Mass Transfer Enhancement in Double-pass Concentric-tube Mass Exchangers under Sinusoidal Wall Fluxes with External Recycle

Chii-Dong Ho1*, Gwo-Geng Lin1, Jing-Min Tang2, Li-Chien Liu1, Li-Pang Lin1 and Jr-Wei Tu1
1Department of Chemical and Materials Engineering, Tamkang University, New Taipei City, Taiwan 25137, R.O.C.
2Department of Aerospace Engineering, Tamkang University, New Taipei City, Taiwan 25137, R.O.C.

Abstract

The influences of external recycle at the ends on double-pass laminar counterflow concentric-tube mass exchangers with sinusoidal wall flux distribution are investigated analytically. An analytical method is proposed to obtain a general solution by using the complex functions in combining the original boundary value problem into an ordinary differential equation with the aid of Frobenius method.

Theoretical results show that a suitable adjustment of the permeable-barrier position can effectively enhance the mass transfer rate, leading to an improved device performance in mass transfer efficiency as compared with that in the single-pass operation (without a permeable barrier inserted in parallel into a circular tube). The mass-transfer efficiency enhancement due to the desirable effect of forced-convection increment in two flow patterns of double-pass devices has been illustrated with the recycle ratio, mass-transfer Graetz number, permeable-barrier location and dimensionless permeable-barrier parameter as parameters.

Key Words: Sinusoidal Wall Fluxes, Orthogonal Expansion Technique, Double-pass Operation, Mass Exchanger, Recycle

1. Introduction

The classical Graetz problem [1–3] describes a fluid flowing hydrodynamically fully established in a bounded conduit of constant properties with negligible axial conduction and diffusion. Extensions to the multi-stream or multiphase systems with interactions of streams or phases are called to conjugated Graetz problems [4–6]. The various boundary conditions were prescribed at the conduit wall in engineering applications such as symmetrical heating with equal wall temperature or heat fluxes [7,8], asymmetrical heating with axially varying heat flux and wall temperature [9–13], and sinusoidal periodic heating in nuclear reactors with the cooling tubes [14,15].

The mass transfer behavior from the higher concentration side to the lower one is analogous to heat transfer characteristics. The single-stream mass exchangers were carried out to achieve either concentrating or diluting mass composition in the flowing stream [16–18] while the improved device performance was designed in conducting double-pass operations such as bioreactors [19], filters [20], gas scrubbers [21], and gas absorbers [22] by inserting a permeable barrier to enhance the convective mass-transfer coefficient. The recycle ratio and the permeable-barrier position in operating double-pass devices with external recycle are the operating parameter and designing parameter, respectively, to be suitably selected for increasing mass-transfer efficiency enhancement of the present study.

The purpose of the present work is to extend the previous work [23] with external recycle in presenting a theoretical formulation of its simplest form under sinusoidal...
wall flux distribution. The new design of recycling double-pass concentric mass exchangers can effectively enhance the mass transfer rate and lead to a device performance improvement. This work includes the influences of recycle effect on mass-transfer efficiency improvement with recycle ratio, permeable-barrier position and mass-transfer Graetz number as parameters. The introduction of external recycle has produced a positive influence on the mass-transfer efficiency improvement.

2. Concentration Distributions

A new recycling double-pass concentric-tube mass exchanger with inserting a permeable barrier into a circular tube of inside diameter $2R$ and length $L$ under sinusoidal mass fluxes, as shown in Figure 1, was investigated theoretically. The thickness of the inner (subchannel $a$) and annular tube (subchannel $b$) are $2\delta R$ and $2(1-\delta)R$, respectively. As comparing to the radius of circular tube $R$, the thickness of the permeable barrier $\delta$ is negligible ($\delta < R$). This new double-pass concentric-tube mass exchanger with external recycle under sinusoidal mass fluxes is an extension of our previous work [23] and two flow patterns are discussed. The case of the working fluid feeding into inner subchannel firstly with volumetric flow rate $V$ and a recycle fluid flowing in outer subchannel with volumetric flow rate $MV$ is called flow pattern A, as shown in Figure 1(a), while flow pattern B is the case of the working fluid feeding into outer subchannel firstly with volumetric flow rate $V$ and a recycle fluid flowing in inner subchannel with volumetric flow rate $MV$, as shown in Figure 1(b). In both flow patterns, the fluid is concentrated by the outer wall with sinusoidal mass flux, $J^* = J^*[1 + \sin(\beta z)]$.

