Mitigation of Resonance for High Speed Train-bridge Systems Considering Overhanging Beam Effects

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Abstract

Short overhanging beams usually exist at both ends of a simply supported bridge. As a train passes through the overhanging beams of the bridge at high speeds, the overhanging beams would be subjected to intensive vibrations, from which the dynamic response of the train-bridge system would be built up. To mitigate the amplified response of the train-bridge system due to the overhanging beam effects, an end rotational spring to restrain the relative rotations is equipped between the free ends of the adjacent beams. By the vehicle-bridge interaction finite element analysis, the numerical studies indicated that the overhanging beams play a key role in amplifying the response of the train-bridge system. Even so, the proposed end restraintscan reduce the amplified response of the VBI system significantly.

Key Words: High Speed Rail, Interaction Dynamics, Overhanging Beam, Resonance, VBI Dynamics, Vibration Reduction

1. Introduction

Speedy delivery, high capacities, potential relieve congestion, energy efficiency, and less amount of land in operation are the advantages of high speed rails (HSR) in intercity express-transport. Focusing on the dynamic behavior of rail bridges due to moving loads, many engineering researchers and scientists have devoted themselves to studying the resonant response of a bridge subjected to a series of moving loads with regular intervals. Based on the considerations of operation and running safety, a rail bridge must be designed to provide sufficient structural strength for a train to travel over it at high speeds [1]. For this, the vehicle-bridge interaction (VBI) dynamics become an important vibration problem of high speed rails [2,3]. In analyzing the VBI problem, two sets of equations of motion are written, one for the supporting bridge and the other for each of the moving vehicles over the bridge. As the contact points between the running vehicles and the bridge move from time to time, the system matrices must be updated and factorized at each time step in an incremental time-history analysis. Considering the complex procedure, the two sets of differential equations, in general, are solved by the following computational approaches [2]: (1) full vehicle-bridge coupling system, (2) iterative scheme, and (3) dynamic condensation method. With the VBI considerations in high speed rails, many interesting topics were investigated, such as wind effects, seismic analysis, and ground movements. However, relatively little information is available on the VBI dynamic problem for a train running on simply supported bridges considering the effects of overhanging beam at support ends (see Figure 1).

In this paper, two objectives will be conducted: (1) to
investigate the influence of the overhanging beam effects on the VBI system of high speed rails; and (2) to reduce the amplified response of the VBI system using an end displacement restraint. By the VBI finite element modeling, a number of multi-span simply supported bridges are modeled as a series of beam elements and a train as a sequence of identical moving two-axle systems. From the present study, the overhanging beams may result in significant amplification on the response of the train-bridge system as the train travels over the bridge with resonant speed. The proposed end restraint is an efficient tool in reducing the amplified response of the VBI system.

2. VBI Problem

Figure 2 shows a train is traveling over a bridge. As indicated, at a certain instant during the passage of the train, some elements of the bridge will be directly acted upon by the two-axle systems, while the others are not. The train-bridge system is coupled and the system matrices are time-varying in performing dynamic response analysis. To overcome the time-varying nature of the problem, Yang et al. [4,5] proposed a condensing method to represent the degrees-of-freedom (DOFs) of the two-axle system by those of the beam element in contact, in which the two-axle systems are discretized in advance by Newmark’s finite difference formulas. The result is a VBI element that possesses the same number of DOFs as the parent beam element, while the properties of symmetry and bandedness are preserved. Such an element is particularly suitable for analyzing the dynamic responses of the vehicle-bridge interaction problems concerning both the bridge and vehicle responses. Readers who are interested in derivation of the VBI element should refer to the paper by Yang et al. [4] for further details.

