Analytical Valuation of Asian Options with Continuously Paying Dividends in Jump-Diffusion Models

Hsien-Jen Lin
Department of Applied Mathematics, Aletheia University,
Tamsui, Taiwan 251, R.O.C.

Abstract

We consider the problem of valuation of certain Asian options in the geometric jump-diffusion models with continuously dividend-paying assets. With the sources of diffusion risks and two primitive tradeable assets, the market in this model is, in general, incomplete, and so, there are more than one equivalent martingale measures and no-arbitrage prices. For this jump-diffusion model, we adopt the minimal martingale measure as the risk-neutral pricing measure for option valuation in a dynamically incomplete market. A partial integro-differential equation satisfied by the no-arbitrage price of an Asian option is obtained by change of numeraire technique under the minimal martingale measure.

Key Words: Asian Options, Jump-diffusion Model, Dividend, Itô’s Formula, Partial Integro-differential Equations

1. Introduction

The theory of option pricing has been a central concern of mathematical economists since the celebrated work of Black and Scholes [1], and the extensions of Merton [2]. An Asian option, or option on the average, is a financial contract whose payoff includes a time average of the underlying asset price. The average may be over the entire time period between initiation and expiration or may be over some period of time that begins later than the initiation of the option and ends with the option’s expiration. An option based on an average price is an attractive feature for thinly traded assets and commodities such as oil, gold and foreign currencies. The averaging complicates the mathematics, but protects the holder against speculative attempts to manipulate the asset price near the expiry date. Hence, Asian options are popular and commonly traded in the financial community. The problem of pricing Asian option is already complicated when the underlying asset is a geometric Brownian motion. The difficulty in pricing this contract stems from the fact that its value is strongly path-dependent, since the payoff depends not only on the terminal spot price, but also on the entire history of the price. Even though in the literature a number of results have been established on the subject of continuously averaged Asian options in a Black-Scholes setting, many empirical research works have shown that some unexpected events (information release of economic downturn conditions, major political changes or natural catastrophes, etc.) can lead to brusque variations in prices. The first application of jump processes in option pricing was introduced by Merton in 1976. He proposed to add to the behavior of asset prices jumps which have normally distributed size [3]. During the last ten years research on models with jumps has become very active. A large number of such models has been proposed; see [4] and the references therein.

The pricing problem for these options has been widely studied and generalized since Geman and Yor [5]. Véčer studied pricing of Asian options (written on an underlying stock without jumps) on a continuously dividend-paying asset in [6]. Recently, Véčer and Xu in [7] considered pricing Asian options under geometric Brow-
nian motion with Poission jump: $dS_t = S_t \left( (\sigma \, dW_t + \left( e^{\lambda t - \frac{1}{2} \sigma^2 t} \right) dM_t \right)$, where $B_t$ is a standard Brownian motion, $M_t$ is a compensated Poisson process, i.e., $M_t = N_t - \lambda t$, and $\phi_t$ is a Gaussian process with independent increments, and is independent of both $B_t$ and $M_t$. Besides, they mentioned two pure jump processes models: One is Carr-Geman-Madan-Yor (CGMY) model [8] and the other is the General Hyperbolic model [9]. Other partial differential equations for pricing Asian options are provided by Andersen [10], Lipton [11], and Rogers and Shi [12]. In our paper we want to study the valuation of Asian options on an asset paying a continuous dividend yield when the underlying asset is driven by a certain geometric jump-diffusion process (see section 2 below), which is simple, but very practical. Obviously, such a market is incomplete and there is not a unique equivalent martingale measure. Here we will not discuss in detail how to choose an equivalent martingale measure that is in respect “closest” to the underlying canonical measure for pricing purpose. Interested readers are referred to [13] for the Föllmer-Schweizer minimal measure. For the proposed model, we adopt the minimal martingale measure as the risk-neutral pricing measure under which we use the martingale approach, which is perfectly described in the book by Musiela and Rutkowski [14], and present an analytical pricing equation for the Asian option.

The paper is organized as follows. In section 2 we introduce our jump-diffusion setting necessary for what follows. In section 3 we consider the Asian option under the geometric jump-diffusion setting. We derive a partial integro-differential equation (PIDE) whose solution leads to the prices of Asian options with paying continuous dividend yields when the underlying asset follows the dynamics of a jump-diffusion model through the change of numeraire technique. Section 4 concludes this paper.

