Optimal Design of Urban and Rural Public Transportation Network Based on Spatio–Temporal Constraints

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Abstract

Urban and rural public transportation is an important link between urban and rural areas. The design of urban and rural public transportation network plays an important role in promoting the integration of urban and rural public transportation. In this paper, the urban and rural public transportation network optimization design considering both space and time constraints is put forward. Using large-scale data technology and internet technology, we determined time and space limits of the public transportation network. Based on the optimization of the direct transit efficiency, the secondary transfer efficiency and the number of lines, the optimization model is constructed. The tabu search algorithm for the model is designed and the efficiency of the algorithm is estimated. The test results show that the algorithm has a fast convergence rate and a stable result. The conclusion of this paper provides a good theoretical support for the rational design of urban and rural public transportation network. It is also conducive to promote the further development of urban and rural public transport.

Key Words: Urban and Rural Transit, Network Optimization, Space-time Constraint, Comprehensive Passenger Transport Hub, Tabu Search Algorithm

1. Introduction

Urban and rural public transport is to connect the city and township public transport. It is an important link between urban and rural areas, serving people’s travel, closely related with people’s production and daily life. With the increase and frequent exchanges between urban and rural areas, urban and rural integration is gradually accelerated. Actively integration of urban and rural public transportation plays an important role in promoting equalization of urban and rural basic public transportation services.

Urban and rural public transportation network design is a key factor to promote the integration of urban and rural public transportation development, it has also become a hot topic for scholars. Using GIS technology, Ramirez and Seneviratne [1] established a urban and rural public transportation network model. In this model each route may be laid on the bus lines are considered for resistance impact, the magnitude of the resistance is determined by the number of people working around the bus station. Wang Wei et al. [2,3] put forward a straight line design idea called “one route for one line” for public transportation network, and explored the network and sites layout methods by using this model. These studies are mainly from the perspective of space to design urban and rural public transport network. Patanik et al. [4] established an urban and rural transportation network optimization model, which aimed for the smallest weighted sum of passenger travel time and business hours. The model is solved by genetic algorithm. He et al. [5] introduced the transfer time matrix, put forward an algorithm to solve the optimal path of public transportation network. The algorithm takes into account the combined effects of...
bus transfer times, transfer point selections and total travel costs. Lee and Vuchic [6] established mathematical models based on the minimum total travel time, including waiting time, travel time, transfer time, walking time and so on, they also considered the starting frequency and route design. The main results of these studies are from the perspective of time to design urban and rural public transport network.

Although the existing research has made some achievements in the design of urban and rural public transportation network, few research results have taken into account the time and space constraints of urban and rural public transport. Obviously, urban and rural public transportation network design should not only consider the public welfare of public transportation services, but also take into account the economic performance of the services. Public welfare is mainly for the coverage of the township as much as possible. Economic performance is mainly to ensure that travel time for each trip more economical. Therefore, in order to satisfy the public welfare and economy of public transportation, it is necessary to consider their time and space constraints in the design of urban and rural public transportation network. Based on this, this paper puts forward the optimal design of urban and rural public transportation network with both time and space constraints. The tabu search algorithm was used to solve the model, and the efficiency of the algorithm was tested.

2. Spatial and Temporal Constraints of Urban and Rural Public Transportation Network Design

2.1 Site Selection
Urban and rural transit station is the basis of urban and rural public transport network. Site selection is to meet the needs of urban and rural residents’ travel as a fundamental starting point. As the urban and rural bus line is the link between cities and towns, therefore, urban and rural public transport terminal is generally selected in the city center near the urban integrated transport hub. Through urban and rural bus terminal station we can achieve seamless connection between the city bus lines and urban and rural public transport lines. Urban and rural transit station is generally selected in the way through township and village, they should not exceed 2 km from the township and village. Each township and village shall have at least one transit station.

2.2 Spatial Constraints
Taking into account the welfare of urban and rural public transport, bus lines should cover all townships within the city as much as possible. According to “Suggestions on the active promotion of integration development of urban and rural transportation” issued by the Ministry of Transport, this paper proposes to build the concept of “20 km-urban and rural public transport service area”. The service area refers to an area within 20 km radius covered, centered by the city comprehensive transportation hub. All the towns and villages in the region can be candidate station for urban and rural transport, none will be considered as transit outside of the region. Therefore, the “20 km-urban and rural public transport service area” is the spatial constraint for design of urban and rural public transportation network.

