Theoretical Analysis of Wind Turbine Tower-Nacelle Axial Vibration Based on the Mechanical Impedance Method

Dong Xiaohui

School of Mechanical Engineering, Yancheng Institute of Technology, Yancheng 224051, P.R. China

Abstract

Based on an established axial vibration model of the wind turbine tower-nacelle system, the mechanical impedance method was applied to construct the mechanical network diagram for the axial vibration of the tower-nacelle system. Then, the axial free vibration and forced vibration of the system were analyzed theoretically with considering the displacement impedance or admittance as the transfer function. The analysis shows: for free vibration, the system performs damped vibration with light damping, the amplitude attenuates exponentially with light damping, the system returns to the equilibrium position directly with over-damping, and the system does not generate reciprocating vibration with critical damping; for forced vibration, the amplitude of the axial displacement response is related to the frequency ratio of rotation rate. The resonance frequency does not occur at the undamped natural frequency \( \omega_0 \). The peak value of the vibration triggered by blade mass imbalance shifts toward the high frequency direction along with the increase of damping ratio \( \xi \), while the peak value of the vibration triggered by tower front spoiler and pneumatic imbalance shifts toward the low frequency direction along with the increase of \( \xi \). If \( \xi > \frac{\sqrt{2}}{2} \), the amplitude frequency has no peak value, and resonance does not occur. The analysis provides a theoretical basis for the design and control of the wind turbine tower.

Key Words: Wind Turbine, Axial Vibration, Mechanical Impedance Method, Amplitude-frequency Characteristic

1. Introduction

Wind energy, a sort of clean and renewable energy, has drawn wide attention all over the world because of its low cost. The cumulative installed capacity reached 318.12 GW globally until the end of 2013. The installed capacity of 24 countries exceeded 1 GW, including 16 countries in Europe, 4 in Asia Pacific, 3 in North America and 1 in Latin America [1]. With the increase of the installed capacity, the issue of system stability has also attracted increasing concerns. Due to the slender shape of blades and the large mass of the nacelle, the structure of the wind turbine can produce vibration easily. In addition, many other factors can also result in violent vibrations to the wind turbine system, such as the periodical load caused by blade rotation, wind load, earthquake, the switching process and the control process. The vibration of wind turbine may damage the bearings, gears, couplings, blades, tower and other components, and reduce the overall reliability and even the service life of the
system. Therefore, the study of wind turbine vibration plays an important role in the enhancement of system reliability.

In the literature, researches of wind turbine vibration mainly concentrate on modal identification. The structural modal identification includes two categories: input known and input unknown [2,3]. When both input and output are known, the frequency response function can be calculated based on the input and output data and the structural modal parameters can be obtained [2,3]. If the input is unknown, a variety of analysis methods have been developed, such as the P-P method based on the frequency domain [3], the frequency domain decomposition (FDD) method [4], the stochastic subspace identification method based on the time domain [5], the random decrement method [6], the ARMA time series model and so on. For large structures like wind turbine, it is difficult to impose artificial excitation. On the contrary, direct measurement appears to be the main approach of wind turbine vibration test because of the convenience of operation [7,8]. However, this method is based on two assumptions [12,13]: 1) the test structure must be a linear time invariant system, 2) the external input excitation must have broadband frequency spectrum and distribute across the whole structure system and there should be no correlation among the inputs of different points. However, the operation of the wind rotor will lead to the change of wind turbine mass and rigidity over time. Thus, the wind turbine tower is not a linear time invariant system during the operation of the wind rotor. Moreover, due to the aerodynamic force generated by the operation of the wind rotor, the inputs of the wind turbine tower are not fully compliant with the second assumption. Therefore, the practical measurement method is subject to a variety of limitations in studying vibration characteristics. It is applicable to the wind turbine in service only, unable to address the vibration issue from the substantive characteristics.

