Response Properties of Self-Anchored Suspension Bridges to Aerostatic Wind

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Abstract

This paper uses ANSYS software with an improved incremental iteration nonlinear method to analyze the nonlinear response of self-anchored suspension bridges to aerostatic wind. Results of the analysis show that significant changes occur in the girder’s vertical displacement and its rotation angle when the initial wind attack angle varies. On the other hand, the wind load on the cable system has a significant influence on the lateral displacement of the girder; the added wind attack angle also has significant influence on the vertical displacement of the girder. Therefore, in the calculation of aerostatic response, the added attack angle of the girder must be considered to prevent non-negligible errors occurring in the displacement responses and in the calculation of critical wind speed for aerostatic instability. Under aerostatic wind action, the first order natural frequency first increases gradually with wind speed then accelerates quickly before a sudden drop once the frequency reaches the aerostatic instability frequency.

Key Words: Self-Anchored Suspension Bridges, Aerostatic Loads, Aerostatic Responses, Aerostatic Instability Wind Velocity

1. Introduction

The aerostatic instability of a bridge refers to the instability of the girder due to bending or torsion under aerostatic loads. The occurrence of aerostatic instability is determined by the change in structural stiffness and the aerodynamic effect of wind gradients, rather than by the ultimate strength of the structure.

In 1967, Prof. Hirai [1] of Tokyo University observed the aerostatic torsional collapse of a suspension bridge model during a wind tunnel experiment. Tunghua University also observed aerostatic bending/torsion instability in cable-stayed bridges [1, 2] in its wind tunnel testing of Shan-Tou’s Bay Bridge #2. As the development of nonlinear finite element method progresses, in 1994, Boonyapinyo, Yamada, and Miyata [3–13] were the first to integrate structural geometric nonlinearity and aerostatic load nonlinearity factors and proposed the establishment of the structural equilibrium equation using nonlinear finite element method. By solving the eigenvalue and combining iterations of updated upper and lower boundaries of wind speeds, they solved the critical wind speed for structural instability. Later, scholars such as Xu Xie [14], Ming-Shan Fang [15], Cheng Jin [16], Zhi-Tian Zhang [17] Xiao-Jiang Zou [18] investigated and developed a greater understanding of the problem of...
aerostatic instability in bridges. They also investigated the mechanisms involved in the instability of suspension bridges and cable-stayed bridges. In fact, the problem of aerostatic wind stability of bridges involves more than solving critical wind speed for aerostatic instability and its competition with dynamic instability. It is a process of constantly reaching aerostatic wind equilibrium and eventually arriving at instability—a full-range analysis.

Masatsugu N. [19] describes the feasibility of 1,400 m steel cable-stayed bridges from both structural and economic viewpoints. For static instability, elastoplastic, finite-displacement analysis under in-plane load and elastic, finite-displacement analysis under displacement-dependent wind load are conducted; for dynamic instability, multimodal flutter analysis is carried out. It is shown that aerostatic critical wind velocity of lateral torsional buckling governs the dimension of the girder. Finally, the writers briefly compare a cable-stayed bridge with suspension bridge alternatives.

Early aerostatic stability analysis made use of linear analysis through the lateral buckling method and the structural torsion collapse method [16]. A subsequent method was to combine the eigenvalue with the iterations. Using this approach, Ming-Shan Fang [15] combined the incremental method with the iteration method and applied it to calculate the second type of aerostatic instability problems. Cheng Jin [16] further improved this method by introducing inside and out increments and using double iteration. In this way, aerostatic instability analysis developed into the method currently used. At the same time, Cheng Jin [16] introduced a practical formula to determine aerostatic torsional collapse for suspension bridges, expressing the collapse in series and, in using correction parameter theory, further introduced a practical method for determining the wind load effect on the lateral buckling of suspension bridges. Zhi-Tian Zhang [17] introduced a low relaxation factor, which increased the speed of convergence. Starting with responses to aerostatic wind and vibration, Xiao-Lun Hu [20] proposed a method of analysis with one main rule and three secondary rules and investigated the effect of aerostatic wind on long spanned cable-stayed bridges. Using the aforementioned aerostatic stability analysis to iteratively calculate the nonlinear added attack angle requires a parameter to end the convergent calculation. A great deal of experience is required to set the magnitude of this parameter correctly and it is sometimes set by the practical environment. If the magnitude of the parameter is too large, then even though the number of iterations is reduced, the error in the calculated result is large.

