Evaluation and Design of Two Level Continuous Sampling Plans

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Abstract

Dodge (1943) introduced a single level attribute continuous sampling plan designated as CSP-1 for the application of continuous production processes. Lieberman and Solomon (1955) developed multi-level (MLP) continuous sampling plans. In this paper, we restrict our discussion to MLP plans with two sampling levels, which plan is designated as MLP-2 plan. Using a Markov chain model, expressions for the performance measures of MLP-2 plan are derived. Tables are also presented for the selection of MLP-2 plan when the AQL or LQL and AOQL are specified.

Key Words: Average Number Inspected, Average Outgoing Quality, Markov Chain

1. Introduction

Continuous sampling plans are used for processes where there is a continuous product flow and units are not grouped into lots. Dodge (1943) innovated the concept of continuous sampling and provided the mathematical rationale and the rules of operation for the first continuous sampling plan, popularly known as CSP-1. This plan is the simplest and most commonly used type of single level continuous sampling plan. This procedure requires periods of both 100% and sampling inspection.

Some of the examples of production processes where the continuous sampling plans can be applied are as follows.

(i) Cars coming off an assembly line
(ii) Soft drink bottles from a continuous glass ribbon machine
(iii) Welded leads emanating from a welding operation
(iv) Manufacturing of many electronic products, such as personal computers, is performed on a conveyo- rized assembly line

Dodge and Torrey (1992) developed additional continuous sampling plans such as CSP-2 and CSP-3. Lieberman and Solomon (1955) extended the concept of CSP-1 as multi-level plans, which has the following features.

(a) Allows for smoother transition between sampling inspection and 100% inspection
(b) Requires 100% inspection only when the quality submitted is inferior,
(c) Allows for a minimum amount of inspection when quality is definitely good.

This aim is accomplished by the introduction of multi-level sampling plan which specifically allows for any number of sampling levels subject to the provision that transitions can only occur between adjacent levels. Multi-level sampling plans provide for an infinite number of sampling inspection levels. In this paper, we consider the MLP plan with two sampling inspection levels, designating it as MLP-2 plan.

MLP-2 plan alternates between screening and sampling inspection with two levels. The procedure starts with 100% inspection of units in the order of production and the inspection continues until the number of consecutive conforming units reaches some pre-specified integer i. Then the procedure proceeds to sampling inspection at
level 1, where only a pre-specified fraction $f_1$ of the units are inspected. Whenever a non-conforming unit is found, immediately revert to 100% inspection. When $i$ consecutive sampled units are found conforming, the procedure proceeds to sampling inspection at level 2, where only a pre-specified fraction $f_2(< f_1)$ the units are inspected. Whenever a non-conforming unit is found at sampling level 2, then the inspection reverts to the sampling inspection at level 1, otherwise continue the sampling level 2. During sampling inspection, sample units are selected one at a time from the flow of the product so as to assure an unbiased sample. Furthermore, all the non-conforming units found are corrected or replaced with conforming units. A flow diagram showing the operation of MLP-2 procedure is given in Figure 1. The chief features of the MLP-2 plan are the following.

![Flow diagram](image_url)

**Figure 1.** Operation of the MLP-2 Plan.
(1) The number of sampling levels is fixed at two.
(2) The sampling fractions $f_1$ and $f_2$ satisfy the relation $f_1 > f_2$.
(3) The reversion to 100% inspection takes place upon finding a non-conforming unit at sampling level 1 only.

Stephens (1979) used a Markov chain model to find certain measures of performance for CSP-1, CSP-2 and CSP-3 plans, assuming that the production process is in statistical control. In this paper the formulation of the MLP-2 procedure as a Markov chain is given. Following the approach of Stephens, the performance measures such as the average outgoing quality (AOQ), the average fraction of units inspected (AFI) and the average fraction total produced units accepted on a sampling basis or the probability of acceptance ($P_a$) derived. For the purpose of selection of MLP-2 plans, two tables are given. Tables 2 and 3 give MLP-2 plans with parameters $i, f_1 = f$ and $f_2 = f^2$. Table 2 can be used to obtain MLP-2 plans when the acceptable quality level (AQL) with the producer’s risk $\alpha = 0.05$ and the average outgoing quality limit (AOQL) are specified. Table 3 can be used to select the MLP-2 plans when the limiting quality level (LQL) with consumer’s risk $\beta = 0.10$ and the AOQL are specified.