### 2. Market Model Formulation

We consider a Black-Scholes type financial market operating in continuous time, which consists of two primitive assets, namely, a riskless asset (e.g. savings account) and a risky asset (e.g., the price of a stock and a currency exchange rate) that pays dividends continuously over time at the rate $\gamma$ per unit time. These assets are tradeable continuously over time in a finite time horizon $[0, T]$, where $T < \infty$. Suppose that the evolution of the risky asset $S_t$ is influenced by jumps of the proportions $(Y_j)_{j \in \mathbb{N}}$ at the jump times $(\tau_j)_{j \in \mathbb{N}}$ of a Poisson process $(N_j)_{j \in [0, T]}$ with intensity $\lambda > 0$. Between two jumps, we further assume that the dynamics of the risky asset follows the Black-Scholes model. To be more rigorous, we describe sources of uncertainties in the economy using a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and a finite time interval $[0, T]$ on which we define a standard Brownian motion $(B_t)_{t \in [0, T]}$, a Poisson process $(N_t)_{t \in [0, T]}$ with intensity $\lambda > 0$ $(N_t + \lambda t$ is the compensated Poisson process), and a sequence $(Y_j)_{j \in \mathbb{N}}$ of independent, identically distributed random variables taking values in the interval $(-1, +\infty)$. Additionally, assume further that the $\sigma$-generated by $(B_t)_{t \in [0, T]}$ respectively, $(N_t)_{t \in [0, T]}$ and $(Y_j)_{j \in \mathbb{N}}$ are independent. The flow of information $(F_t)_{t \in [0, T]}$ is given by the augmented filtration generated by the random variables $B_s, N_s, s \leq t$ and $Y_j | \mathcal{F}_{\tau_j}$ for $j \in \mathbb{N}$. It can be shown that $(B_t)_{t \in [0, T]}$ is a standard Brownian motion with respect to the filtration $(F_t)_{t \in [0, T]}$ that $(N_t)_{t \in [0, T]}$ is a process adapted to this filtration and has independent increments, i.e., for all $t > s$, $N_t - N_s$ is independent of the $\sigma$-algebra $F_s$. Because the random variables $Y_j | \mathcal{F}_{\tau_j}$ for $j \in \mathbb{N}$ are $(F_t)$-measurable, we know that, at time $t$, the relative amplitudes of the jumps taking place before $t$ are known.

Mathematically the dynamics of $S_t$, price process of the risky asset at time $t \in [0, T]$, can now be described in the following manner:

(i) Let $t \in [\tau_j, \tau_{j+1})$. As mentioned previously, the price process of the risky asset between two jumps $\tau_j$ and $\tau_{j+1}$ is assumed to follow the Black-Scholes model:

$$dS_t = S_t \left( (\mu - \gamma)dt + \sigma dB_t \right), \quad S_0 > 0$$ (1)

where $\mu$ is some parameter representing the mean rate of return of the underlying asset, $\gamma$ is a continuous dividend yield, and $\sigma$ is the volatility of the underlying asset.

(ii) At time $t = \tau_j$, the process is assumed to have a jump $\Delta S_{\tau_j}$, i.e.,

$$\Delta S_{\tau_j} = S_{\tau_j} - S_{\tau_j} = S_{\tau_j} Y_j, \quad \text{thus } S_{\tau_j} = S_{\tau_j} (1 + Y_j)$$ (2)

The price process $(S_t)_{t \in [0, T]}$ is obviously right-continuous, adapted and has only finitely many discontinuities on each interval $[0, T]$. Moreover, for all $t \in [0, T]$,
3. Pricing Options

Throughout this paper, for our model we adopt the minimal martingale measure as the chosen risk-neutral pricing measure under which we use the martingale approach, which is perfectly described in the book by Musiela and Rutkowski [14], and present an analytical pricing equation for the Asian option. We assume that \( P \) is the Föllmer-Schweizer minimal measure and at time 0 \( t \leq T \), represents the value of the risky asset underlying the option. This asset can either be a dividend-paying stock or an exchange rate. In the first case \( r \) is the constant interest rate of the market and \( q \) is the dividend rate of the stock, in the other case \( r \) is the domestic interest rate and \( q \) the foreign one. Under these model assumptions, we consider the problem of pricing a fixed-strike Asian call whose payoff at maturity \( T \) is given by

\[
C_{T,T}(K) = \left[ \frac{1}{T} \int_0^T S_t \, dt - K \right]^+ \tag{4}
\]

where the strike price \( K \) is a nonnegative constant. The time-\( T \) price of an Asian call option is