Based on following assumptions: constant physical properties of fluid; fully-developed laminar flow; neglecting the entrance length and the end effects; ignoring the longitudinal mass diffusion and the concentration polarization phenomena on the permeable barrier, the mass balance equations of a double-pass mass exchanger with sinusoidal mass fluxes can be formulated as

$$\frac{v_a(\eta)R^2}{G_{z,n}LD} \frac{\partial \phi_a(\eta, \xi)}{\partial \xi} = \frac{1}{\eta} \frac{\partial}{\partial \eta} \left( \frac{\partial \phi_a(\eta, \xi)}{\partial \eta} \right)$$

and the corresponding boundary conditions are

$$\frac{\partial \phi_a(0, \xi)}{\partial \eta} = 0$$
$$\frac{\partial \phi_a(1, \xi)}{\partial \eta} = 1 + \sin(B\xi)$$
$$\frac{\partial \phi_a(\kappa, \xi)}{\partial \eta} = \phi_a(\kappa, \xi) - \phi_a(\kappa, \xi)$$
$$\frac{\partial \phi_a(\kappa, \xi)}{\partial \eta} = \omega [\phi_a(\kappa, \xi) - \phi_a(\kappa, \xi)]$$

where $\omega$ is the dimensionless permeable-barrier factor, defined by $\omega = \Re\delta$, and $\varepsilon$ is the porosity of permeable barrier.

The velocity distributions in subchannels $a$ and $b$, say $v_a$ and $v_b$, are different in two flow patterns. In flow pattern A, the $v_a$ and $v_b$, respectively, are

![Figure 1. Double-pass concentric-tube mass exchanger: (a) Flow pattern A; (b) Flow pattern B.](image-url)
\[ v_a(\eta) = 2 \frac{(M+1)\eta}{\eta \kappa^2} \left[ 1 - \left( \frac{\eta}{\kappa} \right)^2 \right], \quad 0 \leq \eta_a \leq \kappa \]

\[ v_b(\eta) = -2 \frac{MV}{\pi \eta^2 (1 - \kappa^2)} \left[ 1 - (\eta)^2 + W_1 \ln \eta \right], \quad \kappa \leq \eta_b \leq 1 \]

and in flow pattern B, the velocity distributions \( v_a \) and \( v_b \) are

\[ v_a(\eta) = 2 \frac{MV}{\pi \kappa^2} \left[ 1 - \left( \frac{\eta}{\kappa} \right)^2 \right], \quad 0 \leq \eta_a < \kappa \]

\[ v_b(\eta) = \frac{2(M+1)V}{\pi \kappa^2} \left[ 1 - (\eta)^2 + W_2 \ln \eta \right], \quad \kappa \leq \eta_b \leq 1 \]

where \( W_1 \) and \( W_2 \) are

\[ W_1 = \frac{1 - \kappa^4 - 1 - \kappa^2}{\ln(1/k)} \]

\[ W_2 = \frac{1 - \kappa^4}{\ln(1/k)} \]

The dimensionless groups in Eqs. (1)-(12) are defined as

\[ \eta = \frac{R}{R}, \quad \xi = \frac{z}{G_{m}L}, \quad G_{m} = \frac{4V}{D_{m}L}, \quad B = \beta G_{m}L, \]

\[ \kappa = \frac{R}{R}, \quad \phi_a = \frac{D(C_1 - C_2)}{J_0 R}, \quad \phi_b = \frac{D(C_2 - C_1)}{J_0 R} \]

The general solution of the dimensionless concentration distribution of the double-pass counterflow concentric-tube mass exchangers with sinusoidal wall fluxes can be expressed [14,23] as follows:

\[ \psi_a(\eta) = \psi_{a0}(\xi) + \sum_{n=0}^{\infty} a_n \xi^{n+1}, \quad n \geq 0 \]

\[ \psi_b(\eta) = \psi_{b0}(\xi) + \sum_{n=0}^{\infty} b_n \xi^{n+1}, \quad n \geq 0 \]

