In this paper, the unsteady flow of a dusty viscous incompressible electrically conducting Bingham fluid through a circular pipe is investigated. A constant pressure gradient in the axial direction and a uniform magnetic field directed perpendicular to the flow direction are applied. The particle-phase is assumed to behave as a viscous fluid. A numerical solution is obtained for the governing nonlinear momentum equations using finite differences. The effect of the magnetic field parameter $H_a$, the non-Newtonian fluid characteristics (the flow index $n$), and the particle-phase viscosity $\frac{\eta_{p}}{c_{p}}$ on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle-phases are studied. It is found that all the flow parameters for both phases decrease as the magnetic field increases or the flow index decreases. On the other hand, increasing the particle-phase viscosity increases the skin friction of the particle phase, but decreases the other flow parameters.

Key Words: Pipe Flow, Two Phase Flow, Hydromagnetic Flow, Non-Newtonian Fluid

1. Introduction

The flow of a dusty and electrically conducting fluid through a pipe in the presence of a transverse magnetic field has important applications such as magneto-hydrodynamic generators, pumps, accelerators, and flowmeters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase by the so-called particle-phase viscosity. There have been many articles dealing with theoretical modelling and experimental measurements of the particle-phase viscosity in a dusty fluid [1–4].


A number of industrially important fluids such as molten plastics, polymers, pulps and foods exhibit non-Newtonian fluid behavior [9]. Due to the growing use of these non-Newtonian materials, in various manufacturing and processing industries, considerable efforts have been directed towards understanding their flow characteristics. Many of the inelastic non-Newtonian fluids, encountered in chemical engineering processes, are fluids exhibiting a yield stress that has to be exceeded before the fluid moves [10]. It is of interest in this paper to study...
the influence of the magnetic field as well as the non-Newtonian fluid characteristics on the dusty fluid flow properties in situations where the particle-phase is considered dense enough to include the particulate viscous stresses.

In the present study, the unsteady flow of a dusty non-Newtonian Bingham fluid through a circular pipe is investigated in the presence of a uniform magnetic field. The carrier fluid is assumed viscous, incompressible and electrically conducting. The particle phase is assumed to be incompressible pressureless and electrically non-conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The governing nonlinear momentum equations for both the fluid and particle-phases are solved numerically using the finite difference approximations. The effect of the magnetic field, the non-Newtonian fluid characteristics and the particle-phase viscosity on the velocity of the fluid and particle-phases are reported.

2. Governing Equations

Consider unsteady, laminar, axisymmetric horizontal flow of a dusty conducting non-Newtonian fluid through an infinitely long pipe of radius \( d \) driven by a constant pressure gradient. A uniform magnetic field is applied perpendicular to the flow direction. The magnetic Reynolds number is assumed to be very small and consequently the induced magnetic field is neglected [11]. We assume that both phases behave as viscous fluids [8]. In addition, assume that the volume fraction of suspended particles is finite and constant. Taking into account these and the previously mentioned assumptions, the governing momentum equations can be written as

\[
\begin{align*}
\rho \frac{\partial V}{\partial t} &= -\frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( \mu_r \frac{\partial V}{\partial r} \right) + \frac{\rho \phi}{1 - \phi} N(V_p - V) - \sigma B^2 V
\end{align*}
\]

(1)

\[
\begin{align*}
\rho_p \frac{\partial V_p}{\partial t} &= \frac{1}{r} \frac{\partial}{\partial r} \left( \mu_p r \frac{\partial V_p}{\partial r} \right) + \rho_p N(V - V_p)
\end{align*}
\]

(2)

where \( t \) is the time, \( r \) is the distance in the radial direction, \( V \) is the fluid-phase velocity, \( V_p \) is the particle-phase velocity, \( \rho \) is the fluid-phase density, \( \rho_p \) is the particle-phase density, \( \partial P/\partial z \) is the fluid pressure gradient, \( \phi \) is the particle-phase volume fraction, \( N \) is a momentum transfer coefficient (the reciprocal of the relaxation time, the time needed for the relative velocity between the phases to reduce \( e^{-1} \) of its original value [8], \( \sigma \) is the fluid electrical conductivity, \( B_0 \) is the magnetic induction, \( \mu_p \) is the particle-phase viscosity which is assumed constant, and \( \mu \) is the apparent viscosity of the fluid which is given by,

\[
\mu = \mu_o + \frac{\tau_o}{\partial V \partial r}
\]

where \( \mu_o \) is the plastic viscosity of a Bingham fluid and \( \tau_o \) is the yield stress. In this work, \( \rho, \rho_p, \mu_p, \phi, \) and \( B_0 \) are all constant. It should be pointed out that the particle-phase pressure is assumed negligible and that the particles are being dragged along with the fluid-phase.