\[
C_{T,T}(K) = E_P \left[ e^{-r(T-t)} \left( \frac{1}{T} \int_0^T S_t \, dt - K \right)^+ \right] F_t, \quad 0 \leq t \leq T \tag{5}
\]

where \( E_P[\cdot] \) denotes expectation under \( P \). Next, we are going to introduce the term of a self-financing strategy and the associated value process in the setting of the Black-Scholes model extended by jumps. A portfolio strategy is defined by a pair of processes \((\phi_t, \psi_t)\) for \( 0 \leq t \leq T \), denoting the quantities of riskless and risky asset respectively held in the portfolio at time \( t \); but, to take the jumps as described above into account, we will constrain the processes \((\phi_t)\) and \((\psi_t)\) to be left continuous and, for all \( 0 \leq t \leq T \), \(E_P[\int_0^T \phi_t^2 S_t^2 \, dt] < +\infty \) and \(E_P[(\phi_t)^2] < +\infty \). In our model, we know that the price process \((S_t)\) is itself right-continuous. Thus, from a financial point of view, considering left-continuous processes as trading strategies means intuitively that the market participant can react upon the jumps at best only immediately after their occurrence. This condition is the counterpart of the condition of predictability. The value at time \( t \) of the portfolio strategy is given by \( V_t \) and let the wealth evolve according to the following self-financing strategy

\[
dV_t = \phi_t dS_t + \phi_t r S_t \, dt + \phi_t \gamma S_t \, dB_t + \phi_t \gamma S_t \, dW_t \tag{6}
\]

that is, taking into account equation (3),

\[
dV_t = \phi_t S_t ((\mu - q) dt + \sigma dB_t) + \phi_t \gamma S_t dt + r (V_t - \phi_t S_t) dt + \phi_t S_t ((\mu - q) dt + \sigma dB_t) \tag{7}
\]

Next, for a discussion of random measure, let \( v(dz; dt) \) be a Poisson random measure on \((-1, \infty) \times \{0\} \times \mathbb{R}^+ \) with expectation measure \( v(dz) \, dt \), where \( v \) is the Lévy measure and \( dt \) denotes Lebesgue measure, and we denote by \( h \) the common law of the random variables \( Y_j \)'s.

Let \( Z_t = (\mu - q) + \sigma B_t + J_t \), where \( J_t = \sum_{j=1}^{N_t} Y_j \). Then

\[
dS_t = S_t \, dZ_t \tag{8}
\]

or, equivalently,

\[
S_t = S_0 + \int_0^t S_s ((\mu - q) ds + \sigma dB_s) + \int_0^t \int_0^s S_z \, v(dz; ds) \tag{9}
\]

According to the Lévy decomposition of \( Z \), we obtain

\[
Z_t = ((\mu - q) + \lambda E_p [Y_j]) t + \sigma B_t + M_t \tag{10}
\]

Here

\[
M_t = \int_0^t \int_0^s z(v(dz; ds) - \lambda h(dz) ds) = J_t - \lambda t E_p [Y_j] \tag{11}
\]

is a martingale under \( P \), where \((B_t)\) and \((M_t)\) are independent.

**Lemma 1.** Let \( P \) be a risk-neutral measure. Then under
The discounted portfolio value is a martingale.

Lemma 2. Suppose that $E_p[|Y_1|] = +\infty$ and set $\tilde{S}_t = e^{\alpha t}S_t$.
Then the interest-rate-discounted value $(e^{\alpha t}\tilde{S}_t)$ of an account that initially purchases one share of the underlying asset and continuously reinvests the dividends in the underlying asset is a martingale under $P$ if and only if $\mu = r - \lambda E_p[Y_1]$.

In order for the discounted value of a portfolio that invests in a dividend-paying stock to be a martingale, the discounted value of the stock with the dividends reinvested must be a martingale. It follows from Lemma 2 that $Z_t = (r - \gamma) + \sigma B_t + M_t$. For our model, under $P$, we define

$$\tilde{V}(dz; dt) = V(z; dt) - \lambda h(z)dt$$

where $\tilde{V}(dz; dt)$ is a compensated measure, and $V(z; dt)$ is the Poisson measure associated with the jumps of $Z$ and $\lambda h(z)dt$ is the compensator measure. Hence,

$$d\tilde{S}_t = \tilde{S}_t (\sigma dB_t + dM_t) + (r - \gamma)S_t dt$$

The Doléans-Dade formula gives

$$S_t = S_0 \exp \left\{ \sigma B_t + M_t + \left( r - \gamma - \frac{\sigma^2}{2} \right) t \right\}$$