The recursive relations form of coefficients \( a_n \) and \( b_n \) in flow pattern A are

\[ a_{2n} = \frac{-B(M+1)}{8\kappa \eta^2} \left( a_{2n-2} - \frac{1}{\kappa^2} a_{2n-4} \right), \quad n \geq 1 \]
and those in flow pattern B, respectively, are

$$a_s = \frac{-MBi}{2\kappa n} \left( a_{n-2} - \frac{1}{\kappa^2} a_{n-4} \right), \quad n \geq 1$$

(26)

and

$$h_i = 0, \quad b_s = \frac{(M+1)Bi}{2W_i(1-\kappa^2)n^2} \left[ b_{n-2} - b_{n-4} + Wz \left( 2b_{n-3} - \frac{1}{2} b_{n-4} - \frac{3}{2} b_{n-2} \right) \right]$$

(27)

Moreover, the boundary value problem for solving \(\theta_{0a}, \theta_{1a}(\eta), \theta_{0b}, \text{and} \theta_{1b}(\eta)\) is

$$\frac{d}{d\eta} \left( \eta \frac{d\theta_{0a}(\eta)}{d\eta} \right) - \frac{v_s(\eta)R^2\eta}{Gz_m LD} \theta_{0a} = 0$$

(28)

$$\frac{d}{d\eta} \left( \eta \frac{d\theta_{0b}(\eta)}{d\eta} \right) + \frac{v_s(\eta)R^2\eta}{Gz_m LD} \theta_{0b} = 0$$

(29)

$$\frac{d\theta_{0a}(0)}{d\eta} = 0$$

(30)

$$\frac{d\theta_{0a}(1)}{d\eta} = 1$$

(31)

$$\frac{d\theta_{0b}(\kappa)}{d\eta} = \frac{d\theta_{1b}(\kappa)}{d\eta}$$

(32)

$$\theta_{0a} = -\theta_{0b}$$

(33)

$$\frac{d\theta_{1b}(\kappa)}{d\eta} = \omega \left[ \frac{\theta_{0b} + \theta_{1b}(\kappa) - \theta_{1a}(\kappa)}{Gz_m} \right]$$

(34)

Then, integrating Eqs. (28) and (29) twice with respect to \(\eta\) for \(\theta_{1a}(\eta)\) and \(\theta_{1b}(\eta)\) yields

$$\theta_{1a} = \frac{(M+1)\theta_{0a}}{2\kappa^3} \left[ \frac{1}{4} \eta^4 - \frac{1}{16} \eta^4 + \gamma_{1a} \ln \eta + \gamma_{2a} \right]$$

(35)

and

$$\theta_{1b} = \frac{M\theta_{0b}}{2W_i(1-\kappa^2)} \left[ \frac{1}{4} \eta^4 - \frac{1}{16} \eta^4 \right]$$

$$+ \frac{W_z}{4} \eta^2 \left[ \ln \eta - 1 \right] + \gamma_{1b} \ln \eta + \gamma_{2b}$$

(36)

for flow pattern A and

$$\theta_{1a} = \frac{M\theta_{0a}}{2\kappa^2} \left[ \frac{1}{4} \eta^4 - \frac{1}{16} \eta^4 + \gamma_{1a} \ln \eta + \gamma_{2a} \right]$$

(37)

$$\theta_{1b} = \frac{(M+1)\theta_{0b}}{2W_i(1-\kappa^2)} \left[ \frac{1}{4} \eta^4 - \frac{1}{16} \eta^4 \right]$$

$$+ \frac{W_z}{4} \eta^2 \left[ \ln \eta - 1 \right] + \gamma_{1b} \ln \eta + \gamma_{2b}$$

(38)

for flow pattern B, respectively. The two undetermined constants \(\theta_{0a}\) and \(\theta_{0b}\), and the four integrating constants \(\gamma_{1a}, \gamma_{2a}, \gamma_{1b}\), and \(\gamma_{2b}\) were calculated by the boundary conditions, Eqs. (30)–(34), and the overall mass balance

$$\phi_f = \int_0^1 \frac{4V}{Dp} \left[ 1 + \sin(B\xi) \right] d\xi$$

$$= \frac{8}{Dp} \left[ 1 - \cos \left( \frac{B}{Gz_m} \right) \right]$$

(39)

where \(\phi_f\) is the average outlet concentration.

