Concrete Beams Strengthened with Externally Bonded Glass Fiber Reinforced Plastic Plates

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Abstract

In this work, the flexural behavior of reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates is investigated through the finite element method. The finite element models based on the widely used package ABAQUS are employed in simulating the behavior of reinforced concrete beams strengthened by externally bonded glass fiber reinforced polymer. The numerical results of four-point bending test of reinforced concrete beams strengthened by externally bonded glass fiber reinforced plastic plates in comparing with the experimental results show satisfactory agreement. The results indicate that the flexure strength of reinforced concrete beams can be significantly increased by externally bonded glass fiber reinforced plastic plates. The dynamic behavior of reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates is investigated through the differential quadrature method. In this approach, only eleven sample points are required to obtain convergence.

Key Words: Glass Fiber Reinforced Plastic Plate, Reinforced Concrete Beam, Finite Element Method, Differential Quadrature Method

1. Introduction

Existing cracking reinforced concrete beams may require increasing the shear capacity and cracking. The glass fiber reinforced plastic plate is used to bond the beam in order to increase flexural capacity. Sadatmanesh et al. [1–12] experimentally investigated the static strength of reinforced concrete beams and column strengthened by gluing glass fiber reinforced plastic plates. High strength fiber reinforced plastic straps are wrapped around the column in the potential plastic hinge region to increase confinement and to improve the behavior under seismic forces. Hamoush and Ahmad [13] investigated the strength of steel-plate-strengthened concrete beams. Adhikary et al. [14,15] investigated the shear strengthening of reinforced concrete beams using steel plates bonded on beam web through experiments and numerical analysis. Garden et al. [16,17] experimentally studied the failure modes of reinforced concrete beams strengthened with pre-stressed carbon composite plates and the anchorage length of carbon fiber composite plates used to strengthen reinforced concrete beams. Kankam [18] studied the flexural strength and deformation characteristics of concrete beams reinforced with threaded steel bars that were tensioned against steel plates bearing on the concrete ends by means of tighten nuts. Subedi and Baglin [19] analyzed the ultimate load of plate reinforced concrete beams. Picard et al. [20] investigated the influence of various design parameters such as cube strength, size and position of main embedded tensile bars, width of the beam, plate thickness, and ratio of plate width on the magnitude of ultimate plate peeling moment. Nguyen et al. [21] investigated an analytical model for reinforced concrete beams with bolted side plates accounting for longitudinal and transverse partial interaction. Shen et al. [22] presented an analytical-numerical procedure for
the bending and transverse vibration analysis of moderately thick plates strengthened by a bonded thin composite plate under different kinds of boundary conditions. Ascione and Feo [23] developed a finite element model for predicting shear and normal stresses in the adhesive layer of plated reinforced concrete beams. Raoof et al. [24] investigated the theoretical and experimental study on externally plated reinforced concrete beams. Barnes et al. [25,26] solved the transfer of stress through steel to concrete adhesive bond and analyzed the external steel plate systems for the shear strengthening of reinforced concrete beams based on the equilibrium of forces along the critical section. Sapountzakis et al. [27,28] studied the interface forces and stresses in composite steel-concrete structure. Wang [29] investigated the analytical and experimental study on seismic performance of reinforced concrete T-beams with design deficiency in steel bar curtailment.

ABAQUS standard finite element analysis code is used to model the reinforced concrete beams bonding glass fiber reinforced plastic plates to the tension surface. ABAQUS offers the facility of nonlinear modeling for the reinforced concrete beam. The differential quadrature method (DQM) is used to form the vibration problems of reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates in matrix form. To the author’s knowledge, very few published papers in the literature have presented the dynamic analysis of reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates using the DQM.

2. Flexural Strength of Reinforced Concrete Beams Strengthened by Epoxy Bonded Glass Fiber Reinforced Plastic Plates

Figure 1 shows the geometry of a reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate for four-point bending test. $L$ is the length of the reinforced concrete beam. An epoxy layer is applied to the straps while wrapping for inter laminar bond. The finite element models based on the widely used package ABAQUS are employed in simulating the behavior of reinforced concrete beams strengthened by externally bonded glass fiber reinforced plastic plates. We use the method for measuring the shear strength to get the simple compressive strength and the simple tensile strength by Mohr-Columb criterion. Meshes with triangle type elements are created with an automatic mesh generator. In this two dimensional non-linear finite element model, the boundary nodes are considered simple supported. The modeling approach performed in this study is based on finite element method to describe the total strain. ABAQUS provides an elastic plastic damage theory; including tension cracking, compression crushing, and post crack response. In this study of FEM model, the elastic plastic damage theory is adopted to solve the stress distribution.