This study investigated the axial vibration of the wind turbine tower-nacelle system theoretically by applying the mechanical impedance method. The mechanical impedance method was initially proposed in the 1940s [14]. But until the 1960s, it was used to analyze and calculate integrated structures relying on transfer function analysis equipments and computer programs. In the 1970s, the dynamic system analyzer and a variety of specialized software were used to perform effective dynamic analysis for large and complex integrated structures. In vibration analysis, the mechanical impedance method can simplify the calculation of the inherent frequency and vibration type of integrated structures. Particularly, for some integrated structures that are difficult to handle or unable to conduct accurate calculation, the inherent frequency of the whole structure can be calculated using the mechanical impedance method, as long as the mechanical impedance or admittance at the linkage point of the substructure can be measured. Therefore, based on an established axial vibration model of the wind turbine tower-nacelle system, this study applied the mechanical impedance method to construct the mechanical network diagram for the axial vibration of the system. Then, taking the displacement impedance or admittance as the transfer function, the axial free vibration and forced vibration of the tower-nacelle system were analyzed theoretically to obtain the resonance conditions. This paper will provide a theoretical basis for the design and stability control of the wind turbine.

2. Analysis of Excitation Sources for the Axial Vibration of the Tower-nacelle System

2.1 The Excitation Force Generated by Blade Mass Imbalance

Due to rotation, the weight of the blade is constantly changing relative to the direction of the blade. Thus, the centrifugal force will generate steady-state pull load in the rotating blades, i.e., the centrifugal force F of the three blades is balanced with each other when the rotor rotates. However, if the mass of one blade is different from the other two blades, the incremental mass will produce a 1-time frequency excitation force, as shown in Figure 1 [15].

In Figure 1, F is the centrifugal force generated by the rotation of the rotor. Its value is:
\[ F = m \omega^2 \]

where \( m \) is the mass of the blade, \( \omega \) is the rotation rate of the rotor.

When the mass of one blade \( \Delta m \) is larger than the mass of other two blades, the 1-time frequency excitation force generated by the incremental mass \( \Delta F \) is:

\[ \Delta F = \Delta m \omega^2 \]

The component in the horizontal direction \( F_x(t) \) is:

\[ F_x(t) = \Delta m \omega^2 \sin \omega t \]

\( F_x \) will trigger lateral vibration of the tower-nacelle system.

2.2 The Excitation Force Generated by Tower Front Spoiler

Tower front spoiler can generate strong disturbance to the upwind blade and the tower-nacelle system. For a three-blade tower-nacelle system, a force will be generated when each blade rotates through the tower. The rotor will produce a vibration involving the 3-time frequency component in every loop of rotation. There are also high order components including 6-time frequency and 9-time frequency. The amplitude attenuates with the increase of the order [15]. Thus, the equation for the axial vibration of wind turbine can be written as follows [15]:

\[ F(t) = F_0 + \hat{F}_n \cos(3\omega t) + \hat{F}_{2n} \cos 2(3\omega t) + \hat{F}_{3n} \cos 3(3\omega t) + \ldots \]

(4)

where \( F_0 \) is the average pushing force.

2.3 The Excitation Force Generated by Pneumatic Imbalance

Both manufacturing and installation processes may cause deviation in the angle of attack and result in pneumatic imbalance, which will produce a vibration mainly involving the 1-time frequency component in the axial and lateral directions of the tower-nacelle system. The excitation force can be written as follows:

\[ F_i(t) = F_0 + \hat{F} \cos(\omega t) \]

(5)

3. The Axial Vibration Model of the Wind Turbine Tower-nacelle System

3.1 Definition of Mechanical Impedance and Admittance

Mechanical impedance is the ratio between the complex amplitude of the excitation force and the complex amplitude of the response. Set the excitation force on the system as:

\[ f(t) = |F|e^{i(\omega t + \alpha)} = Fe^{i\alpha} \]

(6)

where \( F = |F|e^{i\alpha} \), the steady-state displacement response is:

\[ x(t) = |X|e^{i(\omega t + \beta)} = Xe^{i\beta} \]

(7)

where \( X = |X|e^{i\beta} \), the displacement impedance is:

\[ Z_D = \frac{|F|e^{i(\omega t + \alpha)}}{|X|e^{i(\omega t + \beta)}} = \frac{|F|}{|X|}e^{i(\alpha - \beta)} = \frac{F}{X} \]

(8)

The displacement admittance is:
$H_D = \frac{1}{Z_D} = \frac{|X|}{|F|} e^{(0-\omega)} = \frac{X}{F}$  \hspace{1cm} (9)

It shows the displacement impedance reflects the dynamic rigidity of a system, while the displacement admittance reflects the dynamic flexibility of a system. A vibration system is constituted of three types of components: spring, damper and mass block. The calculation results of impedance and admittance are shown in Table 1.