In this study, the Euler-beam element is used to simulate the bridge structure, of which the axial displacement is interpolated by linear functions, and the transverse displacements by cubic interpolation (Hermitian) functions. The number of vehicles directly acting on the bridge changes as the train keeps moving, and so do the contact points between each bridge element and the moving vehicles. Typically, a beam element will be acted upon by a wheel-set, as shown in Figure 3. Such an element has been referred to as the planar vehicle-bridge interaction (VBI) element. For this element, two sets of equations of motion can be written, one for the bridge element and the other for the vehicle system:

\[
[m_b]\{\ddot{u}_b\} + [c_b]\{\dot{u}_b\} + [k_b]\{u_b\} = \{p_b\} - \{N\}_e \{f_e\}
\]  

\[
[m_v]\{\ddot{u}_v\} + [c_v]\{\dot{u}_v\} + [k_v]\{u_v\} = \{p_v\} + \{f_e\}
\]

where \([m_b]\), \([c_b]\), \([k_b]\) = the mass, damping, and stiffness matrices of the beam element, and \([p_b]\) and \([f_e]\) = the external nodal loads and the contact forces existing between the sprung mass and the beam element; \([m_v]\), \([c_v]\), \([k_v]\) = the mass, damping, stiffness matrices of the sprung mass, \([p_v]\) = the weight of the lumped part of the vehicle.
and and \( \langle N_i \rangle = \) the interpolation function vector. The preceding two equations (1) and (2) are coupled through the contact forces \{f_i\}, while the coefficients matrices of the planar vehicle system vary according to its acting position on the bridge.

2.1 Planar VBI Element

As shown in Figure 3, the simplified planar vehicle model is represented by a two-axle system with one rigid car body supported by two suspension systems moving on beams [3]. The notations shown in these figures are given: \((u_v, \theta_v)\) are vertical displacement and pitching rotation at midpoint of the rigid car body, \((k_s, c_s)\) the secondary suspension of the bogies, \((m_w)\) the mass of the wheel-set system, \(k_p\) the stiffness on the primary suspension of each axle, \((d, D)\) the axle interval and coach length, and \((M_v, I_v)\) the mass and inertia to rotation of the vehicle body.

The differential equations of the two-axle system are given by [3]:

\[
\begin{bmatrix}
M_v & I_v \\
I_v & m_w
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_v \\
\dot{\theta}_v
\end{bmatrix}
+
\begin{bmatrix}
2c_s & 0 \\
0 & c_s
\end{bmatrix}
\begin{bmatrix}
d^2/2 & -c_s/2 \\
-c_s/2 & c_s/2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_w \\
\dot{\theta}_w
\end{bmatrix}
+
\begin{bmatrix}
2k_s & 0 \\
k_s & k_s\end{bmatrix}
\begin{bmatrix}
d^2/2 & -k_s/2 \\
-k_s/2 & k_s/2\end{bmatrix}
\begin{bmatrix}
\ddot{u}_w \\
\dot{\theta}_w
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

where \(u_{wk} = \) vertical displacement of the \(k\)-th wheel-set.

Following the reference [2], the following power spectrum density (PSD) function is adopted to simulate the vertical profile of the guideway track:

\[
S(\Omega) = \frac{A_v \Omega^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}
\]

where \(\Omega = \) spatial frequency, and \(A_v = 1.5 \times 10^{-7} \) m, \(\Omega_r = 2.06 \times 10^{-6} \) rad/m, and \(\Omega_c = 0.825 \) rad/m are relevant parameters. Using Eq. (4) as the PSD function for the vertical profile of the guide-rail irregularity has the advantage that it won’t become indefinite even when the spatial frequency \(\Omega\) is zero, as can be appreciated from Figure 4 for the profile used in this study.

2.2 Modeling of End Restraint at the Overhanging Beams

To mitigate the free end displacements of the overhanging beams of a simple beam, this study proposed a set of flexural bars to be installed on the overhanging beams of two adjacent beams, as shown in Figure 5. The flexural bar is designed as one movable end along the longitudinal direction of the beam for temperature expansion but restrains relative displacement for providing additional flexural resistance. If the flexural stiffness of the restraint bar is equal to \(12E_sI_s/L_s^3\) then the restraint stiffness can denoted as \(K_r\) in the transverse direction. Here \(E_s, I_s\) is total flexural rigidity of the bars and \(L_s\) the effective length and the stiffness matrix of the end restraint element can be expressed as

\[
k_r = \begin{bmatrix} K_r & -K_r \\ -K_r & K_r \end{bmatrix}
\]

Figure 4. Vertical profile of rail irregularity.