![Figure 1](image.png)

*Figure 1.* Schematic view of the reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate along the tension face.
Figure 2 shows the central deflection of the reinforced concrete beam. The beam size used is 0.20 m (b) \times 0.30 m (h) \times 3.00 m (L). b is the width of the reinforced concrete beam. h is the thickness of the reinforced concrete beam. The beams are reinforced with two D6 bars in the compression zone, whereas two D9 bars are provided in the tension. The diameter of D6 bar is 6.35 mm. The diameter of D9 bar is 9.53 mm. Young’s modulus of the concrete is 21000 MPa. The experimental ultimate load is 91.9 kN \[30\]. The analytical ultimate load is 98.9 kN. The agreement between numerical and experiment curves is satisfactory.

Figure 3 shows the central deflection of the reinforced concrete beam strengthened by epoxy bonded a 2 mm thick glass fiber reinforced plastic plate along the tension face. Young’s modulus of the glass fiber reinforced plastic plate is 37230 MPa. The experimental ultimate load is 172.4 kN \[30\]. The analytical ultimate load is 165.5 kN. The results solved using FEM and experimental results are put together for comparison. Results indicate that the central deflection calculated using the FEM agreed very well with the measured deflection. Figure 4 shows the central deflection of the reinforced concrete beam strengthened by epoxy bonded a 4 mm thick glass fiber reinforced plastic plate along the tension face. The experimental ultimate load is 182.5 kN \[30\]. The analytical ultimate load is 220.0 kN. The numerical results and the results measured experimentally correspond closely. Figure 5 shows the central deflection of the reinforced concrete beam strengthened by epoxy bonded a 6 mm thick glass fiber reinforced plastic plate along the tension face. The experimental ultimate load is 255.7 kN \[30\]. The analytical ultimate load is 248.4 kN. The agreement between numerical and experiment curves is satisfactory.

Figure 6 displays the central deflection of the reinforced concrete beam strengthened by epoxy bonded a 6 mm thick glass fiber reinforced plastic plate along the tension face.
tension face. The beam size used is 0.205 m (b) \times 0.565 m (h) \times 4.875 m (L). The beams are reinforced with two D13 bars in the compression zone, whereas three D25 bars are provided in the tension. The diameter of D13 bar is 12.7 mm. The diameter of D25 bar is 25.4 mm. The experimental ultimate load is 325.0 kN [30]. The analytical ultimate load is 352.0 kN. The agreement between numerical and experiment results is satisfactory. Figure 7 reveals the central deflection of the reinforced concrete beam strengthened by epoxy-bonding a 6 mm thick glass fiber reinforced plastic plate along the tension face.

3. Dynamic Analysis of Reinforced Concrete Beams Strengthened by Epoxy Bonded Glass Fiber Reinforced Plastic Plates

This work demonstrates the dynamic behavior of reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates using the DQM. The influences of glass fiber reinforced plastic plate thickness on the dynamic behavior of reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates are investigated. The DQM is used to form the reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates problems in matrix form. The calculated accuracy and integrity of this problem is demonstrated by a series of case studies using the
DQM. There are many computational methods available for vibration analysis. Bellman et al. [31,32] introduced the DQM. DQM reduces the partial differential equations into a set of algebraic equations and has been used extensively to solve a variety of problems in different fields of science and engineering [33–48].

The efficiency and accuracy of the DQM are not dependent on the number and accuracy of the selected comparison functions. The essence of the DQM is that the derivative of a function at a sample point is approximated by a weighted linear sum of the functional values at all the sample points in the domain. DQM approximates the partial derivatives of the function at a sample point is approximated by a weighted linear sum of the functional values at all the sample points in the domain. DQM approximates the partial derivatives of the function at a sample point as:

\[
\frac{\partial^n}{\partial z^n} f(z_i) \approx \sum_{j=1}^{N} D_y^{(n)} f(z_j) \quad \text{for} \quad i, j = 1, 2, \ldots, N
\]

(1)

where \( f(z_i) \) is the functional value at the sample point \( z_i \), and \( D_y^{(n)} \) are the DQ coefficients of the \( m^{th} \) order differentiation attached to these functional values.