3.2 The Axial Vibration Model and Mechanical Network Analysis of the Tower-nacelle System

The tower-nacelle system can be simplified to a single degree of freedom vibration system with the following assumptions: the blades are rigid, the rotor is a weightless disk, the mass of the rotor and nacelle concentrates on the center of this disk, the tower is elastic in the transverse direction and only the axial vibration is considered. Then, the vibration model is shown in Figure 2(a) [15,16] and Figure 2(b).

In Figure 2, $m$ is the total mass of the nacelle and blades, $c$ is the axial pneumatic and structural damping of the wind turbine in normal state, $k$ is the rigidity of the tower, $\Omega$ is the rotation rate of the rotor, $F(t)$ is the pushing force.

The force imposed on the system is absorbed by three components at the same time, so the force flow is divided into three parts. The velocity of the three components is the same, while the mass is considered grounded. Therefore, a parallel system is obtained, the mechanical network diagram of it is shown in Figure 3.

Because the axial vibration system is a parallel system, the mechanical impedance of the whole system is the summation of the mechanical impedance of the three components. Thus, the displacement impedance of the system can be written as:

$$Z_D = Z_{Dk} + Z_{Dc} + Z_{Dm} = k + i\omega c - \omega^2 m$$  \hspace{1cm} (10)

The displacement admittance of the system can be written as:

$$H_D = 1/Z_{Dk} + Z_{Dc} + Z_{Dm} = 1/(k + i\omega c - \omega^2 m)$$  \hspace{1cm} (11)

Then, the amplitude and argument of the displacement impedance are:

![Figure 2. The axial vibration model of the wind turbine tower-nacelle system.](image)

![Figure 3. The mechanical network diagram for the axial vibration of the wind turbine system.](image)

<table>
<thead>
<tr>
<th>Components</th>
<th>Impedance $Z_D$</th>
<th>Admittance $H_D$</th>
<th>Impedance logarithmic frequency characteristics</th>
<th>Admittance logarithmic frequency characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>$k$</td>
<td>$1/k$</td>
<td>$\log k$ 0°</td>
<td>$-\log k$ 0°</td>
</tr>
<tr>
<td>Damper</td>
<td>$i\omega c$</td>
<td>$1/i\omega c$</td>
<td>$\log \omega + \log c$ 90°</td>
<td>$-\log \omega - \log c$ 90°</td>
</tr>
<tr>
<td>Mass block</td>
<td>$-\omega^2 m$</td>
<td>$-1/(\omega^2 m)$</td>
<td>$-2\log \omega - \log m$ 90°</td>
<td>$2\log \omega + \log m$ 90°</td>
</tr>
</tbody>
</table>

Table 1. The impedance, admittance and logarithmic frequency characteristics of the three basic types of components
The amplitude and argument of the displacement admittance are:

\[ |H_D| = \frac{1}{\omega \sqrt{c^2 + (\omega m - \frac{k}{\omega})^2}} \]  \hspace{1cm} (14)

\[ \phi = \tan^{-1} \left( \frac{c}{k} \frac{1}{\omega m} \right) \]  \hspace{1cm} (15)

The frequency characteristic curve is shown in Figures 4 and 5.