Past studies used CFD techniques to conduct numerical simulations on the cross sections of bridge members with aerostatic wind coefficients, the results of which were verified using wind tunnel tests. This study is a follow-up of these achievements, and for this reason, we did not perform additional verification.

2. Aerostatic Response and Stability Analysis

2.1 Procedures of Analysis

The analysis procedure for the improved incremental nonlinear iteration method used in this paper is given below:

(1) Assume a wind speed and an aerostatic initial wind attack angle. From these, calculate the aerostatic loads based on the three component coefficients of static force in each bridge component. Apply these loads to the finite element model.

(2) Perform a nonlinear static analysis of the bridge structure to calculate the twisting angle at the nodes on the girder. Recalculate the wind loads acting on the girder based on the girder’s three component coefficients of static force corresponding to various aerostatic attack angles (initial attack angle and added attack angle).

(3) Continue the iteration process until a nominated number of iterations are reached or until two successive iterations in displacement or twisting angle responses have differences smaller than a nominal value. At this point, the structural displacement for the considered wind speed can be determined and we can proceed to perform the eigenvalue analysis and to calculate the model’s frequency of vibration.

(4) Increase the wind speed to the next strength and repeat steps (1)–(3). If the nonlinear analysis is convergent, then the displacement response of the structure at that wind speed is confirmed.

(5) If the nonlinear analysis is not convergent, then reduce the wind speed and repeat the analysis. The interpolation method was used to calculate aerostatic instability wind speed.
2.2 Calculation of Aerostatic Loads for Structural Components

2.2.1 Aerostatic Loads on the Girder

The aerostatic loads acting on the girder can be determined based on the three component coefficients of static force obtained from experiments on the model.

In a uniform flow field, the average wind speed is $U$. The aerostatic wind load per unit girder length has three components: lift ($L$), drag ($D$), and lifting moment ($M$). Each component can be expressed separately along the wind axis and structural axis as shown in Figure 1. The structural axis coordinate system is connected to and is fixed to the cross-section of the girder. The coordinate system is formed by the $FH$ axis, the $FV$ axis and the cross-sectional axis denoted by $X$. The wind axis coordinate system is determined by the direction of wind flow.

Following standard practice, the three component coefficients of static force in the model experiment are defined as follows.

The three component coefficients of static force in the structure axis coordinate system are:

$$
C_H = \frac{F_H}{1/2 \rho U_c^2 BL'}; \quad C_V = \frac{F_V}{1/2 \rho U_c^2 BL'}; \quad C_M = \frac{F_M}{1/2 \rho U_c^2 BL'}
$$

where $F_H$, $F_V$ and $F_M$ are the three static force components in the structure axis coordinate system; $\rho$ is air density; $U_c$ is the experimental wind speed; $H$ is the height of the girder; $B$ is the width of girder; $L'$ is the length of the model. The three component coefficients of static force in the wind axis coordinate system are:

$$
C_D = \frac{D}{1/2 \rho U_g^2 HL'}; \quad C_L = \frac{L}{1/2 \rho U_g^2 BL'}; \quad C_M = \frac{M}{1/2 \rho U_g^2 B^2 L'}
$$

where $\alpha$ is the effective attack angle between the wind flow and the structure. It is the sum of the initial attack angle $\alpha_0$ and the structure’s added attack angle $\theta$ caused by the static effect of the wind flow. The transformation relations among corresponding coefficients are:

From the wind axis to the structural axis:

$$
C_H = C_D \cos \alpha + C_V \frac{B}{H} \sin \alpha; \quad C_L = \frac{H}{B} C_M \sin \alpha + C_V \cos \alpha
$$

From the wind axis to the structural axis:

$$
C_H = C_D \cos \alpha - \frac{B}{H} C_V \sin \alpha; \quad C_V = \frac{H}{B} C_D \sin \alpha + C_V \cos \alpha
$$

The aerostatic load applied to the girder differs from the usual constant load in that it is not only a function of wind speed $U$ but also a function of structural deformation (mainly the effective attack angle). Aerostatic loads lead to structural deformation, and the change in effective attack angle can affect the magnitude of the aerostatic load in turn. In addition, when the bridge is at an aerostatic equilibrium, the effective attack angle varies along the length of the girder, i.e., aerostatic load varies along the length of the bridge, which indicates a spatial distribution characteristic of aerostatic load.