The DQ coefficients are difficult to obtain, because they may be ill conditioned. To overcome the numerical ill-conditioning in determining the DQ coefficients \( D_y^{(n)} \), Quan and Chang [42,43] established a set of algebraic expressions to calculate the DQ coefficients, namely:

\[
f(z) = \sum_{i=1}^{N} M(z) \frac{M_i(z)}{(z - z_i)} f(z_i)
\]

(2)

where

\[
M(z) = \prod_{j=1}^{N} (z - z_j),
\]

\[
M_i(z) = \prod_{j=1,j\neq i}^{N} (z_i - z_j) \quad \text{for} \quad i, j = 1, 2, \ldots, N
\]

Substituting equation (2) into equation (1) yields:

\[
D_y^{(n)} = \frac{M_i(z)}{(z_j - z_j)M_i(z_j)} \quad \text{for} \quad i, j = 1, 2, \ldots, N \text{ and } i \neq j
\]

(3)

and

\[
D_y^{(2)} = \sum_{j=1}^{N} \frac{D_y^{(1)}}{D_y^{(1)}} \quad \text{for} \quad i, j = 1, 2, \ldots, N
\]

(4)

The second-order and higher-order derivatives of the DQ coefficients can also be obtained by matrix multiplication [33,34], thus:

\[
D_y^{(2)} = \sum_{j=1}^{N} D_y^{(1)} D_y^{(1)} \quad \text{for} \quad i, j = 1, 2, \ldots, N
\]

(5)

\[
D_y^{(3)} = \sum_{j=1}^{N} D_y^{(2)} D_y^{(1)} \quad \text{for} \quad i, j = 1, 2, \ldots, N
\]

(6)

\[
D_y^{(4)} = \sum_{j=1}^{N} D_y^{(3)} D_y^{(1)} \quad \text{for} \quad i, j = 1, 2, \ldots, N
\]

(7)

\[
D_y^{(n)} = \sum_{j=1}^{N} D_y^{(n-1)} D_y^{(1)} \quad \text{for} \quad i, j = 1, 2, \ldots, N
\]

(8)

The selection of sample points always plays an important role in the solution accuracy of the DQM. The unequally spaced sample points of each beam using the Chebyshev-Gauss-Lobatto distribution in the present computation are chosen as the following equation:

\[
z_i = \frac{1}{2} \left[ 1 - \cos \left( \frac{(i - 1)\pi}{N - 1} \right) \right] \quad \text{for} \quad i, j = 1, 2, \ldots, N
\]

(9)

By assuming the reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate is long, the strain energy of the reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate can be simplified as:

\[
U = \frac{1}{2} \int_0^L E_i I_i \left( \frac{\partial^2 u_1}{\partial z^2} \right)^2 dz + \frac{1}{2} \int_0^L E_z I_z \left( \frac{\partial^2 u_2}{\partial z^2} \right)^2 dz
\]

(10)

\[
+ \frac{1}{2} \int_0^L k (u_1 - u_2)^2 dz
\]

where \( E_i \) is Young’s modulus of the reinforced concrete beam and \( E_z \) is Young’s modulus of the glass fiber reinforced plastic plate. \( u_1 \) is the transverse displacement of the reinforced concrete beam and \( u_2 \) is the transverse deflection of the glass fiber reinforced plastic plate. \( I_i \) is the 2nd moment area of concrete beam section and \( I_z \) is
the 2nd moment area of glass fiber reinforced plastic plate section. \( k \) is the stiffness of the epoxy adhesive. The kinetic energy of the reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate is

\[
T = \frac{1}{2} \rho_1 A_1 \left( \frac{\partial u_1}{\partial t} \right)^2 dz + \frac{1}{2} \rho_2 A_2 \left( \frac{\partial u_2}{\partial t} \right)^2 dz \tag{11}
\]

