the assumptions of laminar flow through the flow channels as well as uniform temperatures and velocities over cross sections of flow channels for mathematical simplicity. The last assumption may lead to somewhat inaccurate results, especially for the temperature in channel b. The outlet temperatures were solved simultaneously from the governing equations (Eqs. (5)–(8) for A-B type and Eqs. (37)–(40) for B-A type) with the use of appropriate boundary conditions (Eqs. (23), (24), (29) and (30) for A-B type and Eqs. (41)–(44) for B-A type). The graphical results for heat-exchanger efficiency are plotted in Figures 3–5 and the numerical results for \( t_{a,\text{in}}, Q_{1,\text{in}}, Q_{1,\text{out}}, Q_{2}, \) and \( Q_2 \) are listed in Tables 1–3. It was found that the heat-transfer rates in all devices increases with \( q_a, q_b, \text{ and } R \), as well as with \( (t_{a,\text{in}} - t_{b,\text{in}}) \) for cooling operating and with \( (t_{b,\text{in}} - t_{a,\text{in}}) \) for heating operation, in phase a, and that the heat-transfer rates are the same for cooling \( (t_{a,\text{in}} > t_{b,\text{in}}) \) and heating \( (t_{a,\text{in}} < t_{b,\text{in}}) \) operations if \( (t_{a,\text{in}} - t_{b,\text{in}}) = (t_{b,\text{in}} - t_{a,\text{in}}) \). Further, the performances in double-pass heat exchanger with or without recycle are the same for concurrent-flow and countercurrent-flow operations.

Considerable improvement in heat transfer is obtainable if heat exchange is operated with double pass and external recycle of higher reflux ratio, arranged in the recycle phase (phase a), under either cooling \( (t_{a,\text{in}} > t_{b,\text{in}}) \) or heating \( (t_{a,\text{in}} < t_{b,\text{in}}) \) operation. In this kind of heat exchanger, double-pass and recycle operations provide the desirable effect of increase in fluid velocity as well as the heat-transfer coefficient, which can compensate for the undesirable effect of remixing as well as the decrease in driving force (temperature difference) of heat transfer, resulting in enhancement of heat-transfer rate, especially for larger reflux ratio, as shown in Table 1–3. It is shown in Table 4 that more than 20% improvement in performance I is obtainable in the system of present interest with the negligible amount of hydraulic-dissipated energy. One may expect that I will further increase when \( q_a, q_b, \text{ or } |t_{a,\text{in}} - t_{b,\text{in}}| \) continuously increases.

Practically, there are two broad types of heat exchangers: direct-contact exchanger and indirect-contact exchanger (recuperator). In recuperators the two flowing streams are separated by a wall and heat has to pass through this wall, like the device of present interest. Recuperators are certainly less effective than the direct-contact exchanger because the presence of the wall hinders the flow of heat. But this type of exchanger is used where the fluids are not allowed to contact each other, as with gas-gas systems, miscible liquids, dissolving solids, or reactive chemicals. It is confirmed that the model with double-pass and external-recycle operations developed in present study, enhances the performance in the flat-plate parallel-flow heat exchangers without recycle. We believe that it would be also applicable to other kinds of recuperators, such as shell-and-tube, cold-finger, multi-plate, spiral-plate and compact exchangers.

6. List of Symbols

\( C_{pa}, C_{pb} \) heat capacity of fluid a, fluid b (kJ/kg \cdot K)

\( D_{eq} \) equivalent diameter (m)

\( E \) enhancement of hydraulic-dissipated power in double-flow device

\( H_1, H_2 \) hydraulic-dissipated power for single-, double-pass (hp)

\( h \) height of flow channel, \( h = h_a = h_b \) (m)

\( h_{b, a}, h_b \) convective heat-transfer coefficient in fluid a, in fluid b (m/s)

\( I \) improvement of heat-transfer rate defined by Eq. (65)

\( k_a, k_b \) thermal conductivity of fluid a, fluid b (kJ/m \cdot s \cdot K)

\( L \) effective length of a heat exchanger (m)
Heat Transfer Performance in Double-Pass Flat-Plate Heat Exchangers with External Recycle

\[ \ell = m_a (1 + R)/m_b \]

\[ m = \text{time rate of heat capacity, } q_a \rho_a C_p_a, q_b \rho_b C_p_b \]

\[ \Delta p = \text{pressure drop through the flow channel (N/m}^2\text{)} \]

\[ Q = \text{total heat-transfer rate (kJ/s)} \]

\[ Q_{1,A}, Q_{1,B} = \text{total heat-transfer rate of single-pass operation in concurrent-flow device, in countercurrent-flow device (kJ/s)} \]

\[ Q_{2, A-B}, Q_{2, B-A} = \text{total heat-transfer rate of double-pass operation, in A-B type device, in B-A type device (kJ/s)} \]

\[ R = \text{reflux ratio} \]

\[ S = \text{overall heat-transfer area of a heat-transfer medium, WL (m}^2\text{)} \]

\[ t_{a,1}, t_{b,1}, t_{a,2}, t_{b,2} = \text{bulk fluid temperature in fluid a, fluid b (K)} \]

\[ t_{a,i}, t_{b,i} = \text{at the inlet (K)} \]

\[ t_{a,i}^o, t_{b,i}^o = \text{mixed inlet temperature in fluid a (K)} \]

\[ t_{a,r}, t_{b,r} = \text{at the outlet (K)} \]

\[ (\Delta t)_m = \text{mean temperature difference between fluid a and fluid b (K)} \]

\[ U = \text{overall heat-transfer coefficient defined by Eq. (9)} \]

\[ W = \text{width of a heat exchanger (m)} \]

\[ x = \text{rectangular coordinate (m)} \]

6.1 Greek Letters

\[ \alpha, \beta = \text{dimensionless group defined by Eqs. (21) and (22)} \]

\[ \xi_{a,b}, \xi_{p,a} = \text{dimensionless outlet concentrations defined by Eqs. (33) and (32)} \]

\[ \xi = \text{x/L} \]

\[ \mu_a, \mu_b = \text{viscosity of fluid in fluid a, in fluid b (Pa s)} \]

\[ \rho_a, \rho_b = \text{fluid density in fluid a, in fluid b (kg/cm}^3\text{)} \]

\[ \eta = \text{heat-exchanger efficiency} \]

References


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