MHD Couette Flow with Temperature Dependent Viscosity and the Ion Slip

Hazem Ali Attia

Department of Mathematics, College of Science, Al-Qasseem University,
P. O. Box 237,Buraidah 81999, Kingdom of Saudi Arabia

Abstract

The steady MHD Couette flow with heat transfer of an electrically conducting fluid is studied considering the ion slip. The viscosity of the fluid is assumed to be temperature dependent. The fluid is subjected to a constant pressure gradient and an external uniform magnetic field perpendicular to the plates which are kept at different but constant temperatures. The effect of the ion slip and the temperature dependent viscosity on both the velocity and temperature distributions is examined.

Key Words: Magnetohydrodynamics, Heat Transfer, Variable Properties, Computational Fluid Dynamics

1. Introduction

The flow of a viscous incompressible electrically conducting fluid between two infinite parallel insulating plates has been studied with heat transfer by many researchers [1-5] due to its important applications in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets sprays. Most of these studies are based on constant physical properties. It is known that some physical properties are functions of temperature [6] and assuming constant properties is a good approximation as long as small differences in temperature are involved. More accurate prediction for the flow and heat transfer can be achieved by considering the variation of the physical properties with temperature. Klemp et al. [7] studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. The MHD fully developed flow and heat transfer of an electrically conducting fluid between two parallel plates with temperature dependent viscosity is studied in [8,9] without taking the Hall effect or the ion slip into consideration.

In the present work, the steady magnetohydrodynamic (MHD) Couette flow of a viscous incompressible electrically conducting fluid with heat transfer between two electrically insulating plates is studied considering the Hall and ion slip effects. The upper plate is moving with a constant speed and the lower plate is kept stationary while the fluid is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected [1,5]. The two plates are kept at two constant but different temperatures while the viscosity of the fluid is assumed to be temperature dependent. Thus, the coupled set of the equations of motion and the energy equation including the viscous and Joule dissipations terms becomes non-linear and is solved numerically using the finite difference approximations to obtain the velocity and temperature distributions.

2. Formulation of the Problem

The fluid is assumed to be flowing between two infinite horizontal plates located at the $y = \pm h$ planes. The upper plate moves with a uniform velocity $U_o$ while the lower plate is stationary. The two plates are assumed to be electrically insulating and kept at two constant tem-
temperatures $T_1$ for the lower plate and $T_2$ for the upper plate with $T_2 > T_1$. A constant pressure gradient $dP/dx$ is applied in the x-direction. A uniform magnetic field $B_o$ is applied in the positive y-direction which is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [1,5]. The Hall and ion slip effects are taken into consideration and consequently a z-component of the fluid in the x-direction, $w = w(y)$ is the velocity component of the fluid in the $z$-direction, $\beta_c = \sigma \beta B_o$ is the Hall parameter [1], $\beta = 1/n_q$ is the Hall factor [1], $n$ is the electron concentration per unit volume, $-q$ is the charge of the electron, and $\beta_i$ is the ion slip parameter [1]. It should be noted that the electric conductivity $\sigma$ and, in turn, $\beta_c$ may vary with temperature, however, it is considered in this study to be constant. The no-slip condition at the plates implies that

$$y = -h: u = w = 0, \quad y = h: u = U_o, \quad w = 0$$ (3)

The energy equation describing the temperature distribution for the fluid is given by [1,10]

$$k \frac{d^2 T}{dy^2} + \mu \left( \frac{du}{dy} \right)^2 + \left( \frac{dw}{dy} \right)^2$$

$$+ \frac{\sigma B_o^2 (1 + \beta_c \beta_i)}{(1 + \beta_c \beta_i)^2 + \beta_i^2} (u^2 + w^2) = 0,$$ (4)

where $T$ is the temperature of the fluid and $k$ is the thermal conductivity of the fluid. The last two terms in the left-hand side of Eq. (4) represent, respectively, the viscous and Joule dissipations. The temperature of the fluid must satisfy the boundary conditions,

$$y = -h: T = T_1, \quad y = h: T = T_2,$$ (5)

The viscosity of the fluid is assumed to vary exponentially with temperature and is defined as, $\mu = \mu f_1(T)$. By assuming the viscosity to vary exponentially with temperature, the function $f_1(T)$ takes the form [7], $f_1(T) = \exp(-\alpha_1(T - T_1))$. In some cases $\alpha_1$ may be negative, i.e. the coefficient of viscosity increases with temperature [8,9].