where \( \rho_1 \) is the density of the material of the concrete beam and \( \rho_2 \) is the density of the material of the glass fiber reinforced plastic plate. The cross-section area of the concrete beam is \( A_1 \) and the cross-section area of the glass fiber reinforced plastic plate is \( A_2 \). Substituting equations (10) and (11) into Hamilton’s equation,

\[
\int_0^L \left( 6T - 8U \right) dt = 0 \tag{12}
\]

the differential equations governing the dynamics of reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate are:

\[
\rho_1 A_1 \frac{\partial^2 u_1}{\partial t^2} + \frac{\partial^2}{\partial z^2} \left( E_1 I_1 \frac{\partial^2 u_1}{\partial z^2} \right) + ku_1 - u_2 = 0 \tag{13}
\]

\[
\rho_2 A_2 \frac{\partial^2 u_2}{\partial t^2} + \frac{\partial^2}{\partial z^2} \left( E_2 I_2 \frac{\partial^2 u_2}{\partial z^2} \right) + ku_2 - u_1 = 0 \tag{14}
\]

The corresponding boundary conditions of the reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate are:

\[
u_i (0, t) = 0 \tag{15}
\]

\[
E_1 I_1 \frac{\partial^2 u_1 (0, t)}{\partial z^2} = 0 \tag{16}
\]

\[
u_i (L, t) = 0 \tag{17}
\]

\[
E_1 I_1 \frac{\partial^2 u_1 (L, t)}{\partial z^2} = 0 \tag{18}
\]

\[
u_i (0, t) = 0 \tag{19}
\]

\[
E_2 I_2 \frac{\partial^2 u_2 (0, t)}{\partial z^2} = 0 \tag{20}
\]

Consider the displacements to be of the form

\[
u_i (z, t) = U_i (z) \exp \left( i \omega t \right) \tag{23}
\]

\[
u_j (z, t) = U_j (z) \exp \left( i \omega t \right) \tag{24}
\]

Equations (13) and (14) can be simplified to

\[
\omega^2 \rho_1 A_1 U_1 = \frac{d^2}{dz^2} \left( E_1 I_1 \frac{d^2 U_1}{dz^2} \right) + k(U_1 - U_2) \tag{25}
\]

\[
\omega^2 \rho_2 A_2 U_2 = \frac{d^2}{dz^2} \left( E_2 I_2 \frac{d^2 U_2}{dz^2} \right) + k(U_2 - U_1) \tag{26}
\]

The corresponding boundary conditions of the reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate are:

\[
U_i (0) = 0 \tag{27}
\]

\[
E_1 I_1 \frac{d^2 U_1 (0)}{dz^2} = 0 \tag{28}
\]

\[
U_i (L) = 0 \tag{29}
\]

\[
E_1 I_1 \frac{d^2 U_1 (L)}{dz^2} = 0 \tag{30}
\]

\[
U_j (0) = 0 \tag{31}
\]

\[
E_1 I_1 \frac{d^2 U_2 (0)}{dz^2} = 0 \tag{32}
\]

\[
U_j (L) = 0 \tag{33}
\]

\[
E_1 I_1 \frac{d^2 U_2 (L)}{dz^2} = 0 \tag{34}
\]

By employing the DQM, the equations of motion of the reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate can be discre-
The elements in the mass matrix are:

\[
[C_g] = \omega^2 [M_g] [W(z_j)]
\]

(35)

where

\[
\begin{bmatrix}
W(z_1) \\
W(z_2) \\
\vdots \\
W(z_{n_2}) \\
W(z_{n_1}+1) \\
W(z_{n_1}+2) \\
\vdots \\
W(z_{n_2}+1)
\end{bmatrix} = 
\begin{bmatrix}
U_1(z_1) \\
U_1(z_2) \\
\vdots \\
U_1(z_{n_2}) \\
U_2(z_1) \\
U_2(z_2) \\
\vdots \\
U_2(z_{n_2})
\end{bmatrix}
\]

(36)

The elements in the mass matrix are:

\[
M_{ii} = \rho_i A_i \quad \text{for } i = 3, 4, \ldots, N - 2
\]

(37)

\[
M_{ii} = \rho_i A_2 \quad \text{for } i = N + 3, N + 4, \ldots, 2N - 2
\]