4. Analysis of Axial Free Vibration of the Tower-nacelle System

Under the condition of free vibration, the axial excitation force of the system is 0. If a loaded wind turbine stops in an emergency, the pushing force on the rotor can generate a static displacement about 0.5 m to the wind turbine. After the emergency stop, the pushing force will suddenly disappear [17]. Therefore, under the initial condition \(x(0) = 0.5 \text{ m}, \dot{x}(0) = v_0, F(0) = 0\), the wind turbine will perform free vibration around the equilibrium position, i.e., \(Z_D = 0\). According to the definition of mechanical impedance, if \(Z_D = 0\), it is the frequency equation of the system, i.e., \(k + i\omega c - \omega^2 m = 0\), replace \(i\omega c\) with the complex variable \(s\), set \(\omega_0 = \sqrt{\frac{k}{m}}\) (undamped natural frequency), \(\xi = \frac{c}{2\sqrt{mk}}\) (viscous damping factor or damping ratio). Then, the frequency equation can be written as follows:

\[ s^2 + 2\xi \omega_0 s + \omega_0^2 = 0 \]  \hspace{1cm} (16)

Two eigenvalues can be calculated from Equation (16).

\[ s_{1,2} = \left( -\xi \pm \sqrt{\xi^2 - 1} \right) \omega_0 \]  \hspace{1cm} (17)

According to Equation (17), the characteristics of free vibration are dependent on the damping ratio \(\xi\). The specific analysis is as follows:

(1) Zero damping (\(\xi = 0\)):

Obviously, \(\xi = 0\) means \(c = 0\), from the equation

\[ s^2 + \omega_0^2 = 0 \]
above, the eigenvalues are imaginary numbers in this case.
\[ s_{1,2} = \pm i\omega_0 \]  
\hspace{1cm} (18)

the response of the system is:
\[ x(t) = X \cos(\omega_0 t - \varphi) \]  
\hspace{1cm} (19)

where,
\[ X = \sqrt{x_0^2 + \left(\frac{v_0 + \xi \omega_0 x_0}{\omega_0 (1 - \xi^2)}\right)^2}, \quad \varphi = \arctan\left(\frac{v_0 + \xi \omega_0 x_0}{x_0 \omega_0 \sqrt{\xi^2 - 1}}\right) \]  
\hspace{1cm} (20)

According to the initial condition of the free vibration, \( x_0 = 0.5 \).

(2) Light damping:
From equation (17), the two eigenvalues are complex conjugate roots in this case.
\[ s_{1,2} = -\xi \omega_0 \pm i \omega_d \]  
\hspace{1cm} (21)

where
\[ \omega_d = \sqrt{1 - \xi^2} \omega_0 \]  
\hspace{1cm} (22)

\( \omega_d \) is the damping natural frequency of the system. Then, the steady-state displacement response is:
\[ x(t) = X e^{-\xi \omega_0 t} \cos(\omega_d t - \varphi) \]  
\hspace{1cm} (23)

where
\[ X = \sqrt{x_0^2 + \left(\frac{v_0 + \xi \omega_0 x_0}{\omega_0 (1 - \xi^2)}\right)^2} \]  
\hspace{1cm} (24)

\[ \varphi = \arctan\left(\frac{v_0 + \xi \omega_0 x_0}{x_0 \omega_0 \sqrt{\xi^2 - 1}}\right) \]  
\hspace{1cm} (25)

According to Equation (23), this value will affect the initial amplitude of the damped free vibration. The vibration amplitude \( A = X e^{-\xi \omega_0 t} \) shows the wind turbine free vibration system performs damped vibration with light damping. Its amplitude attenuates exponentially. The larger the damping ratio \( \xi \) is, the faster the amplitude attenuates. The time course of the attenuation process is shown in Figure 6.

(3) Over-damping \((\xi > 1)\):
From equation (17), the two eigenvalues are real numbers.
\[ s_{1,2} = (-\xi \pm \sqrt{\xi^2 - 1}) \omega_0 \]  
\hspace{1cm} (26)

In this case, both \( s_1 \) and \( s_2 \) are negative real numbers. The system does not generate any vibration, but returns to the equilibrium position directly. When the damping is large, the energy input by the initial excitation will be consumed shortly, and the system does not have sufficient time to generate reciprocating vibration.