$$\prod_{0 \leq s \leq t} \exp(-\Delta M_s)$$

To ensure that $S$ is a positive process, we make the following assumption: $\Delta M_s \geq -1$ for all $s \geq 0$, which is equivalent to saying that $\nu([0, t], (-\infty, -1]) = 0$ for all $t \geq 0$. Next, in terms of random measure, we may rewrite (7) as

$$dV_t = \phi_t \gamma S_t dt + r(V_t - \phi_t S_t) dt + \phi_t S_t ((\mu - \gamma) dt + \sigma dB_t)$$

$$+ \phi_t \int_0^t \nu(v; dz; dt) = \phi_t \gamma S_t dt + r(V_t - \phi_t S_t) dt + \phi_t \nu S_t$$

which implies

$$d\tilde{V}_t = \phi_t (d\tilde{S}_t + \gamma \tilde{S}_t dt) = e^{-\alpha t} \phi_t (\sigma dB_t + dM_t)$$

Since $(B_t)$ and $(M_t)$ are martingales and $(\phi_t S_t)$ is left-continuous, in particular, under the risk-neutral measure $P$, the discounted portfolio process $\tilde{V}_t$ is a martingale.

Proposition 1. Suppose that we have a self-financing portfolio $V$ such that $d(e^{\alpha t}V_t) = q_t [d(e^{\alpha t}S_t) + e^{\alpha t}S_t \eta_t]$ where the predictable process $q_t$ represents the shares invested in stock, $\eta_t$ is the measure representing the dividend yield, and the stock price $S_t$ is a semimartingale. If we set the shares invested in the stock to be

$$q_t = \exp(-\int_0^t \eta(s) - r(T - s) + \int_0^t \mu(s) \eta(s) ds)$$

where $\mu_t$ represents a general weight factor, and start with the initial wealth

$$V_0 = q_0 S_0 - e^{-\gamma T} K$$

where the strike price is a nonnegative constant, then we have

$$V_t = \int_0^t S_t d\mu(t) - K$$

Proof: The proof of the proposition is essentially same as the proof of the Proposition 2.2 in [7].

To study the Asian call with payoff (4), it follows from (16) and Proposition 1 that for continuous averaging, i.e., $d\mu(t) = \frac{1}{T} dt$ and a continuous dividend yield at the rate $\gamma$, i.e., $\eta(s) = \gamma ds$, we take the shares invested in the risky asset at time $t$

$$\phi_t = \frac{e^{-\alpha(T-t)} - e^{-\alpha(T-t)}}{(r-\gamma)T}$$

and we take the initial capital to be

$$V_0 = \frac{e^{-\gamma T} - e^{-\gamma T}}{(r-\gamma)T} S_0 - e^{-\gamma T} K$$

Then we have

$$V_t = \frac{1}{T} \int_0^T S_t ds - K$$

Moreover, we can compute $V_t$ by integrating (16) to obtain
In terms of $V_T$, we may rewrite (5) as

$$V_T = \frac{e^{\gamma(T-t)} - e^{-(T-t)}}{(r - \gamma)T} S_T + \frac{e^{\gamma(T-t)}}{T} \int_0^T S_t ds - e^{-\gamma(T-t)} K$$  \hspace{1cm} (20)$$

Comparing (14) and (27), we see that

$$G_t = \exp\left\{ \sigma B_t + M_t - \frac{\sigma^2}{2} t \right\} \prod_{0 \leq s \leq t} (1 + \Delta M_s) \exp(-\Delta M_s)$$  \hspace{1cm} (27)$$

Comparing (14) and (27), we see that

$$G_t = \exp\left\{ \sigma B_t + M_t - \frac{\sigma^2}{2} t \right\} \prod_{0 \leq s \leq t} (1 + \Delta M_s) \exp(-\Delta M_s)$$  \hspace{1cm} (27)$$

Under the probability measure $\tilde{P}$ defined by

$$\tilde{P}(A) = \int_A G_t d\tilde{P} \quad \text{for all } A \in \mathcal{F}$$  \hspace{1cm} (29)$$

the process

$$\tilde{B}_t = -\sigma t + B_t$$  \hspace{1cm} (30)$$

is a Brownian motion, according to Girsanov’s theorem and the process $M_t + \alpha t$ is a quadratic pure jump process with compensator measure given by $\lambda(1+z)h(dz)dt$, and hence

$$dX_t = (e^{-\gamma \Phi_t} - X_t) \left( \sigma dB_t - \gamma^2 dt + \int_{-1}^1 \frac{z}{1+z} (v(dz; dt) \right)$$

