EM Scattering from PEC Plane Moving at Extremely High Speed: Simulation in One Dimension

Mingtsu Ho

Graduate School of Opto-Mechatronics and Materials, WuFeng University, Chiayi, Taiwan 621, R.O.C.

Abstract

In this paper the one-dimensional numerical results of the simulation on the reflection of electromagnetic fields from perfect conductor traveling at extremely high velocity were presented. Maxwell’s equations were numerically solved through the application of the method of characteristics (MOC) where the perfect conductor was set to travel as fast as 90 percent of the light speed either approaching or receding from the incident electromagnetic fields. In order to accurately predict the behavior of electromagnetic fields interacting with moving object, the relativistic boundary conditions were used in MOC in conjunction with the characteristic variable boundary conditions. Both time- and frequency-domain results were demonstrated. The validation of the numerical accuracy is carried out by comparing the computational results with the theoretical Doppler values.

Key Words: Computational Electromagnetics, Method of Characteristics, Relativistic Boundary Condition, Doppler Effects

1. Introduction

It can be dated back to as early as 1960’s that researchers put efforts on the electromagnetic scattering from moving objects. Several recent practical examples can be found inside the laser resonant tube where laser pulse interacts with over-dense plasma (solid targets) causing the high-order harmonic generation as suggested by Baeva et al. [1] and the Doppler Effects produced by a reflecting charge sheet as interpreted by Bulanov et al. [2]. There were several studies on the scattering of electromagnetic waves from uniformly moving or vibrating mirrors [3–8]. The scattered electromagnetic fields either from mechanical vibrations of objects near the audio-frequency range or from uniformly moving or vibrating mirrors were reported. Harfoush et al. employed the finite-difference time-domain (FDTD) technique for the numerical simulation of the electromagnetic scattering problem around the fundamental frequency through the application of linear interpolation to the incident electric field on every occasion the moving boundary moves away from the grid point. The reason for such arrangement is that the FDTD method defines the field components at the cell nodes in an interlacing fashion and explicitly approximates the spatial derivative by using finite difference scheme alternatively. It is also concluded that objects undergoing translational motion demonstrate Doppler shifts in the reflected fields and that oscillating perfect conductors change not only the phase but the magnitude of the scattered fields. Also can be found are the electromagnetic field scattering by an object in relativistic translational motion [9] and by a perfectly conducting wedge in uniform translational motion [10], and the diffraction of electromagnetic pulse by a moving half-plane [11].

The FDTD technique and method of moment (MoM) have been the two most widely used numerical approaches for the solutions of electromagnetic scattering problems.
since they were proposed in the 1960s. A recently developed numerical solver of the time-domain Maxwell’s equations, the characteristic-based method has been shown to have good agreement with the FDTD method for the case where Gaussian electromagnetic pulse normally illuminates on an infinite perfect conducting strip [12]. It is also shown that the MOC approach is applicable to the simulation of the following: the propagation of electromagnetic pulse through lossless non-uniform dielectric slabs [13], the reflection of electromagnetic pulse from objects vibrating at different frequencies [14], and the solutions of the Maxwell-Minkowski equations for the propagation of electromagnetic pulse onto a moving lossless dielectric half-space [15].

The two conventional methods, MoM and FDTD, define all field components at node points. On the contrary, the characteristic-based method first places all field variables at the center of the grid cell and solves the Maxwell’s curl equations by evaluating all flux quantities and then balancing all field components within each computational cell. It is thus considered to be a more appropriate approach for solving problems involved with time-varying grids, since all field quantities stay in the center of cell despite the motion of grid point. Though so, this paper will not cover the comparisons of the advantages nor the computational efficiency among different numerical techniques.

The main objective of this paper is to demonstrate the one-dimensional numerical simulation results of the reflection of electromagnetic pulses from perfect conductors that may be moving at extremely high velocity. The computational results are obtained by using the method of characteristics (MOC) in conjunction with the relativistic boundary conditions and the characteristic variable boundary conditions. It is necessary to combine these two sets of boundary conditions for the accurate description of the relativistic effects of the moving perfect plane surface on the electromagnetic pulse. In the numerical model, the perfect conductor is set to travel at a constant velocity, varying from 10 up to 90 percent of the speed of light, either toward or away from the incident electromagnetic pulse. The reflected electric fields are recorded and plotted alongside with that from a stationary perfect conductor. Also demonstrated are the corresponding spectra obtained by means of Fourier transform. The numerical scheme accuracy is investigated by comparing the computational results with the theoretical values based on the Doppler Effects. These investigations include the magnitude and pulse width, as well as the highest frequency content of the reflected electric fields.