The girder’s three component coefficients of static force used in this calculation are shown in Figure 2.

2.2.2 Aerostatic Loads on the Towers and Cables

Following the code [21], the transverse wind load applied to the tower, hanger, and main cable can be calculated from the following equation

$$
F_H = \frac{1}{2} \rho V_g^2 C_H A_w
$$

where $V_g$ is the static wind speed; $C_H$ is drag coefficient due to structural components of the side span, and $A_w$ is the projected area of the structural component along the wind direction.

Figure 1. Illustration of the three static force components acting on the girder.
Based on this code [13], the drag coefficient of the tower is chosen as 1.55 and that for cable system is taken as 0.7 in this study.

3. Structural Analysis

3.1 Basic Information for Analysis

The main bridge length of the Jinzhou Straight Bridge is 660 m, its central span is 400 m, and at each side spans a self-anchored suspension bridge, 130 m in length and 23.5 m in width. The main girder is a steel box girder with a height of 3.0 m and a bridge width of 23.5 m. The main cable and hangers are made of galvanized high strength cables with a strength of 1670 MPa. The tower has a reinforced concrete frame structure with a total height of 96.35 m. The superstructure is 76.35 m high. The cross-section of the tower post is half-elliptic with a width of 6 m along the length of the bridge. The width in the transverse direction is 2.5 m at the top and increases gradually to 3.8 m at the bottom. The configuration of the bridge is shown in Figure 3 and the cross-section of the steel box girder is shown in Figure 4.

3.2 Finite Element Model

This study uses ANSYS software with a three-dimensional finite element model for the analysis. The single-beam model is used for the girder and the spacial hanger element is used for cables and hangers. The effect of the initial internal force on the stiffness matrix of the structure is also considered. The Ernst formula is used to modify the elastic moduli to account for the sagging of the bridge cables. The beam element is used for the tower posts and cross beams and 3-D beam elements are used for the piers. It is assumed that, at the intersection of the tower post and transverse beams, both the bending stiffness in the plane along the transverse direction and the

![Figure 2. Distribution curves of the three component coefficients of static force.](image)

![Figure 3. Side view of the Jing-Zhou Bay Bridge (Unit: cm).](image)

![Figure 4. The cross-section of the steel box girder (Unit: cm).](image)
axial stiffness along the longitudinal direction of the tower are increased. There are transverse and longitudinal constraints between the girder and the side pier, as well as longitudinal and vertical constraints on the bridge turning. No limitations in motion are imposed along the longitudinal direction and about the transverse axis. There is a vertical support for the girder at the intersection between the girder and the tower limiting the lateral displacement of the girder. A model with pile foundations was established in this study to investigate the effect of pile foundations on the aerostatic wind response of the self-anchored suspension bridge.

In the dynamic finite element model of the Jinzhou Straight Bridge, it is assumed that both the tower bottom and pier bottom are fixed. The hangers are divided into 8 sections and are represented by a 3D beam model (see Figure 5).

Furthermore, in order to consider the differences in the bridge’s aerostatic response under different initial wind attack angles, the initial wind attack angle is chosen as -3°, 0°, or 3°. To analyze the effect of added attack angle on the static wind response, two more scenarios were analyzed, one where the added attack angle is included, the other where the added attack angle is excluded. In order to investigate how much of the wind load is transferred onto the cable system, which includes the main cable and the hangers, as compared to the whole static wind load acting on the structure, the analysis considered both the presence and absence of the wind load on the cable system. To compare the static wind response subjected to the uniform flow field and the turbulent flow field, the wind speed cross-section parameter is chosen to be 0 and 0.12. To compare the effect of different factors on the static wind response of the Jinzhou Straight Bridge, the wind speed is taken to be 100 m/s for all cases.

4. Results of Analysis and Their Comparison

4.1 Effect of Initial Wind Attack Angle on Structural Displacement Due to Static Wind

Figures 6.1–6.3 show the results obtained for the girder’s vertical and lateral displacements and twisting angle when the initial attack angle is set at -3°, 0° and 3°.

Figures 6.1–6.3 show that as the initial attack angle varies, significant changes occur in the girder’s vertical

![Figure 6.1](image-url)

**Figure 6.1.** Vertical displacement of the girder as initial attack angle varies.

![Figure 6.2](image-url)

**Figure 6.2.** Lateral displacement of the girder as initial attack angle varies.
displacement and twisting angle. When the initial attack angle is at $0^\circ$, the vertical displacement at the mid-span is negative, indicating that the girder deflects downwards.