(4) Critical damping
This case is the dividing line between the two cases above. Set \( c_0 = 2\sqrt{mk} \) as the critical damping coefficient determined by the system parameters. Then, the critical damping free vibration response of the wind turbine tower-nacelle system under the initial condition is:
\[ x(t) = e^{-\xi \omega_0 t} \left[x_0 + (v_0 + \omega_0 x_0)t\right] \]  
\hspace{1cm} (27)

Obviously, the motion is also non-periodic for critical damping.
5. Analysis of Axial Forced Vibration of the Tower-nacelle System

5.1 Analysis of Axial Vibration Caused by Blade Mass Imbalance

Setting the axial displacement of the wind turbine as \( x(t) \), the excitation force caused by blade mass imbalance is shown in Equation (3). According to the definition of the displacement admittance, the axial vibration response of the wind turbine is:

\[
\begin{align*}
    x(t) &= H_\delta(\Omega)F_\delta(t) \\
    & = \frac{\Delta m r}{m \omega_0^2} \sin \left( \frac{2\pi}{\omega_0} \right) \\
    & \times \left[ \frac{\Omega}{\omega_0} \right]^2 \left( \frac{\omega_0}{\omega_0} \right)^2 \left( 1 - \frac{2\pi}{\omega_0} \right) \left( 1 - \frac{2\pi}{\omega_0} \right) \\ 
    & \times \left( 1 - \left( \frac{\Omega}{\omega_0} \right)^2 \right) + \left( 2\pi \right) \left( \frac{\omega_0}{\omega_0} \right)^2 \\
\end{align*}
\]  

(28)

The amplitude is:

\[
|H(\Omega)| = \frac{\Delta m r}{m \omega_0^2} \left( \frac{\Omega}{\omega_0} \right)^2 \left( 1 - \left( \frac{\Omega}{\omega_0} \right)^2 \right) + \left( 2\pi \right) \left( \frac{\omega_0}{\omega_0} \right)^2 \\
\]  

(29)

The amplitude frequency characteristic curve is shown in Figure 7.

Figure 7 shows the axial vibration of the tower-nacelle system caused by blade mass imbalance is featured with the following characteristics:

1. According to Equation (29), if the rotation rate \( \Omega = 0 \), \(|H(0)| = 0\), indicating all curves start from \(|H(0)| = 0\). If the rotation rate is small, i.e., \( \Omega \ll \omega_0 \), \(|H(\Omega)|\) approaches to 0, indicating the amplitude of low frequency excitation is close to 0. At this time, the dynamic effect is very small, so the forced vibration in this dynamic process can be approximately described by the static process. Thus, the characteristics of the vibration system are dependent on elastic components in this region. It is also consistent with the actual operation of the wind turbine.

2. If the rotation rate \( \Omega \) is high, i.e., the frequency ratio \( \Omega/\omega_0 \gg 1 \), \(|H(\Omega)|\) approaches to a constant value \( \frac{\Delta m r}{m \omega_0^2} \), indicating that when the rotor is in high speed, the system does not have sufficient time to respond to high frequency excitation because of the inertial effect. The vibration characteristics are dependent on the joint effects of elastic components and mass components.

3. If the rotation rate \( \Omega \) is close to the undamped natural vibration frequency \( \omega_0 \), i.e., \( \Omega/\omega_0 \approx 1 \), the \(|H(\Omega)|\) curve exhibits a peak value, indicating the dynamic effect is significant at this point in time. However, the \(|H(\Omega)|\) curve varies greatly in this frequency range for different values of damping rate \( \xi \). If \( \xi \) is large, the peak value of \(|H(\Omega)|\) is small, and vice versa. Thus, the characteristics of the vibration system at this time are dependent on damping components.

