A Fast Decomposed Three-dimensional OTSU Algorithm Based on Cuckoo Search for Image Segmentation

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Abstract

Image threshold method is an important technique for image segmentation. The maximum between-class variance of pixel (Otsu) algorithm has been widely applied in the literature. However, the original Otsu method for image segmentation is very time-consuming, and the segmentation results are often unstable for the image with low signal to noise ratio (SNR). In this paper, a fast image segmentation method (D3OTSU-CS), decomposed three-dimensional Otsu based on the technology of the cuckoo search (CS), is proposed. The proposed method starts by overcoming the complex computational by decomposing the original three-dimensional Otsu into a one-dimensional Otsu and a two-dimensional Otsu. The cuckoo search algorithm is employed to find the optimal threshold vector by the global Lévy flight searching, and the between-class variance of the two-dimensional Otsu is investigated as fitness functions. The Experimental results are illustrated to show that the computation time efficiency of the proposed method is increased by about 98.6% than the 3OTSU. In addition, the stability and reliability of the segmentation results by the proposed method outperform 2OTSU and 3OTSU.

Key Words: Image Segmentation, Otsu Algorithm, Cuckoo Search, Lévy Flight, Nature-inspired Strategy

1. Introduction

Image segmentations plays a very important role in the subsequent task [1,2]. The main objective of image segmentation is to divide the given image into non-overlapping regions with specific properties according to the specific criteria [3]. In the literature, there are many segmentation measures, such as threshold selection [4], edge detection [5], graph theory [6] and level set [7]. The thresholding methods are regarded as a well-known method because of its simple theory and is so to implement. The threshold method is categorized according to the information they are exploiting, such as histogram shape, measurement space clustering, entropy, object attributes, spatial correlation, local gray-level surface [8], etc. Among them, histogram shape has been used widely to obtain the image segmentation threshold [8–10]. The output of the thresholding operation is a binary image whose one state will indicate the foreground objects, while the complementary state will correspond to the background [10].

Otsu method based on threshold selection, is extended from one-dimension to two-dimension and three-dimension [9,10]. The segmentation efficiency of Otsu algorithm is improved, and the time complexity is higher. Otsu [11] proposed a 1D Otsu algorithm, which chooses the optimal thresholds by maximizing the between-class variance with an exhaustive search. Jing [12] proposed fast
2D Otsu algorithm. That considers both the pixel gray level and the local statistics of its neighboring the space relevant information among the pixels adequately. To improve the stability and reliability of the algorithm for low SNR image segmentation, Jing [13] proposed a 3D Otsu algorithm. Then Fan [14] pointed out Jing’s mistake and had revamped the error, and proposed a fast 3D Otsu algorithm which the concrete recursive formula has been given. The experimental results show that the computing time of Fan’s algorithm is less than the Jing’s algorithm. 3D Otsu improves the robustness of image segmentation, however Fan’s fast recursive formula reduces the algorithm’s time complexity, the running time is still need about one minute.

The intelligent optimization algorithms are often employed to solve the problem of optimization and optimal search [15]. In addition, optimization is universal engineering math problems, such as artificial bee colony algorithm [16], particle swarm optimization, pulse coupled neural network, Cuckoo Search [14], etc. However, the widely used optimization algorithms involve several parameters, the principles are complex, and are difficult to deal with the problem of large-scale rapid optimization.

In the paper, the more effective method (D3OTSU-CS) is proposed in order to meet the real-time requirements of image segmentation and improve the stability and reliability of the low SNR image segmentation. Firstly, the proposed method decomposes the 3D Otsu into a 1D Otsu and a 2D Otsu. Secondly, the method optimizes the search process of two-dimensional between-classes variance by CS, in order to obtain the optimal segmentation. CS is emerging and gradually extended to solve optimization problems [14]. CS can be extended to implementation global optimization efficiently and rapidly. Therefore, the proposed method investigates the variance of the 2D OTSU as the fitness of CS, combines with global Lévy flight searching, and implementation image segmentation quickly. Ultimately, the experimental results show that, the proposed method enhances the adapt ability of image segmentation, improves the segmentation stability and reliability for low SNR images, and solves the extremely time-consuming process of calculating the variance between-class.