The initial and boundary conditions of the problem are given as

\[
\begin{align*}
V(r,0) &= 0, V_p(r,0) = 0, \\
\frac{\partial V(0,t)}{\partial r} &= 0, \frac{\partial V_p(0,t)}{\partial r} = 0, V(d,t) = 0, V_p(d,t) = 0
\end{align*}
\]

(3a)/(3b)

where “\( d \)” is the pipe radius.

Eqs. (1)–(3) constitute a nonlinear initial-value problem which can be made dimensionless by introducing the following dimensionless variables and parameters

\[
\begin{align*}
\bar{\tau} &= \frac{r}{d}, \bar{T} = \frac{\mu_o}{\rho d^2}, \bar{G}_o = -\frac{\partial P_c}{\partial z}, k = \frac{\rho \phi}{\rho (1 - \phi)}, \bar{\mu} = \frac{\mu}{\mu_o}
\end{align*}
\]

\[
\bar{V}(r, t) = \frac{\mu_o V(r, t)}{G_o d^2}, \bar{V}_p(r, t) = \frac{\mu_p V_p(r, t)}{G_o d^2}.
\]

(\( \alpha = \frac{N d^2 \rho}{\mu_o} \) is the inverse Stokes’ number, \( \beta = \frac{\mu_p}{\mu_o} \) is the viscosity ratio, \( \tau_o / G_o d \) is the Bingham number (dimensionless yield stress), \( \bar{H}_o = B_o d \sqrt{\sigma / \mu_o} \) is the Hartmann number (Sutton et al. 1965)).
By introducing the above dimensionless variables and parameters as well as the expression of the fluid viscosity defined above, Eqs. (1)–(3) can be written as (the bars are dropped),

\[
\frac{\partial V}{\partial t} = 1 + \frac{\partial^2 V}{\partial r^2} + \frac{\mu}{r} \frac{\partial V}{\partial r} + k \alpha (V - V) - H^2 V \quad (4)
\]

\[
\frac{\partial V_p}{\partial t} = \beta \left( \frac{\partial^2 V_p}{\partial r^2} + \frac{1}{r} \frac{\partial V_p}{\partial r} \right) + \alpha (V - V_p) \quad (5)
\]

\[
\mu = 1 + \frac{\tau_p}{r} \frac{\partial V}{\partial r}
\]

\[
V(r,0) = 0, V_p(r,0) = 0, \quad (6a)
\]

\[
\frac{\partial V(0,t)}{\partial r} = 0, \frac{\partial V_p(0,t)}{\partial r} = 0, V(1,t) = 0, V_p(1,t) = 0 \quad (6b)
\]

The volumetric flow rates and skin-friction coefficients for both the fluid and particle phases are defined, respectively, as (Chamkha 1994)

\[
Q = 2\pi \int_0^1 r V(r,t) dr, \quad Q_p
\]

\[
= 2\pi \int_0^1 r V_p(r,t) dr, C = -\frac{\partial V(1,t)}{\partial r}, C_p = -\beta \frac{\partial V_p(1,t)}{\partial r} \quad (7)
\]

3. Results and Discussion

Eqs. (4) and (5) represent a coupled system of nonlinear partial differential equations which are solved numerically under the initial and boundary conditions (6), using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method [12,13] is used at two successive time levels. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas algorithm [12,13]. Computations have been made for \( \alpha = 1, \gamma = 1, \) and \( k = 10. \) Grid-independence studies show that the computational domain \( 0 < t < 10 \) and \( 0 < r < 1 \) can be divided into intervals with step sizes \( \Delta t = 0.0001 \) and \( \Delta r = 0.005 \) for time and space respectively. It should be mentioned that the results obtained herein reduce to those reported by Dube and Sharma [6] and Chamkha [7] for the cases of non-magnetic, inviscid particle-phase, and Newtonian fluid. These comparisons lend confidence in the accuracy and correctness of the solutions.

Figures 1 and 2 present the time evolution of the profiles of the velocity of the fluid \( V \) and dust particles \( V_p \), respectively, for various values of \( \tau_0 \) and for \( H_a = 0.5 \) and \( \beta = 0.5 \). Both \( V \) and \( V_p \) increase with time and \( V \) reaches the steady-state faster than \( V_p \) for all values of \( \tau_0 \). It is clear also from Figure 1 that increasing \( \tau_0 \) decreases both \( V \) and \( V_p \) while its effect on the steady-state time can be neglected.