4. Acquire the derivative of Equation (29) over \( \Omega \) and set \( \Omega \) to 0, then the maximum value of \(|H(\Omega)|\) can be obtained as:

\[
\frac{\Omega_r}{\omega_0} = \frac{\omega_0}{\sqrt{1 - 2\xi^2}} \\
\]  

(30)

Substituting \( \Omega_r \) into Equation (29), the maximum value of \(|H(\Omega)|\) is written as:

Figure 7. The amplitude frequency characteristic curve of the axial vibration caused by blade mass imbalance.
When the rotation rate is equal to \( \frac{\omega}{c_{87}} \), \( |H(\Omega_r)| \) achieves the maximum value \( |H(\Omega_r)| \), and system resonance occurs. The resonance frequency is \( \Omega_r \). The relationships between the resonance frequency \( \Omega_r \), the damped natural frequency \( \omega_d \) and the undamped natural frequency \( \omega_0 \) are as follows:

\[
\Omega_r > \omega_0 > \omega_d
\]  

(32)

As shown in Figure 6, the resonance does not occur at \( \omega_0 \), but it occurs slightly at higher than \( \omega_0 \). The peak value of \( |H(\Omega_r)| \) shifts toward the high frequency direction along with the increase of \( \xi \). It is exactly opposite to the peak position when the system is subjected to harmonic excitation. As the length of blade is quite large, the rotation belongs to the long flexible rotor rotation system. The centrifugal force point of mass imbalance does not coincide with the connection point of rotor and axle, so the system has a certain arm length. Compared to the system subjected to harmonic excitation, the deflection generated by centrifugal force at the connection point is different. Calculating the deflection at the connection point, it is necessary to displace the force. The added equivalent moment will change the deformation of axle, which will increase the deflection and rotation of the axle, and reduce the critical rational rate of the axle. In addition, as the support structure of blades, the tower system is not rigid. Thus, considering the elastic deformation of tower, the support is equivalent to a series connection of the spring and elastic axle. The combined elastic stiffness becomes lower than the elastic stiffness of the axle itself. Because the exciting force caused by mass imbalance varies with rotational rate. When the damping rate decreases, the rotational rate will decrease, the rotor will take a longer time to reach the maximum amplitude, and the peak value will become larger. Due to the presence of external damping, the center of blade wheel responds to mass imbalance with \( \Omega = \omega_0 \) and the magnitude \( |H(\omega_0)| = \frac{\Delta nm_r}{m} \omega_0^2 \frac{1}{2\xi} \). This value is smaller than \( H(\Omega_r) \). According to Eq. (30), there still exists an increasing trend as the rotational rate \( \omega_0 < \Omega < \Omega_r \). Therefore, if \( \Omega = \omega_0 \), the system response to mass imbalance does not reach the peak value due to the presence of damping. The peak value occurs if \( \Omega > \omega_0 \).

Moreover, Equation (30) also shows if \( 1 - 2\xi^2 < 0 \)
i.e., \( \xi > \frac{\sqrt{2}}{2} \), \( \Omega_r \) does not exist, \( |H(\Omega)| \) does not have a peak value, and the system does not resonate.

The characteristics above suggest the following conclusions:

1. The amplitude of the axial displacement response caused by blade mass imbalance is related to the frequency ratio of rotation rate \( \eta = \frac{\Omega}{\omega_0} \). If \( \eta < \frac{\sqrt{2}}{2} \) and \( \Omega_r = \frac{\omega_0}{\sqrt{1 - 2\xi^2}} \), system resonance occurs, i.e., the system resonates as the rotation rate \( \Omega \) is slightly higher than \( \omega_0 \). The resonance amplitude of the system is dependent on the damping coefficient \( \xi \). If \( \xi > \frac{\sqrt{2}}{2} \), the system does not resonate.

2. The system response amplitude is related to \( \frac{\Delta nm_r}{m} \). Thus, the rotor must be properly controlled to lower the value of \( \frac{\Delta nm_r}{m} \).

3. In the tower design, the natural frequency \( \omega_0 \) should be either higher or lower than the operational rotation rate of the wind turbine to avoid the occurrence of resonance.

5.2 Analysis of Axial Vibration Caused by Tower Front Spoiler

The tower front spoiler and the tower wake in certain cases can produce a periodic excitation force in the axial direction. Setting the axial displacement of the wind turbine as \( X(t) \), the front spoiler excitation force can be calculated from Equation (4). According to the definition of displacement admittance, the axial vibration response of the wind turbine is:
where \( A_0 \) is the static displacement caused by the average pushing force \( F_0 \). \( A_{1b} \) is the static displacement caused by a constant force equivalent to the excitation force \( \bar{F}_{bn} \). \( b(b = 1, 2, 3, \ldots) \) is the order number.