The problem is simplified by writing the equations in the non-dimensional form. To achieve this, we define the following non-dimensional quantities,

$$\hat{x}, \hat{y}, \hat{z} = \left( \frac{x}{h}, \frac{y}{h}, \frac{z}{h} \right), \quad \hat{P} = \frac{P}{\rho U_o h}, \quad (\hat{u}, \hat{v}, \hat{w})$$

$$= \left( \frac{u}{U_o}, \frac{v}{U_o}, \frac{w}{U_o} \right), \quad \hat{T} = \frac{T - T_1}{T_2 - T_1}, \quad G = -\frac{d\hat{P}}{dx},$$

$$\hat{f}_1(\theta) = \exp(-\alpha_1(T_2 - T_1)) = \exp(-\alpha \theta), \quad \alpha$$ is the viscosity exponent,

$$R = \rho U_o h / \mu_o$$ is the Reynolds number,

$$Ha^2 = \sigma B_o^2 h^2 / \mu_o$$ is the Hartmann number,

$$Pr = \mu_o c_p / \kappa_o$$ is the Prandtl number,

$$Ec = U_o / (c_p(T_2 - T_1))$$ is the Eckert number.

$$\tau_{u_l} = (d\hat{u} / d\hat{y})_{\hat{y}=1}/R$$ is the axial skin friction coefficient at the lower plate,

$$\tau_{u_u} = (d\hat{u} / d\hat{y})_{\hat{y}=1}/R$$ is the transverse skin friction coefficient at the lower plate,

$$\tau_{w_l} = (d\hat{w} / d\hat{y})_{\hat{y}=1}/R$$ is the axial skin friction coefficient at the upper plate,

$$\tau_{w_u} = (d\hat{w} / d\hat{y})_{\hat{y}=1}/R$$ is the transverse skin friction coefficient at the upper plate,

$$Nu_{l_k} = (d\hat{\theta} / d\hat{y})_{\hat{y}=1}$$ is the Nusselt number at the lower plate,

$$Nu_{u_k} = (d\hat{\theta} / d\hat{y})_{\hat{y}=1}$$ is the Nusselt number at the upper plate.

where $\rho$ is the density of the fluid and $c_p$ is the specific heat at constant pressure of the fluid. In terms of the above non-dimensional quantities Eqs. (1) to (5) read (the hats are dropped for convenience)

$$G + f_1(\theta) \frac{d^2 u}{dy^2} + \left( \frac{d f_1(\theta)}{dy} \right) u - \frac{Ha^2}{(1 + \beta_i \beta_i) + \beta_i^2}$$

$$\times ((1 + \beta_c \beta_i) u + \beta_i w) = 0,$$ (6)
y = 0 \quad \text{and} \quad y = 1 \quad \text{and} \quad y = -1 \quad \text{with} \quad u = w = 0, y = 1 : u = 1, w = 0 \quad (8)

Eqs. (6), (7), and (9) represent a system of coupled non-linear ordinary differential equations which can be solved numerically under the boundary conditions (8) and (10) using the finite difference approximations. The Crank-Nicolson implicit method is used [11]. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time levels. The non-linear terms are first linearized and then an iterative scheme is used at every time step to solve the linearized system of difference equations. All calculations have been carried out for \( \mathrm{G} = 5 \), \( \mathrm{R} = 1 \), \( \mathrm{Pr} = 1 \), and \( \mathrm{Ec} = 0.2 \). Grid-independence studies show that the computational domain \(-1 < y < 1\) can be divided into intervals with step size \( \Delta y = 0.005 \). Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns \( u, w, \theta \) and their first derivatives for the last two approximations differ from unity by less than \( 10^{-6} \) for all values of \( y \) in \(-1 < y < 1\). In order to examine the accuracy and correctness of the solutions, the results are compared and shown to have complete agreement with those reported by Attia [5] for \( \beta_e \) and \( \alpha = 0 \) and for those given by Attia and Kotb for \( \beta_e = 0 \) [9].

### 3. Results and Discussion

Figures 1 and 2 present the profiles of the velocity components \( u \) and \( w \) for various values of the parameters \( \beta_e, \beta_i \) and \( \alpha \) and for \( \mathrm{Ha} = 3 \). Figure 1 shows that increasing \( \beta_e \) or \( \beta_i \) increases \( u \) for all values of \( \alpha \). This is because the effec-
tive conductivity decreases with increasing \( \beta_e \) or \( \beta_i \), which reduces the magnetic damping force on \( u \). In Figure 2, the velocity component \( w \) increases with increasing \( \beta_e \) since \( w \) is a result of the Hall effect. On the other hand, increasing the ion slip parameter \( \beta_i \) decreases \( w \) for all values of \( \beta_e \) as a result of decreasing the source term of \( w \) while increasing its damping term. The influence of the ion slip on \( w \) becomes more pronounced for higher values of \( \beta_e \). Also, the influence of the Hall parameter \( \beta_e \) on \( w \) is more apparent for lower values of \( \beta_i \). Figures 1 and 2 show that increasing the viscosity exponent \( \alpha \) increases both \( u \) and \( w \) as a result of decreasing the viscosity. It is also seen in Figures 1 and 2 the influence of the viscosity exponent \( \alpha \) on the symmetry of the profiles of \( u \) and \( w \) about the plane \( y = 0 \). It is clear from Figures 1 and 2 that the effect of \( \beta_e \) or \( \beta_i \) on \( u \) and \( w \) is more pronounced for higher values of \( \alpha \).