After all functions of \(\theta_{0a}, \theta_{0b}, \theta_{1a}, \theta_{2a}, \theta_{2b}, \theta_{3a}\), and \(\theta_{3b}\) were determined and then substituted into \(\phi_a\) and \(\phi_b\), the complete solutions of the mass distribution in a double-pass concentric-tube mass exchanger with external recycle were obtained.

### 3. Mass-transfer Efficiency Improvement

The mass transfer efficiency of a double-pass concentric-tube mass exchanger with external recycle can be determined by the local Sherwood number as

$$Sh(\xi) = \frac{k_m D_h}{D}$$

(40)

where \(D\) is the diffusivity of solute, \(D_h\) is the equivalent diameter of conduit, \(D_h = 2R\), and \(k_m\) is average convection mass transfer coefficient which is defined as

$$k_m = \frac{D}{R J_0 \phi_f(1, \xi) - \frac{D}{R} \left( 1 + \sin(B\xi) \right)}$$

(41)
Substituting Eq. (41) into Eq. (40) yields

$$Sh(\xi) = \frac{2[1 + \sin(B_2^2 \xi)]}{\phi_b(1, \xi)}$$ (42)

Moreover, the average Sherwood number of the double-pass concentric-tube mass exchanger with external recycle was determined by

$$\overline{Sh} = G_{zm} \int_0^{1/G_{zm}} Sh(\xi) \, d\xi = G_{zm} \int_0^{1/G_{zm}} \frac{2[1 + \sin(B_2^2 \xi)]}{\phi_b(1, \xi)} \, d\xi$$ (43)

The mass-transfer efficiency improvement by employing a double-pass operation was defined as the percent increase in mass transfer rate based on that in a single-pass device with the same working dimensions and operating parameters

$$I_m = \frac{\overline{Sh} - Sh_0}{Sh_0} \times 100$$ (44)

where $Sh_0$ is the average Sherwood numbers of single-pass device [23].

The power consumption increment define as in flow pattern A

$$I_p = \frac{(M + 1)^2}{k^4} + \frac{M^2}{(1 - k^2)(1 - k^2)^2} - 1$$ (45)

and in flow pattern B

$$I_p = \frac{M^2}{k^4} + \frac{(M + 1)^2}{(1 - k^2)(1 - k^2)^2} - 1$$ (46)

### 4. Results and Discussions

The recycling double-pass counterflow concentric-tube mass exchangers with sinusoidal wall fluxes were developed making mass balance equations and solved with the aid of the linear superposition of Eqs. (14) and (15). The convergence of Taylor series of $\ln \eta$ for $N = 2$ and 3 with $k = 0.75$ and $\omega = 0.3$ in both flow patterns A and B are shown in Table 1. The calculated results show that the truncation after $N = 2$ of Taylor series for approximation of $\ln \eta$ is good enough. In conclusion, the terms of $n = 70$ and $N = 2$ for the Frobenius method and Taylor series, respectively, were employed in the calculation procedure in this study.

It is of importance for an engineer to grasp at wall concentration distributions beforehand in designing a mass exchanger under the case of Neumann boundary conditions. The present study is to apply the sinusoidal wall fluxes for double-pass mass exchangers with external recycle, as referred to conjugated Graetz problems, and the wall concentration distributions were determined solving the mathematical formulations analytically. The methods for improving the device performance in double-pass mass exchangers are either the desirable effect of the convective heat-transfer coefficient enhancement (volumetric flow rate) or the undesired effect concentration driving-force reduction due to the premixing at inlet. This study shows that the desirable effect of the convective heat-transfer coefficient (volumetric flow rate) suppresses the undesired effect of the concentration gradient reduction, which brings about the mass transfer improvement. The theoretical predictions are illustrated in Figures 2 and 3 for flow patterns A and B, respectively.