2. The MLP-2 Procedure as a Markov Chain

Let $[X_m]$ (m = 1, 2,...) denote a discrete-parameter Markov chain with finite state space($S_j$) (j = 1, 2, ..., 4i+4). The states of the process are defined, in a way similar to Roberts (1965) and Lasater (1970), as follows:

- $S_k = A_{k-1}$ (k = 1, 2,..., i+1) = 100% inspection is being conducted and the last (k-1) consecutive units inspected were conforming.
- $S_{4i+2} = I_d$ = Sampling inspection at level 2 is in use and the last unit submitted for inspection was found to be non-conforming.
- $S_{4i+3} = I_n$ = Sampling inspection at level 2 is in use and the last unit submitted for inspection was conforming.
- $S_{4i+4} = N$ = Sampling inspection at level 2 is in use and the last unit submitted for inspection was not inspected.

The set of (4i+4) states defined above completely describe the mutually exclusive phases of inspection for the MLP-2 plan. A flow chart showing the description of the process by means of states and transition is given Figure 2 and one step transition probability matrix for the process is given in Table 1. The transition probability matrix reveals that the process is ergodic (See Ross (1996)).

3. The Performance Measures of MLP-2 Plan

(1) The average number inspected (ANI) under the screening or 100% inspection is

$$ANI_{100}(p) = \frac{1 - q'}{pq}$$

(2) The average number inspected (ANI) under the sampling procedure is determined by

$$ANI_{SAM}(p) = f_2 + \left(f_1 - f_2\right)q'\frac{1}{p(1-q')}$$

(3) The average fraction of total produced units inspected in the long run is determined by

$$AFI(p) = \frac{f_1 f_2 \left(1 - q' \left(1 - q^2\right)\right)}{f_1 f_2 \left(1 - q^2\right)^2 + f_2 q' \left(1 - q^2\right) + f_2 q''}$$

(4) The average fraction of the total production accepted on a sampling basis (the operating characteristic function) is

$$P_a(p) = \frac{q' \left(f_2 \left(1 - q'\right) + f_2 q'\right)}{f_1 f_2 \left(1 - q^2\right)^2 + f_2 q' \left(1 - q^2\right) + f_2 q''}$$
The average outgoing quality is

\[ \text{AOQ}(p) = \frac{pq'[f_2(1-f_1) + q'(f_2 - f_1)]}{f_1f_2(1-q')^2 + f_2q'(1-q') + f_1q'^2} \]  

(5) The average outgoing quality limit (AOQL), the maximum of the AOQ for all values of p, may be found graphically using the AOQ function given in (5). A derivation of these measures, based on a Markov chain formulation of the plan, is shown in Appendix.

4. Selection of MLP-2 Plans

Tables 2 and 3 can be used to select the MLP-2 plans with parameters \( i, f_1 = \frac{f}{f}, f_2 = \frac{f^2}{f^2} \). The following examples illustrate the selection of the plans from these tables.

Suppose that one requires an MLP-2 plan with parameters \( i, f_1 = \frac{f}{f}, f_2 = \frac{f^2}{f^2} \) satisfying the desired condition \( \text{AQL} = 1\% \) with \( \alpha = 5\% \) and \( \text{AOQL} = 1.4\% \). Table 2 is used for the selection of the plan. The ratio \( \text{AOQL/AQL} \) is equal to 1.4. Corresponding to \( \text{AQL} = 0.01 \) and \( \text{AOQL/AQL} = 1.4 \), Table 2 gives \( i = 75 \) and \( f = \frac{1}{n} = \frac{1}{5} \). Thus, the required plan is \( (i,f_1,f_2) = (75, \frac{1}{5}, \frac{1}{25}) \).