2. Governing Equations and Boundary Conditions

The characteristic-based method is an implicit method numerically approximating the time-domain Maxwell’s equations. Electromagnetic scattering problems in free space can be completely described by the Maxwell’s curl equations

\[
\frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E} = 0
\] (1)

\[
\frac{\partial \vec{D}}{\partial t} - \nabla \times \vec{H} = 0
\] (2)

where \(\vec{E}\) and \(\vec{D}\) are the electric field intensity and flux density, \(\vec{H}\) and \(\vec{B}\) are the magnetic field intensity and flux density. Since the present simulation employs a transversely polarized plane Gaussian pulse as an excitation source, a two-dimensional formulation is sufficient to describe the problem. In the numerical model, the incident fields have only two components, \(\vec{D} = \hat{z}D_z\) and \(\vec{B} = -\hat{y}B_y\), and accordingly propagate in the positive-x direction. In addition, objects used in the model are perfect electrically conducting (PEC) planes.

There are two arrangements needed to be done prior to numerically solving the Maxwell’s curl equations via the application of the MOC approach. The governing equations have to be recast in the form of the Euler equation which is then transformed from the Cartesian system \((t, x, y)\) to a curvilinear system \((\tau, \zeta, \eta)\) as

\[
\frac{\partial Q}{\partial \tau} + \frac{\partial F}{\partial \zeta} + \frac{\partial G}{\partial \eta} = 0
\] (3)
where vectors \( Q, F, \) and \( G \) are functions of metric terms and variable vectors \( q, f, \) and \( g \). They are respectively defined as follows

\[ Q = Jq \]  (4)
\[ F = J\xi f + J\eta g \]  (5)
\[ G = J\eta f + J\eta g \]  (6)

\[ q = \begin{bmatrix} B_x \\ B_y \\ D_z \end{bmatrix}, f = \begin{bmatrix} 0 \\ -E_x \\ -H_y \end{bmatrix}, g = \begin{bmatrix} E_z \\ 0 \\ H_z \end{bmatrix} \]  (7)

The definition of the Jacobian \( (J) \) of the reverse transformation is given as

\[ J = \frac{\partial(x, y)}{\partial(\xi, \eta)} \]  (8)

and as an example the symbol \( \xi \) is defined as \( \xi = \frac{\partial x}{\partial x} \). By applying the central difference operator

\[ \delta_x(\psi) = \psi_{k+\frac{1}{2}} - \psi_{k-\frac{1}{2}} \]  (9)

to (3), we have

\[ \frac{Q^{n+1} - Q^n}{\Delta \tau} + \frac{\delta E}{\Delta \xi} + \frac{\delta G}{\Delta \eta} = 0 \]  (10)

In (9), the symbol \( \delta \) stands for the vector \( F \) or \( G \) as defined in (5) and (6), the subscript \( (k) \) represents cell index along either \( \xi \) or \( \eta \)-direction in the curvilinear coordinate system; the one-half index indicates that fluxes are evaluated at the two interfaces of the \( k^{th} \) cell along the k-direction. In (10), with the superscripts \( (n+1) \) and \( (n) \) on quantity \( Q \) being two successive time levels, i.e., the numerical time step \( \Delta \tau \), the first term designates the approximation the derivative of vector \( Q \) with respect to time. It is the difference \( (Q^{n+1} - Q^n) \) that the present numerical method solves for and the newly-updated \( Q^{n+1} \) become available by adding that difference to the previous \( Q^n \). Notice that the flux differences in second term \( \xi \)-direction and third term \( \eta \)-direction in (10) are evaluated separately, one direction at a time. To solve the system of linear equations, the implicit numerical formulation was incorporated with the flux vector splitting technique and the lower-upper approximate factorization scheme.

In order to accurately model the relativistic effects of the perfect conducting plane surface on the electromagnetic pulse since it is traveling at extremely high speed, the relativistic boundary conditions and the characteristic variable boundary conditions were combined and then the resulting boundary conditions were employed in the numerical formulation. The relativistic boundary conditions are given by

\[ \hat{n} \times \vec{E} = (\hat{n} \cdot \vec{v}) \vec{B} \]  (11)

where \( \vec{v} \) is the velocity of the perfect plane and \( \hat{n} \) is the unit vector normal of the moving plane, \( \vec{E} \) and \( \vec{B} \) are the electric field intensity and the magnetic flux density that are arriving at the conductor surface from the adjacent cell. Note that the velocity \( \vec{v} \) in (11) is defined to have a positive number if the conductor and the incident fields travel in the same direction; negative if they approach each other. In MOC, the characteristic variable \( \text{(CV)} \) is defined as the product of eigenvector matrix and instantaneous variable vector. As a result, each characteristic variable is characterized by one particular eigenvalue which carries the information of direction and speed at which that corresponding characteristic variable propagates. Hence the characteristic variable arriving at the perfect conducting plane is the one carries information and propagates from the adjacent cell

\[ \text{CV} = \hat{n} \times \vec{B} + \eta_0 \vec{D} \]  (12)

where \( \vec{D} \) and \( \vec{B} \) are the electric and magnetic flux densities of the adjacent cell and \( \eta_0 \) is the characteristic impedance of vacuum. Setting the field components used in the relativistic boundary conditions as (11) to be those on the conductor surface, designating \( \vec{E}_b^h \) and \( \vec{B}_b^h \) as the boundary values, and then solving \( \vec{E}_b^h \) and \( \vec{B}_b^h \) from (11) and (12), we have the boundary values in terms
of CV and the velocity $v$ as

$$\vec{b}^o = \frac{1}{v-1} CV$$

(13)

$$\vec{e}^o = \frac{v}{v-1} CV$$

(14)