Acquire the derivative of the amplitude over \( \Omega \) and set it to 0. The maximum points of each order of the amplitude are:

\[
\Omega_r = \frac{\omega_0}{3b} \sqrt{2\xi^2 - 1}
\]  

(34)

Resonance occurs as the rotation rate is equal to \( \Omega_r \). At this time, the relationships between the resonance frequency \( \Omega_r \), the damped natural frequency \( \omega_d \) and the undamped natural frequency \( \omega_0 \) are as follows:

\[
\Omega_r < \frac{\omega_d}{3b} < \frac{\omega_0}{3b}
\]  

(35)

Equation (35) shows that system resonance does not occur at \( \frac{\omega_0}{3b} \), but it occurs at slightly smaller than \( \frac{\omega_0}{3b} \). The peak value of the amplitude frequency shifts toward the low frequency direction along with the increase of \( \xi \). If \( \xi > \frac{\sqrt{5}}{2} \), \( \Omega_r \) does not exist, the amplitude frequency has no peak value, the system does not resonate, and the dynamic displacement is smaller than the static displacement.

5.3 Analysis of Axial Vibration Caused by Pneumatic Imbalance

Setting the axial displacement of the wind turbine as \( x(t) \), the excitation force of pneumatic imbalance can be calculated from Equation (5). According to the definition of the displacement admittance, the axial vibration response of the wind turbine is:

\[
X(t) = A_0 + \sum_{n=1}^{\infty} \left( \frac{A_n}{1 - \left( \frac{3b\Omega}{\omega_0} \right)^2} \right) \cos \left( \frac{2\xi \Omega t}{\omega_0} \right)
\]  

(36)

where \( A_0 \) is the static displacement caused by the average pushing force \( F_0 \). \( A_1 \) is the static displacement caused by a constant force equivalent to the excitation force \( \bar{F} \). Acquire the derivative of the amplitude over \( \Omega \) and set it to 0. The maximum points of each order of the amplitude are:

\[
\Omega_r = \omega_0 \sqrt{2\xi^2 - 1}
\]  

(37)

System resonance occurs as the rotation rate is equal to \( \Omega_r \). At this time, the relationships between the resonance frequency \( \Omega_r \), the damped natural frequency \( \omega_d \) and the undamped natural frequency \( \omega_0 \) are as follows:

\[
\Omega_r < \omega_d < \omega_0
\]  

(38)

Equation (38) shows that the system resonance does not occur at \( \omega_0 \), but it occurs at slightly smaller than \( \omega_0 \). The peak value of the amplitude frequency shifts toward the low frequency direction along with the increase of \( \xi \). If \( \xi > \frac{\sqrt{5}}{2} \), \( \Omega_r \) does not exist, the amplitude frequency has no peak value, the system does not resonate, and the dynamic displacement is smaller than the static displacement.

5.4 Calculation Example

Literature [18] defined the maximum deviation of blade imbalance in the actual production. On such basis, this study constructed a model with respect to the factor of blade imbalance. The specific model parameters are shown in Table 2. MATLAB was used to simulate the amplitude frequency characteristics of blade mass imbalance. The results are shown in Figure 8.
As shown in Figure 8, the resonance does not occur if $\Omega = 3\text{rad/s}$, but it occurs if $\Omega > 3\text{rad/s}$; the $|H(\Omega)|$ curve differs a lot for different values of damping ratio $\xi$. If $\xi$ is large, the peak value of $|H(\Omega)|$ is relatively small; if $\xi$ is small, the peak value of $|H(\Omega)|$ is relatively large. If the rotational rate is 3 times or greater than the natural frequency, the amplitude approaches to a constant value. The result of this calculation example is consistent with the conclusion of the section 5.1.