2. Related Work

2.1 Three-dimensional Histogram

\[
g(m,n) = \frac{1}{k^2} \sum_{i=-\lceil k/2 \rceil}^{\lceil k/2 \rceil} \sum_{j=-\lceil k/2 \rceil}^{\lceil k/2 \rceil} f(m+i, n+j)\]  

where the size of the domain is \( k \times k \).

\[
h(m,n) = \text{Med} \left\{ \sum_{i=-\lceil k/2 \rceil}^{\lceil k/2 \rceil} \sum_{j=-\lceil k/2 \rceil}^{\lceil k/2 \rceil} f(m+i, n+j) \right\}
\]

we can define the three-dimensional histogram by \( f(m, n) \), \( g(m, n) \) and \( h(m, n) \). The \( H \) is three-dimensional histogram is description as using the following equation:

\[
H = \{ f(m, n), g(m, n), h(m, n) \}
\]

Hence, the given image \( I \) is shown in Figure 1(a). The three-dimensional histogram which is shown in Figure 1(b), is composed of \( f(m, n) \), \( g(m, n) \) and \( h(m, n) \), where the vertical axis \( f(m, n) \) is the gray value of the pixel \((m, n)\), the horizontal axis \( g(m, n) \) is the domain mean, and the vertical axis \( h(m, n) \) is the domain median of the pixel \((m, n)\).

2.2 The Between-class Variance

Let define a vector \((i, j, k)\) corresponding to \( f(m, n), g(m, n) \) and \( h(m, n) \) of any pixel \((m, n)\). Let the number of pixel \((i, j, k)\) be \( n_{ijk} \) and \( M \times N \) be the total number of pixels in the given image \( I \), the probability of occurrence of \((i, j, k)\) is defined as:

\[
p_{ijk} = \frac{n_{ijk}}{M \times N}
\]

where \(0 \leq i, j, k < L\), and that is \( \sum_{n_{ij}} \sum_{n_{jk}} \sum_{n_{ik}} = 1 \).

In case of the thresholding vector \((s, t, r)\), we assume that the image \( I \) is divided into two classes. The probabilities of the two classes computed as:

\[
o_0(s, t, r) = \sum_{i=0}^{s-1} \sum_{j=0}^{t-1} \sum_{k=0}^{r-1} p_{ijk}
\]

\[
o_1(s, t, r) = \sum_{i=s+1}^{L-1} \sum_{j=t+1}^{M-1} \sum_{k=r+1}^{N-1} p_{ijk}
\]

Then, the three dimensional histogram of image which
can be divided into eight regions by a threshold in the scope of histogram as shown in Figure 2. The major information of image I is mainly in the region 1 and region 8; the edge information and the noise information mainly in the other six regions. We suppose the pixels from region 2 to region 7 are ignored. Thus, the total mean of each component of the image can be easily calculated as:

\[
\omega_0 + \omega_1 = 1
\]

Using Eq. (5)–Eq. (7) discriminant analysis, the means for the background and target are:

\[
m_0 = \left( m_{i0}, m_{j0}, m_{k0} \right)^T = \frac{\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \sum_{k=0}^{L-1} \omega_i(s, t, r) \omega_j(s, t, r) \omega_k(s, t, r) \mathbf{i} p_{i,j,k}}{\omega_0(s, t, r) + \omega_1(s, t, r)}
\]

Based on the above description the mean vector of the three-dimensional histogram is summarized:

\[
m_2 = \left( m_{i2}, m_{j2}, m_{k2} \right)^T
\]

Thus, it is easy to show that

\[
m_2 = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \sum_{k=0}^{L-1} \omega_i(s, t, r) \omega_j(s, t, r) \omega_k(s, t, r) \mathbf{i} p_{i,j,k}
\]

\[
m = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \sum_{k=0}^{L-1} \omega_i(s, t, r) \omega_j(s, t, r) \omega_k(s, t, r) \mathbf{i} p_{i,j,k}
\]

\[
m = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} \sum_{k=0}^{L-1} \omega_i(s, t, r) \omega_j(s, t, r) \omega_k(s, t, r) \mathbf{k} p_{i,j,k}
\]

The trace of the variance between the background and the target can be defined as:

\[
h = t_{\sigma_1}(s, t, r) = \omega_0(s, t, r) \omega_1(s, t, r)
\]

where, \(0 \leq s, t, r < L\) being the optimal segmentation vector, so the maximal between-class variance in form of the following:

\[
h = t_{\sigma_1}(s, t, r) = \max_t \left[ t_{\sigma_1}(s, t, r) \right]
\]

In the case of the original Otsu algorithm, computation is about \(O(L^6)\), that is very time-consuming.

2.3 Cuckoo Search Optimization

Cuckoo search (CS) is based on several species of cuckoo parasite their eggs to the others’ nest to increase their reproductive rate. Once the host bird discovers the invaded egg with probability \(P_a \in [0, 1]\), the bird can throw the egg away or abandon the nest, then rebuild a new nest in other place [15].