(38)

\[
M_{ii} = 0 \quad \text{for } i = 1, 2, N - 1, N, N + 1, N + 2, 2N - 1, 2N
\]

(39)

\[
M_{ij} = 0 \quad \text{for } i \neq j, i = 1, 2, \ldots, 2N \text{ and } j = 1, 2, \ldots, 2N
\]

(40)

The elements in the stiffness matrix are:

\[
C_{ij} = \frac{d^2E_iI_j}{dz^2}\left|_{z = x} \right. \left( \frac{D^{(i)}_j}{L^2} + 2 \frac{dE_iI_j}{dz} \left|_{z = x} \right. \right) + \left. \frac{D^{(i)}_j}{L} \right)
\]

(41)

\[
C_{ij} = \frac{d^2E_iI_j}{dz^2}\left|_{z = x} \right. \left( \frac{D^{(i)}_j}{L^2} + 2 \frac{dE_iI_j}{dz} \left|_{z = x} \right. \right) + \left. \frac{D^{(i)}_j}{L} + k \right)
\]

for \( i = 3, 4, \ldots, N - 2 \)

(42)

\[
C_{ij} = 0
\]

for \( i \neq j, i = 3, 4, \ldots, N - 2 \) and \( j = N + 1, N + 2, \ldots, 2N \)

(43)

\[
C_{ij,N} = -k
\]

for \( i = 3, 4, \ldots, N - 2 \)

(44)

The boundary conditions of the simple supported reinforced concrete beam strengthened by epoxy bonded a glass fiber reinforced plastic plate can be rearranged into the matrix form as

\[
\begin{bmatrix}
1 & 0 & 0 & \ldots & 0 & 0 \\
D^{(2)}_{11} & \frac{D^{(2)}_{12}}{L^2} & \frac{D^{(2)}_{13}}{L^2} & \ldots & \frac{D^{(2)}_{1N-1}}{L^2} & \frac{D^{(2)}_{1N}}{L^2} \\
& \frac{D^{(2)}_{21}}{L^2} & \frac{D^{(2)}_{22}}{L^2} & \frac{D^{(2)}_{23}}{L^2} & \ldots & \frac{D^{(2)}_{2N-1}}{L^2} & \frac{D^{(2)}_{2N}}{L^2} \\
& \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
& \frac{D^{(2)}_{N1}}{L^2} & \frac{D^{(2)}_{N2}}{L^2} & \frac{D^{(2)}_{N3}}{L^2} & \ldots & \frac{D^{(2)}_{NN-1}}{L^2} & \frac{D^{(2)}_{NN}}{L^2} \\
& & & & & 1 & 0 \\
0 & 0 & 0 & \ldots & 0 & 1 \\
0 & 0 & 0 & \ldots & 0 & 1
\end{bmatrix}
\]

(49)

(50)

(51)

(52)
Figure 8 illustrates the frequencies of the reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates with different thicknesses. The beam size used is $0.20 \text{ m} \times 0.30 \text{ m} \times 3.00 \text{ m}$ (L). The beams are reinforced with two D6 bars in the compression zone, whereas two D9 bars are provided in the tension. The numerical results show that higher frequencies are found for the beam with a higher thickness glass fiber reinforced plastic plate. The frequency of the reinforced concrete beam strengthened by epoxy-bonding a 2 mm thick glass fiber reinforced plastic plate along the tension face is greater than the frequency of the reinforced concrete beam strengthened by no epoxy-bonding glass fiber reinforced plastic plate for mode 1. The frequencies of the reinforced concrete beam strengthened by epoxy-bonding a 2 mm thick glass fiber reinforced plastic plate along the tension face is smaller than the frequency of the reinforced concrete beam strengthened by no epoxy bonded glass fiber reinforced plastic plate for mode 2 to mode 6.

4. Conclusion

The results conduct that the flexure strength of reinforced concrete beams can be significantly increased by externally bonded glass fiber reinforced plastic plates. The calculated results agreed closely with the measured results in the literature. This paper demonstrates the dynamic behavior of reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates using the DQM. The results indicate that bonding composite plates to concrete beams increase the stiffness, yield moment, and flexural strength of the beams. The simplicity of this formulation makes it a desirable candidate for modeling reinforced concrete beams strengthened by epoxy bonded glass fiber reinforced plastic plates.

References


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