Suppose that one requires an MLP-2 plan with parameters \( i, f_1 = \frac{f}{f}, f_2 = \frac{f^2}{f^2} \) satisfying the desired condition \( \text{LQL} = 0.25\% \) with \( \beta = 10\% \) and \( \text{AOQL} = 0.06\% \). Table 3 is used for the selection of the plan. The ratio \( \text{LQL/AOQL} \) is equal to 4.16. Table 3 does not have an entry for which

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**Figure 2.** States and Transitions of the MLP-2 Procedure.
LQL/AOQL = 4.17. Hence the ratio LQL/AOQL is just greater than 4.2 is considered for the purpose of selection of the plan, since the higher the value of LQL/AOQL, the lower will be the AOQL when the LQL is fixed. Corresponding to LQL = 0.25% and LQL/AOQL = 4.2, Table 3 gives i = 1525 and f = 1/4. Thus the required plan is (i, f1, f2) = (1525, 1/4, 1/16).

5. Construction of Tables

The operating characteristic (OC) function of MLP-2 plan given in (4) can be rewritten as

\[ P_2(p) = \frac{q'[f_2(1-q') + f_1q']}{D} \] (6)

Where \( D = f_2(1-q')^2 + f_1q'(1-q') + f_1q'^2 \)

By taking \( f_1 = f \) and \( f_2 = f^2 \), equation (6) becomes a quadratic equation in \( f \). Since \( f \) cannot be negative, the positive root of the quadratic equation is given by

\[ f = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \] (7)

where \( A = (1-q')P_d(p) \), \( B = q'(1-q')[P_d(p) - 1] \) and \( C = q^2[P_d(p) - 1] \)

The following procedure is used for the construction of Tables 2 and 3.

(1) For each integer \( i \) in the interval (2, 2000) find the integer \( n \) and the average outgoing quality limit \( P_L \) as follows.
   (i) If the AQL condition is specified, take \( p = AQL \) and \( P_a(p) = 0.95 \). If the LQL condition is specified, take \( p = LQL \) and \( P_a(p) = 0.10 \).
   (ii) Find \( f \) using equation (7) if the parameters of the MLP-2 plan are \( i, f_1, f_2 = (1525, 1/4, 1/16) \).
   (iii) Take \( n = \text{int}(1/f + 0.99) \) if the AQL condition is specified and \( n = \text{int}(1/f) \) if the LQL condition is specified. This type of integer conversion will make \( f_1 \) and \( f_2 \) to be less than or equal to 5% and 10% respectively.
   (iv) Take revised \( f \) as \( f = 1/n \).

(2) For the desired AOQL, find \( i \) and \( f \) for which \( D = \text{AOQL} - P_L \) is non-negative and minimum. These \( i \) and \( 1/f \) values are tabulated in the Tables.
Table 2 gives MLP-2 plans for preferred AQL values with $\alpha = 0.05$ and fixed values of AOQL/AQL. Table 3 gives MLP-2 plans for specified LQL values with $\beta = 0.10$ and fixed values of LQL/AOQL.

### 6. Conclusions

In this paper, a two level continuous sampling plan has been considered. Its measures of performance have been derived using the Markov chain model. The attractive feature of the two level sampling plan is that a smaller inspection effort is required at good incoming quality levels when compared to CSP-1 sampling procedure. Tables provided in this paper will be useful for selecting the plan parameters for given various quality levels in terms of (AQL, AOQL) and (LQL, AOQL).

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Appendix

Glossary of Symbols

- \( p \) = the probability of a unit produced by the process being non-conforming
- \( q \) = (1-\( p \))
- \( i \) = the clearance number
- \( f_1 \) = the sampling frequency at inspection level 1
- \( f_2 \) = the sampling frequency at inspection level 2
- \( S_j \) = \( j^{th} \) state of the process