2.3 Time Constraints
Taking into account the economy of urban and rural public transport, it is necessary to optimize each bus travel time. If the bus travel time per trip is too short, the number of passengers served along the way will be reduced, thus it will short the income. If the bus travel time per trip is too long, it will less attraction to passengers, while increase operating costs. According to the intention survey of the bus trip, 1 hour is the maximum length that urban and rural passengers can bear. Therefore, this paper proposed to build “1 hour-urban and rural transit circle”. The board range is 1 hour-driving distance from the city’s comprehensive transportation hub. Urban and rural transit stations can not be laid out the border. Therefore, “1 hour-urban and rural transit circle” is the time constraints for urban and rural transit network design.

2.4 Space-time Constraints
In the optimization of design for urban and rural public transportation network, this paper proposed both welfare and economic for urban and rural public transport. It is necessary to meet not only the space constraints of “20 km-urban and rural public transport service area”, but also the time constraints of “1 hour-urban and rural
transit circle”. With the rapid development of large data technology and internet technology, we can use ArcGIS Desktop 10.2 to achieve these two constraints on open maps.

1. Using ArcGIS Desktop 10.2 Buffer analysis tool to achieve the space constraint of “20 km-urban and rural public transport service area”: calculate the coverage of the area in which urban integrated transport hub as the center, and 20 km as radius, see Figure 1.

2. Using ArcGIS Desktop 10.2 Isochronal line analysis tool to achieve the time constraint of “1 hour-urban and rural transit circle”: direct call to GAODEI map API interface, based on real-time road traffic from GAODEI map, choose urban integrated transport hub as the starting point of the line, select the bus travel mode, set travel time for 1 hour, then calculate the “1 hour-urban and rural transit circle”, see Figure 2.

Only area that in the intersection of regions of “20 km-urban and rural public transport service area” and “1 hour-urban and rural transit circle” can be considered in the designed urban and rural public transportation network. This can be easily determined by the processing of ArcGIS technique described above. As is shown in Figure 3.

3. Optimized Design Model of Urban and Rural Public Transportation Network

3.1 Problem Description

Urban and rural public transportation network optimization design can be described as follows: in accordance with the set optimization goals, under the time and space constraints, set the urban comprehensive transport hub as urban and rural public transport terminal station, start from which the transport bus lines will be, heading to the surrounding towns and villages. By connecting a number of urban and rural transit station along the way they can be interwoven into a network. As shown in Figure 4.

The design of urban and rural bus lines should meet the following requirements:

1. Each urban and rural bus station must be laid under the time and space constraints;
2. Each urban and rural bus lines are from the city com-
prehensive transport hub, urban comprehensive transport hub is the terminal station of urban and rural public transport;
(3) Each city integrated transport hub can radiate a number of urban and rural bus lines;
(4) Each urban and rural transit bus stop only corresponds to an urban and rural bus line;
(5) Optimization of network design should meet the needs of direct travel between sites as much as possible, to reduce the demand for transfer, and try to reduce the number of lines laid.

3.2 Optimization Method

From the description of this problem, it has a great similarity with vehicle routing problem (VRP). They are both from the fixed nodeto the surrounding radiation and aim to determine the appropriate transport routes. Urban and rural public transportation design involves the urban and rural public transport terminal station, passengers, urban and rural transit station, urban and rural public transport network, bus and other elements. They are similar to the distribution center, goods, customers, transport networks, and vehicles in the vehicle routing problem [7, 8].

In view of the commonality of the two problems, combined with the characteristics of urban and rural public transport, urban and rural public transportation network optimization design problem can be regarded as a special type of open vehicle routing problem. Because each bus station has on and off demand, which is similar to a pickup and delivery problem: each passenger gets on at one bus node and gets off at another bus node, which is similar to a pairs of delivery VRP problem [9, 10]. Therefore, this paper adopts the optimization method for VRP to construct an optimization model of urban and rural public transportation network.