In addition, when the initial attack angle changes from negative to positive, the girder changes from being downwardly loaded to being lifted and the girder’s twisting angle changes gradually from negative to positive. Furthermore, we observe that the girder’s vertical displacement tends to vary but the rotation angle tends to remain constant. The trend of change in the girder’s lateral displacement is different to both of these parameters. When the initial attack angle is $0^\circ$, the girder’s lateral displacement is at its minimum. As the initial attack angle moves away from $0^\circ$, either in a positive or negative direction, the lateral displacement also increases.

4.2 Effect of the Cable System Wind Load on the Structure’s Aerostatic Displacement

In order to find the percentage of the wind load that is taken by the cable system compared to the whole static wind load acting on the entire structure, an analysis has been performed to compare the results between the case where the cable system wind load is present and a case without the cable system wind load. The displacement at the girder’s mid-span for each of the three different initial attack angles $-3^\circ$, $0^\circ$ and $3^\circ$ are listed in Table 1.

Table 1 shows that the cable system wind load on the Jinzhou Straight Bridge has a significant effect on the lateral displacement of the girder but the effect on other displacements is less significant. Also related to girder’s lateral displacement, Table 1 shows that the wind load on the cable system constitutes more than 20% of all wind loads, which is a significant finding.

4.3 Effect of Added Attack Angle on the Displacements of the Structure

This analysis determines the effect of added attack angle at three different values $-3^\circ$, $0^\circ$, and $3^\circ$ on the structural aerostatic responses. Results for the calculation of displacements at the girder’s mid-span are listed in Table 2.

Table 2 shows that the added attack angle in the Jinzhou Straight Bridge has a significant effect on the girder’s vertical displacement and it has a relatively large effect on the twisting of the girder but a lesser effect on the lateral displacement of the girder. It may be concluded that the added attack angle reduces the sensitivity of girder’s lift coefficient, lift moment coefficient, and drag coefficient.

<table>
<thead>
<tr>
<th>Table 1. Effect of the cable systems wind load on the girder displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Item</strong></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>Vertical displacement of the girder (m)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Lateral displacement of the girder (m)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Angular displacement of the girder (deg)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
In calculating the aerostatic response, the girder’s added attack angle should be considered else it can lead to non-negligible errors in the displacement response and the aerostatic instability wind speed, such as over-estimating girder’s rotation angle by 12%.

### 4.4 Effect of Aerostatic Load on the Structure’s Natural Frequency

Under the action of aerostatic wind loading, analyses are performed using initial attack angles of $-3^\circ$, $0^\circ$ and $3^\circ$ with an added attack angle of 0 and wind speed cross-section parameter of 0.12. The results are shown in Figures 7.1, 7.2, and 7.3 for the girder’s first order symmetric frequencies of bending about vertical axis and transverse axis, and of torsion, due to change in wind speed. These frequencies are well known as the frequency of the girder’s first symmetric mode for bending about vertical axis, transverse axis and for torsion.

<table>
<thead>
<tr>
<th>Item</th>
<th>Initial attack angle</th>
<th>$-3^\circ$</th>
<th>$0^\circ$</th>
<th>$3^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal displacement of the tower (m)</td>
<td>Cable system wind load included</td>
<td>0.142</td>
<td>0.028</td>
<td>-0.0797</td>
</tr>
<tr>
<td></td>
<td>Cable system wind load not included</td>
<td>0.143</td>
<td>0.029</td>
<td>-0.0793</td>
</tr>
<tr>
<td></td>
<td>Relative difference %</td>
<td>0.35</td>
<td>2.14</td>
<td>-0.50</td>
</tr>
<tr>
<td>Lateral displacement of the tower (m)</td>
<td>Cable system wind load included</td>
<td>0.2937</td>
<td>0.2918</td>
<td>0.2924</td>
</tr>
<tr>
<td></td>
<td>Cable system wind load not included</td>
<td>0.265</td>
<td>0.263</td>
<td>0.264</td>
</tr>
<tr>
<td></td>
<td>Relative difference %</td>
<td>-9.70</td>
<td>-9.77</td>
<td>-9.75</td>
</tr>
<tr>
<td>Rotational displacement of the tower about the vertical axis (deg)</td>
<td>Cable system wind load included</td>
<td>-0.023</td>
<td>0.025</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>Cable system wind load not included</td>
<td>-0.024</td>
<td>0.024</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>Relative difference %</td>
<td>-3.84</td>
<td>3.47</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Figure 7.1 shows that the first order symmetric frequency for bending about vertical axis, subject to various initial attack angles and various wind speed cross-section parameters.
section parameters gradually increases with increasing wind speed initially, then the frequency increase rapidly accelerates followed by a sudden drop as soon as the frequency approaches the aerostatic instability frequency.