6. Conclusions

This study analyzed the excitation sources that can trigger axial vibration to the wind turbine tower-nacelle system. An axial vibration model was established by simplifying the tower-nacelle system as a single degree of freedom vibration system. According to this model, the mechanical impedance method was applied to construct the machinery network diagram for the tower-nacelle system. Then, the displacement impedance and admittance of the system were calculated. The axial free vibration and forced vibration of the tower-nacelle system were analyzed theoretically by considering the displacement admittance as the transfer function. The following conclusions were drawn:

(1) In the case of light damping, the axial free vibration of the wind turbine tower-nacelle system performs damped vibration, and its amplitude attenuates exponentially. The system does not produce periodic reciprocating vibration in over-damping and critical damping states.

(2) The amplitude of the axial displacement response caused by blade mass imbalance is related to the frequency ratio of rotation rate $\eta = \frac{\Omega}{\omega_0}$. System resonance occurs if the rotation rate $\Omega$ is slightly higher than $\omega_0$. The resonance amplitude of the system is dependent on the damping coefficient $\xi$. Its peak value shifts toward the high frequency direction along with the increase of $\xi$.

(3) The amplitude of the axial displacement response caused by tower front spoiler is related to the frequency ratio of rotation rate $\eta = \frac{\Omega}{\omega_{0.5}}$. System resonance occurs as the rotation rate $\Omega_{br} = \frac{\omega_0}{3b} \sqrt{\frac{\xi^2 - 1}{\xi^2 - 2}}$.

The peak value of the amplitude frequency shifts toward the low frequency direction along with the increase of $\xi$.

(4) The amplitude of the axial displacement response caused by pneumatic imbalance is related to the frequency ratio of rotation rate $\eta = \frac{\Omega}{\omega_0}$. System resonance occurs as the rotation rate $\Omega_r = \omega_0 \sqrt{\frac{\xi^2 - 1}{\xi^2 - 2}}$.

The peak value of the amplitude frequency shifts toward the low frequency direction along with the increase of $\xi$.

(5) If $\xi > \sqrt{\frac{\xi^2 - 2}{2}}$, $\Omega_r$ does not exist, the amplitude frequency has no peak value, and the system does not resonate.

<table>
<thead>
<tr>
<th>Table 2. The list of parameters for blade mass imbalance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Blade 1 mass/kg</td>
</tr>
<tr>
<td>Blade 2, 3 mass/kg</td>
</tr>
<tr>
<td>Distance from the gravity center of blade to the gravity center of wheel hub $r/m$</td>
</tr>
<tr>
<td>System non-damping free vibration frequency $\omega_0/\text{rad} \cdot \text{s}^{-1}$</td>
</tr>
</tbody>
</table>

Figure 8. Calculation the example of the tower-nacelle axial vibration caused by blade mass imbalance.
Acknowledgment

This work was supported by Science and Technology Program of Ministry of Housing and Urban-Rural Development of People's Republic of China (2014-K7-020).

Nomenclature

Latin Upper Case

$A$ vibration amplitude
$A_0$ static displacement caused by the average pushing force
$A_{1b}$ static displacement caused by a constant force equivalent to the excitation force
$F$ centrifugal force
$F(t)$ excitation force
$F_0$ average pushing force
$H_D$ displacement admittance
$Z_D$ displacement impedance

Latin Lower Case

$c$ damper
$c_0$ critical damping coefficient
$f(t)$ excitation force
$k$ rigidity of the tower
$m$ the mass
$r_s$ distance from the center of gravity to the center of rotation of the blades
$x(t)$ displacement response
$x_0$ static displacement $x(0) = 0.5$ m,
$v_0$ $x(0)$, initial velocity

Greek

$\eta$ the frequency ratio of rotation rate and undamped natural frequency
$\xi$ viscous damping factor or damping ratio
$\phi$ argument
$\Omega$ the rotation rate of the rotor
$\omega_0$ undamped natural frequency
$\omega_d$ the damping natural frequency
$\Omega_r$ the resonance frequency

Subscripts

$b$ order number

References


Manuscript Received: Jun. 30, 2015
Accepted: Nov. 9, 2015