3.3 Model Construction

Let \( G = (V, E) \) be a connected network, \( V \) is the node set, and \( V = \{v_1, v_2, \ldots, v_n\} \) represents \( n \) urban and rural transit stations. Among them, \( V_1 \) is the urban and rural public transport terminal station. \( E \) is the edge set, and \( E = \{e_1, e_2, \ldots, e_m\} \) is the edge connecting \( m \) urban and rural transit stations. \( R \) is for the road (chain) set, \( R = \{r_1, r_2, \ldots, r_M\} \) denotes \( M \) urban and rural lines. \( r_k \) denotes the k-th urban and rural public transport lines, \( r_k = \{r_{k1}, r_{k2}, \ldots, r_{kn_k}\}, \) where \( r_{ki} \) is the order of urban and rural transit stations on the k-th line as \( S. \) \( rd_1 \) represents the first station of the k-th line, \( rd_{nk} \) denotes the end station of the k-th line, and \( n_k \) denotes the number of urban and rural transit stations on \( r_k \). \( D = (d_{ij})_{n \times n} \) represents the transit station distance matrix. \( d_{ij} \) represents the distance between the transit station \( v_i \) and the transit station \( v_j, i, j = 1, 2, \ldots, n; \) OD = \( (q_{ij})_{n \times n} \) represents the transit demand matrix. \( q_{ij} \) represents the OD demand between the transit stations \( v_i \) and \( v_j, i, j = 1, 2, \ldots, n; \) Let D and OD be symmetric matrices.

The model is constructed with the following objectives:
(1) Direct transportation efficiency

The direct transportation efficiency index is taken as the first target of the model. It is defined as the ration of the direct passenger flow and the total line length in the urban and rural public transportation network. It reflects the level of direct passenger traffic carried by the line per unit length.

\[
\eta = \frac{\sum_{k=1}^{M} \sum_{j=1}^{n_k} d_{r_kj} q_{r_kj}}{\sum_{k=1}^{M} \sum_{i=1}^{n_k} d_{r_ki} q_{r_ki}}
\]  

(1)

(2) The second transfer efficiency

The secondary transfer efficiency index is taken as the second target of the model. It is defined as the proportion of the total passenger transport demand that must be met through the urban integrated transport hub to achieve the purpose of trips by more than two transfers. Which is:
In the above formula, \( h_{ij} \) is determined by the second transfer matrix \( H = (h_{ij})_{n \times n} \) of the urban and rural public transport network. \( h_{ij} \) is variable 0 or 1, while \( h_{ij} = 1 \), it means from \( i \) to \( j \) need secondary or more transfer. While \( h_{ij} = 0 \), it means that from \( i \) to \( j \) is direct arrival or only need one transfer. \( p_{ij} \) is determined by the reach matrix \( P = (p_{ij})_{n \times n} \) of the urban and rural public transport network. \( p_{ij} \) is variable 0 or 1, while \( p_{ij} = 1 \), that means it is connected from \( i \) to \( j \), including either direct arrival or transfer, while \( p_{ij} = 0 \), that means \( i \) to \( j \) is not accessible.

(3) The number of bus lines

The number of bus lines \( M \) is set as the third target of the model.

Therefore, the mathematic model of urban-rural public transport network optimization design can be described as:

Objective function:

\[
\begin{align*}
\min & \ M \\
\max & \ \eta \\
\min & \ \phi
\end{align*}
\]

Restrictions:

\[
\begin{align*}
V &= W \cap X \\
r_i \cap r_j &= v_i, \quad i, j = 1, 2, \cdots, M, \quad \forall i \neq j \\
r_{n_k} &= r_i \\
q_n &= \sum_{k=1}^{n-1} \sum_{s=1}^{n-1} q_n_{rs}
\end{align*}
\]

Equation (6) shows that the urban and rural public transit stations are constrained by time and space, \( W \) refers to the regional collection which satisfies “20 km-urban and rural transit service area”, and \( X \) is the regional collection of “1 hour-urban and rural transit circle”; Equation (7) indicates that each urban and rural bus line starting from the city comprehensive transport hub (urban and rural public transport terminal station), only where each urban and rural bus lines intersection; Equation (8) is to ensure that this line is an open style (non-closed line); Equation (9) indicates the direct traffic of the Kth urban and rural line \( r_k \).