Figure 5. Flexural bars connecting the overhanging beams of two adjacent beams.
Then one can assemble the rotational stiffness matrix with the VBI element described in section 2.1 by the direct stiffness method [6].

2.3 VBI Analysis Using FEM

Because the VBI element and its parent element are fully compatible, conventional element assembly process can be applied with no difficulty to form the equations of motion for the entire vehicle-bridge system, that is

\[ [M][\ddot{U}_b] + [C][\dot{U}_b] + [K][U_b] = [P_b] \]  \hspace{1cm} (6)

where \([M],[C],[K]\) respectively denote the mass, damping, and stiffness matrices of the entire vehicle-bridge system, \([U_b]\) the bridge displacements, and \([P_b]\) the external loads acting on the bridge. The preceding equations are typical second-order differential equations, which can be solved by a number of time-marching schemes. In this study, the Newmark \(\beta\) method with constant average acceleration is employed again to render the preceding equations into a set of equivalent stiffness equations, from which the bridge displacements \([U_b]\) can be solved for each time step. Once the bridge displacements \([U_b]\) are made available, the bridge accelerations and velocities can be computed accordingly. By a backward procedure, the response of the sprung masses can be computed as well on the element level, which serves as indicator of the riding comfort [4].

3. Resonant Speed and Speed Parameter

Owing to regular feature of identical intervals \(D\) of bogie-set arrangement for a train traveling over a bridge at speed \(v\), the bridge may experience a periodic action of successive moving loads with an exciting passage frequency \(v/D\). Once the exciting frequency matches any of circular natural frequencies \((f)\) of the bridge, the resonant response will be developed on the bridge [7,8]. Let us denote the resonant speed as \(v_{res}\), which is equal to \(fD\). In reality, sub-resonance of acceleration response may also be generated on the bridge as the moving loads pass through the bridge at the speed of \(v_{sub, res} = fD/j\), where \(j\) represents the number of complete cycles of oscillation of the beam occurring during the passage of two adjacent loads [9], which has duration of \(D/v\). In this study, such a resonant speed with \(j \geq 2\) is called sub-resonant speed.

Let \(L\) represent the span length of a simply supported bridge. The speed parameter \(S\) is defined as the ratio of the first exciting frequency of a moving load with constant speed \(v\), i.e., \(\pi v/L\), to the fundamental frequency \(\Omega\) (= 2\(\pi f\)) of the bridge [2],

\[ S = \frac{\pi v}{\Omega L} \]  \hspace{1cm} (7)

According to the resonant speed given previously, that is, \(v_{res} = fD\), the corresponding resonant speed parameter is denoted as \(S_r\) and can be expressed as \(S_r = D/2L\). As for the sub-resonant parameters, they are given as \(S_r/j\). Since the resonant response may result in the ballast destabilization and diminishing of operational safety of a running trains on track structures, the maximum acceleration will be employed to evaluate the dynamic interaction of vehicle-bridge system for the beams with overhanging arms in the following numerical examples [1].

4. Finite Element Modeling of the VBI System

As indicated in Figure 6, Figure 6(a) represents a conventional simply supported bridge for typical structural analysis of railway bridges, and Figure 6(b) the bridge with short overhanging beams at both ends for practical considerations. They are named Case I and Case II, respectively.

The properties of the beams and planar two-axle sys-

Figure 6. VBI model (a) Simple beam (Case I); (b) Simple beam with overhanging beams (Case II).
tem are listed in Tables 1 and 2, respectively. Here, $f_1$ denotes the fundamental frequency of the first mode and $v_{res} = f_1 D$ the first resonant speed of the beam under the moving train loads. In the following examples, 16 coaches are considered for the moving train and the simple beam shown in Figure 6(a) is modeled by 10 beam elements and the beam in Figure 6(b) by 2 beam elements for each of the overhanging beams and 6 beam elements at the central span. Based on Newmark’s method of direct integration with constant average acceleration of $\beta = 1/4$ and $\gamma = 1/2$ [2], numerical solutions for the dynamic response of the bridge due to successive moving train loads have been computed using the incremental-iterative method associated with the Newmark method [10] with the time step of 0.005s.