Figure 7.2 shows that the first order symmetric frequency for bending about the transverse axis, subject to various initial attack angles and various wind speed cross-section parameters also increases gradually with the increasing wind speed initially. Once the frequency approaches the aerostatic instability frequency, the first order symmetric frequency increases rapidly. This occurs only when the attack angle is at +3° and the wind speed cross-section parameter is set to 0.

Figure 7.3 shows that when the attack angle is at +3°, the first order symmetric torsion frequency somewhat increases with increasing wind speed initially and then dips into a wavy behavior; when the attack angle is at -3° and 0°, the first order symmetric torsion frequency generally increases with the increasing wind speed.

The main reason for the change in first order symmetric frequencies of bending about vertical and transverse axes as well as in torsion under aerostatic wind load shown in Figures 7.1–7.3 is due to the action of aerostatic wind load, which changes the stiffness of the structure.

4.5 Critical Wind Speed to Induce Aerostatic Instability

The initial attack angles considered in the calculation are -3°, 0°, and 3° and the wind speed cross-section parameters considered are 0 and 0.12. The analysis results are shown in Table 3.

Table 3 shows that, when the initial attack angle is at +3° and the wind speed cross-section parameter is set to 0.12, the minimum critical wind speed for aerostatic instability is 182 m/s. This minimum value indicates that aerostatic instability will not occur in the lifetime of the Jinzhou Straight Bridge.

5. Conclusions

This paper uses an improved nonlinear incremental iteration method to analyze the nonlinear aerostatic response of a self-anchored suspension bridge to aerostatic wind as well as to find the critical wind speed for aerostatic instability. The Jinzhou Straight Bridge was analyzed under aerostatic load to investigate the effect of factors such as the initial attack angle, cable system wind load, added attack angle, and wind speed cross-section parameters on the displacements, critical aerostatic instability wind speed, and the structure’s first order natural frequency. The following conclusions were obtained:

1. When different initial wind attack angles are considered, the changes in the girder’s vertical displacement and its rotation angle are most profound.

2. The cable system’s aerostatic wind load has a significant effect on the lateral displacement of the girder but only a small effect on other displacements. In the analysis of the Jinzhou Straight Bridge, the transverse wind load on the cable system is more than 20% of the total transverse wind load on the bridge. This ratio is considerably large and is therefore noteworthy.

3. The added attack angle has a significant effect on the girder’s vertical displacement. It has a large influence on the girder’s twisting angle but a very small effect on its transverse displacement. In the calculation of the bridge’s aerostatic response, the girder’s added attack angle should be considered otherwise non-negligible errors could occur in the displacement response and in the resulting critical aerostatic instability wind speed.

4. Under the action of aerostatic wind load, the order symmetric frequency for bending about the vertical axis increases gradually initially with increasing wind speed. It then increases rapidly before dropping off suddenly as soon as the frequency approaches the aerostatic instability frequency. The first order symmetric frequency for bending about the lateral axis

### Table 3. Critical wind speed of aerostatic instability for Jinzhou Straight Bridge

<table>
<thead>
<tr>
<th>Initial attack angle (deg)</th>
<th>Wind speed cross-section parameter</th>
<th>Critical wind speed to induce aerostatic instability (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
<td>341</td>
</tr>
<tr>
<td>-3</td>
<td>0.12</td>
<td>319</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>271</td>
</tr>
<tr>
<td>0</td>
<td>0.12</td>
<td>253</td>
</tr>
<tr>
<td>+3</td>
<td>0</td>
<td>201</td>
</tr>
<tr>
<td>+3</td>
<td>0.12</td>
<td>182</td>
</tr>
</tbody>
</table>
also increases gradually with increasing wind speed. At $+3^\circ$ attack angle, the first order symmetric torsion frequency initially increases a little as wind speed increases before decreasing and entering into a wavy behavior pattern.

References


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