<table>
<thead>
<tr>
<th>$G_{zm}$</th>
<th>$N$</th>
<th>Flow pattern A</th>
<th>Flow pattern B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_{2b}$</td>
<td>$\theta_{3b}$</td>
<td>$\overline{Sh}$</td>
</tr>
<tr>
<td>1</td>
<td>7.96 x 10^{-2}</td>
<td>-1.06</td>
<td>0.17</td>
</tr>
<tr>
<td>2</td>
<td>7.96 x 10^{-2}</td>
<td>-1.06</td>
<td>0.17</td>
</tr>
<tr>
<td>10</td>
<td>2.86 x 10^{-2}</td>
<td>-0.16</td>
<td>1.35</td>
</tr>
<tr>
<td>2</td>
<td>2.86 x 10^{-2}</td>
<td>-0.16</td>
<td>1.35</td>
</tr>
<tr>
<td>50</td>
<td>-1.32 x 10^{-2}</td>
<td>-6.1 x 10^{-2}</td>
<td>4.29</td>
</tr>
<tr>
<td>3</td>
<td>-1.32 x 10^{-2}</td>
<td>-6.1 x 10^{-2}</td>
<td>4.29</td>
</tr>
<tr>
<td>100</td>
<td>-1.77 x 10^{-2}</td>
<td>-3.19 x 10^{-2}</td>
<td>5.95</td>
</tr>
<tr>
<td>3</td>
<td>-1.77 x 10^{-2}</td>
<td>-3.19 x 10^{-2}</td>
<td>5.95</td>
</tr>
</tbody>
</table>
larger mass-transfer Graetz number $G_{zm}$ (either high volumetric flow rate or short conduit length) comes out with a shorter residence time of working fluid in conduit, resulting in the decreasing dimensionless wall concentration and the flatter profiles in both flow patterns, as demonstrated in Figures 2 and 3. Moreover, the convective mass transfer coefficient increases with increasing the permeable barrier location $\kappa$ and creating a higher average velocity in the annular channel, and thus, wall concentration oscillations decrease between $\xi = 0$ and $\xi = 1$ as compared that to the single-pass device, as shown in Figures 2 and 3.

The local Sherwood numbers distributions of the double- and single-pass concentric-tube mass exchangers under sinusoidal wall fluxes are shown in Figures 4 and 5 for flow patterns A and B, respectively. The local Sherwood number was expressed in a dimensionless quantity to provide a magnitude of convection heat transfer occurring on device wall surface. The local Sherwood number increases with increasing the mass-transfer Graetz number $G_{zm}$ and the permeable-barrier location $\kappa$, as confirmed by Eq. (42) to be inversely proportion to the dimensionless wall concentration at the outer channel. Moreover, the local Sherwood numbers of double-pass mass exchangers with external recycle in both flow patterns A and B are higher than that of a single-pass device for $G_{zm} > 30$, as observed from Figures 4 and 5.

The comparisons of the average Sherwood number vs. $G_{zm}$ among the present and previous works [23] are illustrated in Figure 6. Figure 6 indicates that the present recycling double-pass mass exchanger obtains a good mass transfer rate with larger Sherwood numbers as compared to both the previous work [23] and single-pass devices, especially in larger $G_{zm}$.

![Figure 2. Dimensionless wall concentration distribution with $\kappa$ as a parameter for $\omega = 0.3$ (flow pattern A).](image)

![Figure 3. Dimensionless wall concentration distribution with $\kappa$ as a parameter for $\omega = 0.3$ (flow pattern B).](image)

![Figure 4. Local Sherwood number distribution with $\kappa$ as a parameter for $G_{zm} = 10$, 30 and 50 (flow pattern A).](image)

![Figure 5. Local Sherwood number distribution with $\kappa$ as a parameter for $G_{zm} = 10$, 30 and 50 (flow pattern B).](image)
The mass-transfer efficiency improvement $I_m$ of the double-pass devices was defined by Eq. (44) and the theoretical predictions of $I_m$ were shown in Tables 2 and 3 for flow patterns A and B, respectively. The mass-transfer efficiency improvement $I_m$ increases with increasing the mass-transfer Graetz numbers $Gz_m$, permeable-barrier location $\alpha/c_{107}$ and the dimensionless permeable-barrier parameter $\omega$ for both flow patterns A and B, as demonstrated in Tables 2 and 3. The minus signs of in Tables 2 and 3 indicate that no improvement in mass transfer efficiency can be achieved, in this case, the single-pass device is preferred to be employed rather than using the recycling double-pass device.