5. Numerical Examples

In this study, only the acceleration response of the VBI system is of concern since the vertical acceleration is regarded as an indicator of the riding comfort of a traveling high-speed train and the operational safety of oscillating ballasts on a rail bridge.

5.1 Dynamic Response Analysis of the VBI Systems

To demonstrate the influence of overhanging beams on the VBI system for a train traveling on a bridge, the track irregularities would be neglected in performing VBI analysis in this example. Let the train model (see Table 2) travel the two bridge models shown in Figure 6 with their corresponding resonant speeds (see Table 1), respectively. The vertical acceleration at the right end of the running coach is denoted as:

$$a_v = \ddot{w} + \frac{D}{2} \dot{\theta}_v$$  \hspace{1cm} (8)

The time-history responses of the midpoint acceleration of the two bridges and the first and last coaches have been plotted in Figures 7 and 8, respectively. The numerical results indicated that the overhanging effects may amplify both dynamic responses of the running vehicles and the bridge. As shown in Figure 8, due to the resonance phenomenon occurring in the vibrating beam, the dynamic response of the last two-axle system ($N = 16$)
running on the beam has been dramatically amplified in comparison with that of the first one since it may experience resonant response transmitted from the vibrating beam.

Figures 9 and 10 present the maximum acceleration for the beam and the running train against the speed parameter \( S \), which has been defined in Eq. (7), respectively. Here, the maximum vertical acceleration of the running two-axle systems shown in Figure 10 is defined as:

\[
a_{r, \text{max}} = \max \left( \left| \ddot{u}_r \pm \frac{D\dot{\theta}_r}{2} \right| \right)
\]  

(9)

As indicated, both the responses of the bridge and running two-axle systems are significantly amplified due to the effects of overhanging beams, especially at the resonant speed parameter of \( S_r = 0.357 \) \((= D/2L)\). Even so, the amplification at sub-resonant speeds is much smaller than that of the resonant speed since the main resonant speed is much higher the sub-resonant ones.

### 5.2 Response Mitigation for the VBI Systems

Figure 11 depicts the schematic diagrams of multi-unit simple beams with various end conditions. Figure 11(a) represent conventional simply-supported beam, Figure 11(b) the simple beam with overhanging beams, and Figure 11(c) the simple beam with end restraints of \( K_t = 6(12EI/L^3) \) at overhanging beams. In addition, the track irregularities are taken into account in the VBI dynamic analyses. Let the train, as shown in Table 2, travel over the multiple beams shown in Figure 11(a)-(c), respectively. The numerical results for the maximum accela-
tion response of the midpoint of the beams and the running coaches are shown in Figures 12 and 13, respectively. As indicated, the end restraints installed at the ends of overhanging beams provide significant response reduction for the VBI system. This concludes that the proposed end restraint equipment is available to reduce train-induced vibration of rail bridges with overhanging beams.

6. Conclusions

In this study, the amplification effect of overhanging beams on dynamic response of a simply supported beam subjected to a high speed train was investigated. To mitigate the resonant response of the train-bridge system due to overhanging beam effects, this study presented an end restraint device installed on the overhang beams of two adjacent beams. The numerical results indicated that the proposed end restraint is effective to reduce the vibration of the VBI system at resonance. The following conclusions are drawn:

1. As a train travels over a simply supported bridge with resonant speed, the influence of overhanging beams on the dynamic response of the VBI system should be taken into account in performing VBI dynamic analysis.
2. The proposed end restraint approach is easy and inexpensive to be installed on overhanging beams for reducing train-induced vibration of the rail bridges.
3. The present study may be regarded as a preliminary work to investigate the VBI dynamics for railway bridges with overhanging beams. A future work considering train-track-bridge interactions can be explored.

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References


Figure 12. Maximum midpoint acceleration vs. S plot.

Figure 13. $a_{r,max}$ vs. S plot.


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