Figures 3, 4, 5, and 6 present the influence of the magnetic field parameter \( H_a \) on the transient behavior of the fluid-phase volumetric flow rate \( Q \), the particle-phase volumetric flow rate \( Q_p \), the fluid-phase skin friction coefficient \( C \), and the particle-phase skin friction coefficient \( C_p \) for various values of \( \tau_0 \) and for \( \beta = 0.5 \). Initially, both phases are at rest, and suddenly; they are set to motion through the application of a constant pressure gradient. As a result, the shear stress at the surface of the pipe increases. This explains the obvious increases in \( Q \), \( Q_p \), \( C \), and \( C_p \) shown in Figures 3 through 6, respectively, for all values of the parameter \( \tau_0 \). These parameters continue to increase until the flow stabilizes and steady-state conditions are attained. The influence of the magnetic field, as shown in Figures 3 through 6, is to retard the flow of both phases causing their average velocities and wall shear stresses in the pipe as well as their steady-state times to decrease. Also, it can be concluded from the figures that increasing \( \tau_0 \) decreases greatly the parameters \( Q, Q_p, C \), and \( C_p \) shown in Figures 3 through 6, respectively, for all values of the parameter \( \tau_0 \). These parameters continue to increase until the flow stabilizes and steady-state conditions are attained. The influence of the magnetic field, as shown in Figures 3 through 6, is to retard the flow of both phases causing their average velocities and wall shear stresses in the pipe as well as their steady-state times to decrease. Also, it can be concluded from the figures that increasing \( \tau_0 \) decreases greatly the parameters \( Q, Q_p, C \), and \( C_p \) but slightly increases their steady-state time.

Figures 7, 8, 9, and 10 present the influence of the particle-phase viscosity \( \beta \) on the transient behavior of \( Q, Q_p, C \), and \( C_p \) for various values of \( \tau_0 \) and for \( H_a = 1 \). It is clear from the figures that the inclusion of the particle-phase viscous stresses causes \( Q, Q_p, C \) to decrease
Figure 1. Time development of $V$ for various values of $\tau_D$.

Figure 2. Time development of $V_p$ for various values of $\tau_D$. 
Figure 3. Time development of $Q$ for various values of $\tau_0$ and $Ha$.

Figure 4. Time development of $Q_p$ for various values of $\tau_0$ and $Ha$. 
Figure 5. Time development of $C$ for various values of $\tau_0$ and $Ha$.

Figure 6. Time development of $C_p$ for various values of $\tau_0$ and $Ha$. 
Figure 7. Time development of $Q$ for various values of $\tau_0$ and $\beta$.

Figure 8. Time development of $Q_p$ for various values of $\tau_0$ and $\beta$. 
Figure 9. Time development of $C$ for various values of $\tau_0$ and $\beta$.

Figure 10. Time development of $C_p$ for various values of $\tau_0$ and $\beta$. 
and $C_p$ to increase (see definition of $C_p$; Eq. (7)) for all values of $\tau_0$ and $t$. Also, the approach to steady-state conditions is much accelerated than that of the case of inviscid particle-phase ($\beta = 0$) as clear from Figures 7, 8, and 9. Figure 10 shows that increasing $\beta$ increases $C_p$ but decreases its steady-state time.

4. Conclusions

The transient MHD flow of a particulate suspension in an electrically conducting non-Newtonian Bingham fluid in a circular pipe with an applied uniform transverse magnetic field is studied. The governing nonlinear partial differential equations are solved numerically. The effect of the magnetic field parameter $H_a$, the non-Newtonian fluid characteristics (the Bingham number $\tau_0$), and the particle-phase viscosity $\beta$ on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle-phases are studied. It was found that all these parameters decrease as the strength of the magnetic field or the yield stress increases. The particle-phase viscosity has an apparent effect on increasing the skin friction of the particle-phase while decreasing the rest of the parameters. The approach to steady-state conditions is much decreased when increasing $\beta$ or $H_a$ but it is not greatly affected by changing $\tau_0$.

References


*Manuscript Received: Jul. 21, 2004*  
*Accepted: Dec. 16, 2004*
Call for Papers

Tamkang University was founded in 1950 and Tamkang Journal was published since 1962. Starting from 1998, Tamkang Journal was divided into two, which are Tamkang Journal of Science and Engineering and Tamkang Journal of Humanity and Social Science. Tamkang Journal of Science and Engineering became an international journal since 2000 and four issues are published each year. Tamkang Journal of Science and Engineering devotes to address the following broad research topics, including Computer Science of various aspects including software technologies, parallel/distributive computing, multimedia techniques and bio-informatics, Civil Engineering, Aerospace, Architecture Engineering, Mechanical Engineering, Electrical Engineering, Physics, Chemistry, Chemical Engineering, Biological Technology, Statistics and Environmental Science. Each issue will focus on a particular topic so that this journal can be the only journal that is persistently pursuing advanced researches from all areas. Each article published in this journal is included in COMPENDEX PLUS (EI). As such, we would like to sincerely invite you, the prestige and devoted researchers to submit your significant research results to Tamkang Journal of Science and Engineering. Your submission not only can significantly promote your research effort but also will superb our journal.