2.3.1 Lévy Flights

Lévy flight has been applied to optimization and optimal search [15]. Lévy flight can find the global optimal
solution reliably. The Lévy flight drawn from the Lévy distribution, defined as:

Lévy ~ \( u = \Gamma^{-\lambda}, \; (1 < \lambda \leq 3) \) \tag{15}

the Lévy distribution is a stationary random process, which is from a long-tailed distributions. \( sl \) is the random step length of the Lévy flight:

\[ sl = \frac{u}{v^{\alpha}} \] \tag{16}

where, \( u \) and \( v \) are both drawn from the normal distribution, defined as:

\[ u \sim N(0, \sigma_u^2), \; v \sim N(0, \sigma_v^2) \] \tag{17}

And, the \( \sigma_u \) and \( \sigma_v \) are the standard deviation of the \( u \) and \( v \), can be determined by the following equations:

\[
\sigma_u = \left( \frac{\Gamma(1 + \beta) \sin(\pi \beta/2)}{\Gamma((1 + \beta)/2) \beta^{-1/2}} \right)^{1/\beta}, \; \sigma_v = 1
\] \tag{18}

\( sl_0 \in [0.1, 1] \).

While \( \beta = 1.5 \), \( l = 50 \), the path of Lévy flight for 50 consecutive steps.

2.3.2 Cuckoo Search

In order to simplify the Cuckoo search, we suppose three idealized conditions [14]:
1. Cuckoo lays only one egg at a time, and the egg randomly parasitic to other nest;
2. The optimal nest will be retained to the next generation;
3. The number of optional host nests is fixed, and the poor quality nests are replaced by the new nest with probability \( Pa \).

For solving the optimization problems, we can utilize the objective function to evaluate the fitness. CS optimization update the nests via the global Lévy flight, and it can be calculated by the formula given as:

\[
x^{(t+1)} = x^{(t)} + \alpha \odot \text{Lévy}(\lambda) \; \; t = 0, 1, 2, \ldots, T
\] \tag{19}

Then, we can obtain the new vector by the \( tth \) updating, which is performed \( x^{(t+1)} = (x_1, x_2, \ldots, x_n)^T \). In the study, the \( \alpha \) is the step factor, for avoiding flying far away in the search process, we use \( \alpha = 0.01 \).

The flat of CS optimization:

Inputting: the number of the optimal nests is \( n \), the poor quality nests will be replaced by the probability \( Pa \), the searching scale is set \( L_b \) to \( U_b \);

Outputting: the optimal nest.

Step 1. The objective function \( f(x) \)

Initializing location of the \( n \) nests \( x_i \) (\( i = 1, 2, \ldots, n \));

Step 2. While \( t < \text{maximum time of iterations} \)

Obtaining the nest \( i \) via Lévy flight;

Randomly selecting one nest \( j \) from the \( n \) nests;

Evaluating the fitness \( F_i \) and \( F_j \);

If \((F_i > F_j)\)

Using the nest \( i \) instead of the nest \( j \);

end

Abandoning and updating the poor quality nest by probability \( Pa \);

Finding the current optimal nest;

end

Step 3. Outputting the global optimal nest.

3. The D3OTSU-CS Algorithm

3.1 Decomposed Otsu Algorithm

In the paper, the three-dimensional Otsu algorithm is decomposed a one-dimensional histogram by the pixel gray-value and a two-dimensional histogram by the domain mean and the domain median. The histogram by the pixel gray-value gets the segmentation threshold \( s_0 \), taking advantage of the spatial position relationship between the gray-value and the pixels. The other two-dimensional histogram gains the segmentation threshold pairs \((t_0, r_0)\), taking advantage of effectively integrating competitive, redundant and complementary of information in form of domain mean and domain median.

3.1.1 The Two-dimensional Between-class Variance

The joint probability of two-dimensional histogram of the image \( I \) can be easily calculated as:

\[
P_{jk} = \frac{n_{jk}}{M \times N} \tag{20}
\]

where, \( n_{jk} \) is the total number of \((j, k)\).

Hence, the trace of the variance between region mean and region median can be defined as:
where, the related parameters are calculated using the following:

\[
\tau \sigma^2_g(t, r) = \\
\left[ \left( \omega_0^g(t, r) m_2^g - m_2^l(t, r) \right)^2 + \left( \omega_0^g(t, r) m_2^r - m_2^l(t, r) \right)^2 \right] \\
\frac{\omega_0^g(t, r)}{\omega_0^g(t, r) [1 - \omega_0^g(t, r)]}
\]  

(21)

3.2 The Segmentation Threshold

In the case of section 3.1, the segmentation threshold vector \((s_0, t_0, r_0)\) can be summarized using the following equations:

\[
\omega_0^g(t, r) = \sum_{j=0}^{t} \sum_{k=0}^{r} p_{jk}
\]  

(22)

\[
m_0^g(t, r) = \sum_{j=0}^{t-1} \sum_{k=0}^{r} k p_{jk}
\]  