**Derivation of Performance Measures of MLP-2 Plan**

The formulation of the MLP-2 procedure as a Mar-

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kov chain with states \((S_j) (j = 1,2,3,...,4i+4)\) implies that the Markov chain is a DFRIA Markov chain. Furthermore, for an infinite number of items submitted for inspection under this plan, a unique vector of steady-state probabilities exists, all elements of which are positive and independent of the initial state. These steady-state probabilities \(\varphi(S)\) satisfy the following conditions:

\[
\varphi(S_m) \geq 0, \quad \text{for } m = 1,2,\ldots,4i+4
\]

\[
\varphi(S_m) = \sum_{j=1}^{4i+4} \varphi(S_j) p_{jm}, \quad \text{for } m = 1,2,\ldots,4i+4 \tag{A1}
\]

\[
\sum_{j=1}^{4i+4} \varphi(S_j) = 1 \tag{A2}
\]

Conditions (A1) and (A2) give the following equations:

\[
\varphi(A_m) = p \left[ \varphi(A_m) + \sum_{i=1}^{4i+4} \varphi(A_j) + \sum_{j=1}^{i} \varphi(Id_j) \right] \tag{A3}
\]

\[
\varphi(A_m) = p \left[ \varphi(A_m) + \sum_{j=1}^{i} \varphi(Id_j) \right] \tag{A4}
\]

\[
\varphi(A_m) = q^{m-1} \varphi(A_1), \quad \text{for } m = 1,2,\ldots,i \tag{A5}
\]

\[
\varphi(Id_j) = \frac{pq^{j-2}}{(1-q^j)} \varphi(A_j), \quad \text{for } j = 1,2,\ldots,i \tag{A6}
\]

\[
\varphi(In_j) = \frac{q^{j-1}}{(1-q^j)} \varphi(A_j), \quad \text{for } j = 1,2,\ldots,i \tag{A7}
\]

\[
\varphi(N_j) = \frac{(1-f_1)q^{j-2}}{f_1(1-q^j)} \varphi(A_j), \quad \text{for } j = 1,2,\ldots,i \tag{A8}
\]

\[
\varphi(Id) = \frac{q^{2i-1}}{p(1-q^i)} \varphi(A_i) \tag{A9}
\]

\[
\varphi(In) = \frac{q^{2i}}{p(1-q^i)} \varphi(A_i) \tag{A10}
\]

\[
\varphi(N) = \frac{(1-f_1)q^{2i-1}}{f_2p(1-q^i)} \varphi(A_i) \tag{A11}
\]

By solving the equations (A3)-(A12), one can get

\[
\varphi(A_m) = \frac{f_1f_2p(1-q^i)^2}{f_1f_2(1-q^j)^2 + f_2q^j(1-q^i) + f_1q^{2i}}
\]

By letting \(D = f_1f_2(1-q^j)^2 + f_2q^j(1-q^i) + f_1q^{2i}\), the steady state probabilities can now be written as follows:

\[
\varphi(A_m) = \frac{f_1f_2p(1-q^i)^2}{D}
\]

\[
\varphi(A_m) = \frac{f_1f_2pq^{m-1}(1-q^i)^2}{D}, \quad \text{for } m = 1,2,\ldots,i
\]

\[
\varphi(Id_j) = \frac{f_1f_2p^2q^{j-i-1}}{D}, \quad \text{for } j = 1,2,\ldots,i
\]

\[
\varphi(In_j) = \frac{f_1f_2pq^{j-i-1}}{D}, \quad \text{for } j = 1,2,\ldots,i
\]

Let \(\varphi(100) = \sum_{m=0}^{i} \varphi(A_m)\)

\[
\varphi(S) = \sum_{j=1}^{i} \left[ \varphi(Id_j) + \varphi(In_j) + \varphi(N_j) \right] + \varphi(Id) + \varphi(In) + \varphi(N)
\]

and

\[
\varphi(FN) = \sum_{j=1}^{i} \left[ \varphi(N_j) \right] + \varphi(N)
\]

Then, \(ANI100(p) = \varphi(100)[\varphi(A_i)]^{-1}\)

\[\text{ANISAM}(p) = \varphi(S) \left[ n_1n_2\varphi(A_i) \right]^{-1}\]

\[\text{AFI}(p) = \frac{ANI100(p) + \text{ANISAM}(p)}{ANI100(p) + n_1n_2\text{ANISAM}(p)}\]
By simplifying the above equations, we can get the performance measures of the MLP-2 plans which have been given in the equations (1) to (5).

References


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