(13) and (14) are used as the boundary conditions in the code. Note that $v$ is a signed number: positive when the PEC recedes from the incidence and negative when approaches. Also for a motionless PEC plane, i.e., $v = 0$, leading to the fact that $\vec{E}^o = 0$. It is physics that the electric field on the PEC surface must vanish.

3. The Problem

The schematic presentation of the present problem is shown in Figure 1 where only the electric field component of the incident electromagnetic pulse was depicted. Initially, the peak of the incident electromagnetic pulse is located at 3.3 meters from the PEC plane. The electric field intensity is normalized to have a peak value of one V/m. The numerical electromagnetic excitation is set to be plane Gaussian electromagnetic pulse, has only two components, $-B_y$ and $D_z$, and therefore propagates in the positive-x direction. Also given in the sketch are the follows: the definition of the pulse width $T$ of a Gaussian pulse and possible directions of movement the traveling perfect plane together with the sign designation of the movement. For practical reasons, a rectangular window with a truncated level of 100 dB was applied to the incident Gaussian pulse. The Gaussian pulse used in the present simulation has a pulse width of 2.084 nanoseconds such that the spatial end-to-end span is measured 6 meters and the corresponding highest frequency content is about 366.46 MHz. The sampling point of the reflected electric fields is located at 6.6 meters away from the perfect plane.

Figure 2 illustrates how the grid system was adapted for problems involving moving boundary. For the case featured with motionless boundary as in Figure 2(a), both the total grid number and cell size are constants. When the boundary is approaching the incident pulse, the cell adjacent to the moving boundary is gradually truncated as time evolves and the total grid number is decreasing as in Figure 2(b). On the other hand, when the conductor is trailed by the incident pulse, Figure 2(c) shows that extra cells are introduced into the grid system and consequently that the total grid number is increasing. The rate of change is proportional to the speed at which the conductor travels. When the grid density is 100 cells per meter and the conductor’s speed is 0.9 C, every meter the electromagnetic fields propagate the total grid number is changed by 90 cells. If the conductor moves away from the incident fields at this speed, the number of cell is increased about 6000 cells by the time when the inter-

![Figure 1](image1.png)

**Figure 1.** Computational domain with z-polarized electric field in Gaussian form normally illuminate on a perfect conducting plane that is uniformly traveling at constant speed.

![Figure 2](image2.png)

**Figure 2.** The total grid number varies with motion of the perfect plane and is (a) fixed, (b) decreasing, and (c) increasing.
action between a six-meter-span electromagnetic pulse and the moving conductor is completed. In contrast, if the conductor moves toward the incident pulse, before the interaction completes, there are more than 300 cells diminished.

In order to exhibit the relativistic effects of the moving perfect plane on the electromagnetic pulse, the perfect plane is set to travel at a constant velocity as 10, 30, 50, 70, or 90% of the speed of light \( C = 3 \times 10^8 \text{ m/s} \) either toward or away from the incident pulse. When a perfect conductor undergoes a uniform translation movement, the reflected pulse reveals Doppler shifts not only in the field strength but the pulse width, and consequently the highest frequency content of the corresponding spectrum. With symbol \( \beta \) being the ratio of the velocity of the perfect conductor to that of light, the reflected pulse width is modified by the factor

\[
\frac{1+\beta}{1-\beta}
\]

whereas the reflected field magnitude and highest frequency content are changed by the factor

\[
\frac{1-\beta}{1+\beta}
\]

Note that \( \beta \) is negative when the perfect plane and the incident pulse are approaching each other and positive when they move in the same direction. For various values of \( \beta \) the corresponding results of (16) were listed in Table 1.