Figure 3 presents the profile of the temperature \( \Theta \) for various values of the parameters \( \beta_e \), \( \beta_i \) and \( \alpha \) and for \( Ha = 3 \). The figure indicates that increasing \( \beta_e \) or \( \beta_i \) increases \( \Theta \) for all values of \( \alpha \). This can be attributed to the fact that, increasing \( \beta_e \) increases both \( u \) and \( w \) and, in turn, increases the Joule and viscous dissipations. Also increasing \( \beta_i \), although it decreases \( w \), it increases the more effective velocity \( u \) of the main flow and consequently increases the viscous and Joule dissipations. It is clear from the figure that the effect of \( \beta_e \) or \( \beta_i \) on \( \Theta \) is more pronounced for higher values of \( \alpha \). Figure 3 shows also that increasing \( \alpha \) increases \( \Theta \) as a result of increasing the velocities and their gradients which increases the viscous dissipation.

Table 1 presents the variation of the axial and the transverse skin friction coefficients and the Nusselt number at both walls of the channel with the ion slip parameter \( \beta_i \) and the viscosity exponent \( \alpha \). The results are estimated for \( Ha = 3 \) and \( \beta_e = 1 \). It is clear from the table that increasing the ion slip parameter \( \beta_i \) increases the magnitude of the axial skin friction coefficient and the Nusselt number while decreases the magnitude of the transverse component of the skin friction. It is of interest to notice the influence of the ion slip parameter in changing the sign of the axial skin friction coefficient at the upper plate. It is also shown that the variation of the skin friction coefficients and the Nusselt number at both plates depends on the ion slip parameter. Increasing the viscosity exponent \( \alpha \) increases the magnitude of \( \tau_{x_i} \) for all \( \beta_e \) and increases \( \tau_{x_i} \) for \( \beta_i = 0 \) while decreases it for \( \beta_i > 0 \). Increasing \( \alpha \) decreases \( \tau_{x_i} \) and increases the magnitude for all values of \( \beta_i \).
Also, increasing \( \alpha \) decreases the magnitude of \( Nu_L \) for zero and small values of \( \beta_i \), but it increases \( Nu_L \) for higher values of \( \beta_i \).

### 4. Conclusion

The steady MHD Couette flow of a conducting fluid under the influence of an applied uniform magnetic field has been studied with temperature dependent viscosity, considering the Hall and ion slip effects. It is found that the effect of the Hall current or ion slip on the velocity components \( u \) and \( w \) and the temperature \( \theta \) is more pronounced for higher values of the viscosity exponent \( \alpha \). It is

**Table 1.** Variation of the steady state skin friction coefficients and the nusselt number at both walls of the channel with \( \beta_i \) for: (a) \( \alpha = 0 \), (b) \( \alpha = 0.5 \), and (c) \( \alpha = -0.5 \)

<table>
<thead>
<tr>
<th></th>
<th>( \beta_i = 0 )</th>
<th>( \beta_i = 1 )</th>
<th>( \beta_i = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{xL} )</td>
<td>1.8518</td>
<td>2.4419</td>
<td>3.2036</td>
</tr>
<tr>
<td>( \tau_{zL} )</td>
<td>0.7846</td>
<td>0.5369</td>
<td>0.2978</td>
</tr>
<tr>
<td>( Nu_L )</td>
<td>1.0039</td>
<td>1.2507</td>
<td>1.5658</td>
</tr>
<tr>
<td>( \tau_{xU} )</td>
<td>0.4676</td>
<td>-0.4372</td>
<td>-1.5834</td>
</tr>
<tr>
<td>( \tau_{zU} )</td>
<td>-1.6917</td>
<td>-0.9402</td>
<td>-0.4346</td>
</tr>
<tr>
<td>( Nu_U )</td>
<td>-0.3300</td>
<td>-0.4146</td>
<td>-0.5237</td>
</tr>
</tbody>
</table>

**Figure 3.** Effect of \( \beta_e \) and \( \beta_i \) on the profile of \( \theta \). (a) \( \alpha = 0 \); (b) \( \alpha = 0.5 \); (c) \( \alpha = -0.5 \).
also shown that increasing the Hall parameter increases the velocity components $u$ and $w$. On the other hand, increasing the ion slip parameter increases $u$ but decreases $w$. The influence of the Hall current on $w$ decreases greatly as the ion slip increases while the influence of the ion slip on $w$ is more apparent for higher values of the Hall parameter. The ion slip or the viscosity exponent has a marked effect on the axial and transverse components of the skin friction and the Nusselt number at both walls of the channel. It is of interest to find that the influence of the viscosity exponent on these parameters depends on the ion slip.

References


*Manuscript Received: May 28, 2004  
Revision Received: Jul. 21, 2004  
Accepted: Nov. 31, 2004*