4. Algorithm Analysis

Tabu search (TS) algorithm is a global step-by-step optimization algorithm. Glover [11] proposed a set of algorithms in 1986, tabu search algorithm by introducing a flexible storage structure and the corresponding tabu criteria to avoid round about search, the contempt criterion is used to forgive some tabu good state, thus ensuring effective diversification search to eventually achieve global optimization. Based on this idea, this paper designs a tabu search algorithm to solve this model.

4.1 The Representation of the Solution

The representation of the solution directly determines the performance of the algorithm. First of all, the form of the solution is determined as composed of urban integrated passenger hub (urban and rural public transport terminal station) and urban and rural transit station arrangement. Then, natural number coding rules are used to generate \( n + 1 \) non-repetitive natural number permutations for \( n + 1 \) stations. Among them, 0 means the city integrated passenger hub, other natural numbers represent urban and rural transit stations. Such as \{01543 0289 067\}, which means 3 urban and rural bus lines, 01543, 0289 and 067.

4.2 Initial Solution

The algorithm generates the initial solution in a random way.

4.3 Neighborhood Structure

The algorithm designs four neighborhoods, which are reassignment, vertex exchange, 2-opt and “tail” exchange. Randomly select two sites on the same line or different lines, and randomly perform one of the four neighborhood transformations [12].

4.4 Evaluation of the Solution

The evaluation of the solution is used to determine
the merits of a solution, it is achieved by constructing an evaluation function \( Z \). \( Z \) is constructed as a function proportional to the direct transport efficiency, inversely proportional to the secondary transfer efficiency and the bus line, and penalizes those who do not satisfy the constraint. See Eq. (10).

\[
Z = \frac{\eta}{\phi \cdot M} \times P
\]  

(10)

Among them, \( P \) represents the penalty value when the constraint condition is not satisfied. \( P \in (1, 0) \), when satisfying the constraint condition, take 1; conversely, take 0 respectively. \( Z \) is the evaluation value of the solution, the larger it indicates the higher quality of the solution.

4.5 Taboo Tabulation

The taboo list constructed by this algorithm is used to record the transformation characteristics of the solutions in the last five iterations. The taboo case is recorded by a set of \((n + 1) \times (n + 1)\) order matrices. If the point \( i \) is selected for vertex reassignment, the taboo case is stored in the element \((i, j)\) of the matrix. If point \( i \) and \( j \) is selected and the following three transformations are performed: vertex exchange, 2-opt, “tail” exchange, the taboo case is stored in the element \((i, j)\) of the matrix, and at each iteration, the previous transform is populated in the taboo list, and the other elements in the table are decremented by 1 until they are 0.

4.6 Termination Criteria

The algorithm terminates when the total number of iterations reaches a given value, or the current best solution has no change within a given number of consecutive iterations. The algorithm sets the maximum number of iterations to \(10^5\), the maximum number of consecutive iterations to \(10^3\) while the best solution remains the same, and the maximum number of candidate solutions to 10.

5. Case Study

Table 1. Distribution of stations

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6. Conclusions

(1) In this paper, we proposed the urban and rural public transportation network optimization design method, considering both space and time constraints. That is, the optimized design should meet the space constraint of “20 km-urban and rural transit service area”, and the time constraint of “1 hour-Urban and rural transit circle”. Further more, we used big data processing technology and internet technology to achieve these constraints.

(2) This paper describes the optimization design of ur-
ban and rural public transportation network based on space-time constraints, resolving it as a special kind of vehicle routing problem, an optimization model is constructed, which targets the direct transfer efficiency, secondary transfer efficiency and bus line number.

(3) In this paper, the tabu search algorithm is designed to construct the model. The results show that the proposed algorithm has a fast convergence rate, a stable result, and good performance.

(4) However, we only considered the design for a comprehensive urban transportation hub. In the actual network design, there may exist several urban comprehensive transportation hubs in one city. Therefore, in the future research, the route design for multi-city comprehensive transportation hubs will be considered.

**Acknowledgement**

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**References**


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**Table 2. OD demand between stations passenger number/day**

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