(23)

\[
m_0^l(t, r) = \sum_{j=0}^{t} \sum_{k=0}^{r} j p_{jk}
\]  

(24)

\[
m_0^r(t, r) = \sum_{j=0}^{t} \sum_{k=0}^{r} k p_{jk}
\]  

(25)

3.3 Based on CS Method

In the case of section 3.1, the segmentation threshold vector \((s_0, t_0, r_0)\) can be summarized using the following equations:

\[
s_0 = \arg \max_{0 < s < L} \sigma_1 \phi(s)
\]  

(26)

\[
(t_0, r_0) = \arg \max_{0 < t < L, 0 < r < L} \sigma_2 \phi(t, r)
\]  

(27)

The decomposed Otsu method is employed to solve the disadvantage of time-consuming caused by complex computational and instability of segmentation results. This improvement reduces the computational from \(O(L^6)\) to \(O(L^4 + L)\).

3.3.2 D3OTSU-CS

The probability of getting a poor quality in the nests set is \(r_1\). Process of algorithm is described as the following:

1. Calculate each pixel gray variance between classes, and based on 1OTSU to obtain an optimal threshold segmentation \(s_0\),

2. Initialize location of the n nests and calculate the fitness of the n nests according to two dimensional variance,

3. Find out the optimal solution set,

4. Update the nests via levy flight and evaluate the fitness of the new nests,

5. Generate random numbers,

\[
\text{If } (r_1 > Pa) \text{ update the nests via levy flight and evaluate the fitness of the new n nests}
\]

\[
\text{else update historical optimal solution set,}
\]

6. If the conditions are met

\[
\text{get the best segmentation threshold vector } (t_0, r_0);
\]

\[
\text{else return to step 4;}
\]

7. Output optimal segmentation vector \((s_0, t_0, r_0)\).

4. Experiments and Results

4.1 Experimental Setup

The size of the nests is set as \(n \in [15, 40]\), and the probability of the host bird abandons the nest is set \(P_a = 0.25\). In the paper, we set \(n = 25\) and \(P_a = 0.25\). The \(Nc\) is regarded as the number of iterations. \(Nc = 50\), the experimental results are shown in Figure 3.

In Figure 3(a)–(f), the vertical axis is the convergence value when the optimal value is searched, and the horizontal axis is the number of iterations. From the Figure, when the optimal value converges, the convergence value is basically same, the algorithm can converge to the optimal value when \(Nc < 30\). Therefore, we setup \(Nc = 30\).
4.2 Experimental Results

4.2.1 Comparison 2OTSU and 3OTSU

From the Figure 4, the proposed method can improve the segment results, and remain more the edge and detail information compared with 3OTSU and 2OTSU.

4.2.2 Comparison with 3OTSU

The experimental results are shown in Figure 5 and Figure 6.

From the above Figures, we can see that, the segmentation stability and reliability are much higher by D3OTSU-CS method.

4.2.3 Comparison with the Fan’s 3OTSU

The experimental results are shown in Figure 7.

From the Figure, the proposed method has better segmentation effect and less noise points in segmentation results of the low SNR image compared with the Fan’s
3OTSU. The proposed method can remain more edge and detail information.

4.3 Time Complexity

However, time complexity of the original 3OTSU is high, we proposed the method solves the disadvantage by dimensionality reduction and intelligent optimization. The time complexity of each algorithm is shown in Figure 8 and Figure 9.

From the Figure 8, with the segmentation method being extended from 2OTSU to 3OTSU, the time complexity increases about 130 times, the time complexity of the Fan’s 3OTSU is about 30 times that of 2OTSU, and the time complexity of the proposed method increases about 20% than 2OTSU. From the Figure 9, the time complexity of the proposed method decreases about 98.6% than 3OTSU. That is to say, D3OTSU-CS solves the problem of time-consuming.

5. Conclusions

In the paper, we have formulated a new fast image
segmentation method, decomposed three-dimensional Otsu based on the cuckoo search optimization. Compared with the original 3OTSU, the proposed method has improved the stability and reliability of image segmentation, firstly, decomposed 3OTSU into a one-dimensional and a two-dimensional to reduce the computational complexity. Then, it employed CS technology to optimize the search process. In addition, the number of parameters is less, and the speed of convergence is fast in the process of implementation. Results are shown that, D3OTSU-CS solves the problem of the original threshold methods are sensitive to noise, and improves the robustness to the images of different SNR.

However, the OTSU method implements image segmentation by the single-threshold, and it is some disadvantages to the implementation of the multi-object image segmentation. So further studies can focus on extending single-threshold to multi-threshold, so that implement image segmentation appreciatively.

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