Figure 7 shows the mass transfer efficiency improvement $I_m$ with various recycle ratio $M$ as parameter. The mass-transfer efficiency improvement increases with increasing Graetz number in both flow patterns, as indicated in Figure 7. The minus signs in Figure 7 represent that the concentric-tube mass exchanger performs a lower efficiency improvement than that of a single-pass device. The influences of recycle ratio $M$ on the mass-transfer efficiency improvement $I_m$ of two flow patterns are in reverse order. The mass-transfer efficiency improvement $I_m$ increases with increasing recycle ratio $M$ in flow pattern A while the mass-transfer efficiency improvement $I_m$ increases with decreasing recycle ratio $M$ in flow pattern B. However, the transfer efficiency in flow pattern B is better than that in flow pattern A under all operation condition.

The double-pass design and external recycle operation not only enhance the mass transfer efficiency of a mass exchanger with sinusoidal wall mass fluxes but also increase the power consumption. In an economic concept, the comparison of the mass transfer efficiency improvement $I_m$ and the power consumption increment $I_p$ is considered in the form of $I_m/I_p$ and is shown in Figure 8. The values of $I_m/I_p$ proportion to the mass-transfer Graetz number for both flow patterns A and B. However, the influence of permeable-barrier location $\kappa$ on the $I_m/I_p$ is different on flow patterns A and B. The values of $I_m/I_p$ increase with permeable-barrier location $\kappa$ in flow pattern A, but flow pattern B performs the highest values of $I_m/I_p$ as $\kappa = 0.5$ is selected, as illustrated in Figure 8. Comparing both flow patterns by examining $I_m/I_p$, flow pattern B shows higher performance than flow pattern A.

The mass-transfer efficiency improvement $I_m$ of the double-pass devices was defined by Eq. (44) and the theoretical predictions of $I_m$ were shown in Tables 2 and 3 for flow patterns A and B, respectively. The mass-transfer efficiency improvement $I_m$ increases with increasing with the mass-transfer Graetz numbers $Gz_m$, permeable-barrier location $\alpha/c_{107}$ and the dimensionless permeable-barrier parameter $\omega$ for both flow patterns A and B, as demonstrated in Tables 2 and 3. The minus signs of in Tables 2 and 3 indicate that no improvement in mass transfer efficiency can be achieved, in this case, the single-pass device is preferred to be employed rather than using the recycling double-pass device.

Figure 7 shows the mass transfer efficiency improvement $I_m$ with various recycle ratio $M$ as parameter. The mass-transfer efficiency improvement increases with increasing Graetz number in both flow patterns, as indicated in Figure 7. The minus signs in Figure 7 represent that the concentric-tube mass exchanger performs a lower efficiency improvement than that of a single-pass device. The influences of recycle ratio $M$ on the mass-transfer efficiency improvement $I_m$ of two flow patterns are in reverse order. The mass-transfer efficiency improvement $I_m$ increases with increasing recycle ratio $M$ in flow pattern A while the mass-transfer efficiency improvement $I_m$ increases with decreasing recycle ratio $M$ in flow pattern B. However, the transfer efficiency in flow pattern B is better than that in flow pattern A under all operation condition.

The double-pass design and external recycle operation not only enhance the mass transfer efficiency of a mass exchanger with sinusoidal wall mass fluxes but also increase the power consumption. In an economic concept, the comparison of the mass transfer efficiency improvement $I_m$ and the power consumption increment $I_p$ is considered in the form of $I_m/I_p$ and is shown in Figure 8. The values of $I_m/I_p$ proportion to the mass-transfer Graetz number for both flow patterns A and B. However, the influence of permeable-barrier location $\kappa$ on the $I_m/I_p$ is different on flow patterns A and B. The values of $I_m/I_p$ increase with permeable-barrier location $\kappa$ in flow pattern A, but flow pattern B performs the highest values of $I_m/I_p$ as $\kappa = 0.5$ is selected, as illustrated in Figure 8. Comparing both flow patterns by examining $I_m/I_p$, flow pattern B shows higher performance than flow pattern A.