### 4. Results

To illustrate the interaction between the electromagnetic pulse and the moving perfect conductor, there are three sequences of snapshots of the computational electric fields given in Figure 3 respectively for three distinctive velocities in terms of \( \beta \), namely, \( \beta = 0 \) (3a), \( \beta = -0.9 \) (3b), and \( \beta = +0.9 \) (3c). It is observed from these plots that Doppler Effect on both magnitude and pulse width of the reflected fields are obvious, and that due to the application of the relativistic boundary conditions to the conductor’s surface on which the electric fields are not always vanished as shown in Figures (3b) and (3c), unlike those in the motionless case (3a). As shown in these plots, time is expressed in the unit of \( \text{m/C} \) for the reason of easy examinations. The reference time zero is set to be the time instance when the peak of the incident pulse is 3.3 meters from the PEC plane that is initially located at \( x = 6.6 \) meters. It is noticed that all snapshots were taken at different time intervals in order to fully demonstrate the interaction between the electromagnetic pulse and perfect plane, and finally that in every snapshot only the electric fields with strength greater than 60 dB level were displayed.

Since the reflected electric fields for the two extreme cases are differed by about 400 times in magnitude, for

<table>
<thead>
<tr>
<th>Speed</th>
<th>( 1-\beta )</th>
<th>Reflected electric intensity</th>
<th>Error %</th>
<th>Reflected pulse width (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{1+\beta}{1-\beta} )</td>
<td>Theoretical</td>
<td>Calculated</td>
<td>Error %</td>
</tr>
<tr>
<td>-0.9 C</td>
<td>19.0000</td>
<td>-19.0000</td>
<td>-18.8046</td>
<td>1.03</td>
</tr>
<tr>
<td>-0.7 C</td>
<td>5.6667</td>
<td>-5.6667</td>
<td>-5.6178</td>
<td>0.86</td>
</tr>
<tr>
<td>-0.5 C</td>
<td>3.0000</td>
<td>-3.0000</td>
<td>-2.9869</td>
<td>0.44</td>
</tr>
<tr>
<td>-0.3 C</td>
<td>1.8571</td>
<td>-1.8571</td>
<td>-1.8541</td>
<td>0.16</td>
</tr>
<tr>
<td>-0.1 C</td>
<td>1.2222</td>
<td>-1.2222</td>
<td>-1.2220</td>
<td>0.02</td>
</tr>
<tr>
<td>0</td>
<td>1.0000</td>
<td>-1.0000</td>
<td>-1.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>+0.1 C</td>
<td>0.8182</td>
<td>-0.8182</td>
<td>-0.8179</td>
<td>0.03</td>
</tr>
<tr>
<td>+0.3 C</td>
<td>0.5385</td>
<td>-0.5385</td>
<td>-0.5373</td>
<td>0.22</td>
</tr>
<tr>
<td>+0.5 C</td>
<td>0.3333</td>
<td>-0.3333</td>
<td>-0.3301</td>
<td>0.97</td>
</tr>
<tr>
<td>+0.7 C</td>
<td>0.1765</td>
<td>-0.1765</td>
<td>-0.1717</td>
<td>2.70</td>
</tr>
<tr>
<td>+0.9 C</td>
<td>0.0526</td>
<td>-0.0526</td>
<td>-0.0472</td>
<td>10.32</td>
</tr>
</tbody>
</table>

Error % = 100% × | Theoretical − Calculated | ÷ Theoretical.
easy observations these reflected electric fields were split into two groups. Figures 4(a) and 4(b) display the reflected pulses of all simulated cases respectively for $\beta \leq 0$ and $\beta \geq 0$ along with the resulted peak value given inside the parentheses. Both the calculated reflected field intensities and the exact values were listed in Table 1. Also tabulated are the calculated pulse widths of the reflected fields which are accompanied by the error percentages. It is encouraging that the numerical results and exact values are in good agreement except for the cases where the perfect plane is moving away the incident at a speed of 0.9 C. Taking $\beta = +0.9$ as an example, the Doppler Effect broadened the reflected pulse from 6 meters to 114 meters measured end to end, or a 19 times expansion in pulse width. Meanwhile, the field magnitude was smoothed out by 19 times with respect to the original strength which is about 25 dB in difference. Consequently, these will reveal on the cut-off level of the corresponding spectrum as shown below.

As an ending of this section, the observed Doppler Effect changes in the highest frequency content of the reflected electric pulses are computed through proper Fourier transform from the computational time-domain results and shown in Figure 5. In the plot the symbol $\otimes$ is used to indicate the theoretical values. It is clear that most cases are in fair agreement although several of them have to be extrapolated.

5. Conclusions

The method of characteristics, in conjunction with the application of the combined boundary conditions of
the relativistic boundary condition and characteristic variable boundary conditions, has been shown to successfully predict the reflection of electromagnetic pulse from perfect plane that is traveling at extremely high, constant speed. The numerical results of the reflected electromagnetic pulses reveal Doppler Effects in the magnitude, pulse width, and highest frequency content and are in good agreement with the theoretical values. The key to the success is two folds: the method of characteristics is capable of solving scattering problems involved with moving boundary without major modification to the numerical formulation; and the combination of the relativistic and characteristic variable boundary conditions can accurately describe the interaction between the electromagnetic fields and moving PEC planes.

**References**


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