### Table 2.
The mass-transfer efficiency improvement with $\kappa$ as a parameter for $M = 1$ (flow pattern A)

<table>
<thead>
<tr>
<th>$Gz_m$</th>
<th>$\omega = 0.3$</th>
<th>$\omega = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = 0.3$</td>
<td>$\kappa = 0.5$</td>
<td>$\kappa = 0.7$</td>
</tr>
<tr>
<td>1</td>
<td>-77</td>
<td>-73</td>
</tr>
<tr>
<td>10</td>
<td>-48</td>
<td>-34</td>
</tr>
<tr>
<td>50</td>
<td>9</td>
<td>58</td>
</tr>
<tr>
<td>100</td>
<td>31</td>
<td>118</td>
</tr>
</tbody>
</table>

### Table 3.
The mass-transfer efficiency improvement with $\kappa$ as a parameter for $M = 1$ (flow pattern B)

<table>
<thead>
<tr>
<th>$Gz_m$</th>
<th>$\omega = 0.3$</th>
<th>$\omega = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = 0.3$</td>
<td>$\kappa = 0.5$</td>
<td>$\kappa = 0.7$</td>
</tr>
<tr>
<td>1</td>
<td>-52</td>
<td>-43</td>
</tr>
<tr>
<td>10</td>
<td>-4</td>
<td>42</td>
</tr>
<tr>
<td>50</td>
<td>53</td>
<td>164</td>
</tr>
<tr>
<td>100</td>
<td>61</td>
<td>214</td>
</tr>
</tbody>
</table>
5. Conclusions

A new external recycle design of a double-pass co-counterflow concentric-tube mass exchanger with sinusoidal wall fluxes was made to improve the mass transfer efficiency in this study. Two flow patterns A and B were investigated and the theoretical mathematical model of mass transfer was developed by using mass balance equations. The analytical solutions were obtained by using a general solution form to separate the original boundary value problem into a partial differential equation and an ordinary differential equation. The effects of mass-transfer Graetz number $G_zm$, permeable-barrier location $\kappa$ and dimensionless permeable-barrier factor $\omega$ on the wall concentration distribution, local Sherwood number, average Sherwood number and mass-transfer efficiency improvement in a double-pass mass exchanger with external recycle were also investigated. The theoretical results show that the mass transfer efficiency increases with increasing $G_zm$, permeable-barrier location $\kappa$ and dimensionless permeable-barrier factor $\omega$ for both flow patterns. Comparing the flow patterns A and B, one finds that flow pattern B achieves a better device performance than flow pattern A. Moreover, in an economic sense, the best selection of operating condition based on the $I_m/I_p$ is $\kappa = 0.5$ with flow pattern B design as $G_zm > 10$, as indicated in Figure 8.

Nomenclature

- $a_n$: coefficient in Eq. (22)
- $B$: constant, defined in Eq. (4)
- $b_n$: coefficient in Eq. (23)
- $C$: concentration in the stream, $mol/m^3$
- $D$: diffusion coefficient of solute, $m^2/s$
- $D_h$: equivalent diameter of the conduit, $m$
- $f$: friction factor
Double-pass Concentric-tube Mass Exchangers

411

\[ G_{m} \] mass-transfer Graetz number
\[ h_f \] friction loss in conduit, J/kg
\[ I_m \] mass-transfer efficiency improvement, defined by Eq. (44)
\[ I_p \] power consumption increment, defined by Eq. (45) and (46)
\[ k_m \] average convection mass transfer coefficient, m/s
\[ L \] conduit length, m
\[ M \] recycle ratio
\[ N \] wall mass flux, mol/m²·s
\[ P \] power consumption, J/S
\[ R \] outer tube radius, m
\[ \overline{\text{Sh}} \] average Sherwood number
\[ V \] input volume flow rate, m³/s
\[ v \] velocity distribution of fluid, m/s

Greek Symbols
\[ \delta \] permeable barrier thickness, m
\[ \varepsilon \] porosity of the permeable barrier
\[ \eta \] transversal coordinate, = r/R
\[ \theta \] defined by Eq. (14)
\[ \kappa \] subchannel radius ratio
\[ \lambda \] constant
\[ \mu \] fluid viscosity, kg/m s
\[ \xi \] longitudinal coordinate, = z/L
\[ \phi \] dimensionless concentration, \( D(C – C_i) / N_{re} R \)
\[ \omega \] dimensionless permeable barrier parameter, = Re / \delta

Subscripts
\( 0 \) in a single-pass device without recycle
\( a \) in inner channel
\( b \) in annulus channel
\( F \) at the outlet
\( i \) at the inlet
\( w \) at the wall surface

Acknowledgement
The authors wish to thank the Ministry of Science and Technology of the Republic of China for its financial support.

References


*Manuscript Received: Jan. 21, 2019
Accepted: May 16, 2019*