$$-(1+z)\lambda h(dz) dt)$$  \hspace{1cm} (31)$$

The change of variable was used in [6] to price Asian options without jumps on a continuously dividend-paying asset. The process $X_t$ is not a martingale under $P$ because its differential (23) has a $dt$ term. However, we can change measure so that $X_t$ is a martingale, and this will simplify (23). To this end, we define a new probability measure $\tilde{P}$ by

$$\left| \frac{d\tilde{P}}{dP} \right|_{|_{|_t}} = G_t$$  \hspace{1cm} (24)$$

and the Radon-Nikodým derivative process $G$ is now given by

$$dG_t = G_t \left( \sigma dB_t + dM_t \right)$$  \hspace{1cm} (25)$$

or, equivalently,

$$G_t = 1 + \int_0^t G_s \left( \sigma dB_s + dM_s \right)$$  \hspace{1cm} (26)$$

where $E_{\tilde{P}}[\cdot]$ denotes expectation under $\tilde{P}$. Since $X$ is Markov under $\tilde{P}$, there must be some function $g(t, x)$ such that $g(t, X_t) = E_{\tilde{P}}[X_T | F_t]$. Therefore, the risk-neutral price of the call $e^{\gamma T} g(t, X_t)$ can be computed. Now we are ready to derive the partial integro-differential equation that $g(t, x)$ must satisfy. First, applying the Itô differentiation on $(t, X_t)$ gives

$$g(t, X_t) = g(0, X_0) + \int_0^t g_t(s, X_s) ds + \int_0^t \int_0^s \left[ g \left( s, X_s \right) \left( e^{-\gamma \Phi_s} - X_s \right) \left( \frac{z}{1+z} \right) \right]$$

$$+ g_s(s, X_s) \left( e^{-\gamma \Phi_s} - X_s \right) \left( \frac{z}{1+z} \right) + g(s, X_s) \left( v(dz; ds) \right)$$

Note that we have replaced the Brownian motion in the classical Black-Scholes setting by the martingale part of the noise process $Z$. Notice (26) has the explicit solution

$$G_t = \frac{e^{\gamma(T-t)} - e^{-(T-t)}}{(r - \gamma)T} S_T + \frac{e^{\gamma(T-t)}}{T} \int_0^T S_t ds - e^{-\gamma(T-t)} K$$  \hspace{1cm} (20)$$

Comparing (14) and (27), we see that

$$G_t = \frac{e^{\gamma(T-t)} - e^{-(T-t)}}{(r - \gamma)T} S_T + \frac{e^{\gamma(T-t)}}{T} \int_0^T S_t ds - e^{-\gamma(T-t)} K$$  \hspace{1cm} (20)$$
Next, note that the final $ds$ integral is a continuous (local)-martingale of finite variation under the new measure $\tilde{P}$. Since $g(t, X_t)$ is a $\tilde{P}$-martingale, the $ds$ integral above must be identical to zero almost surely. This gives

$$
-(1+z)\lambda h(\phi) ds + \int_0^T \left( g_x(s, X_s)(e^{-\gamma} \phi - X_s) \left( \frac{z}{1+z} \right) - g(s, X_s) \right) ds
$$

$$
= \int_0^T \left[ g_x(s, X_s)(e^{-\gamma} \phi - X_s) \left( \frac{z}{1+z} \right) \right] (1+z) h(\phi) ds = 0
$$

We conclude that $g(t, x)$ satisfies the partial integro-differential equation

$$
g_x(t, x) + \frac{1}{2} g_{xx}(t, x)(e^{-\gamma} \phi - x)^2 \sigma^2 + \int_0^T \left[ g_x(s, X_s)(e^{-\gamma} \phi - X_s) \left( \frac{z}{1+z} \right) \right] h(\phi) ds = 0
$$

for $0 \leq t \leq T$ and $x \in \mathbb{R}$

as we know, $\phi$ is in (17).

By means of the previous results, a conclusion can be obtained as follows.

**Theorem 1.** Suppose the underlying asset follows the dynamics of a jump-diffusion model as described in Section 2. Then for $0 \leq t \leq T$, the risk-neutral price at time $t$ of the Asian call option is

$$
C_{t,T}(K) = e^{rT} S_t g \left( t, \frac{V_t}{e^{rT} S_t} \right)
$$

where $g(t, x)$ satisfies (35) and $V_t$ is given by (20).

Note that if there is no jump ($\lambda = 0$), that is, in the geometric Brownian model, $dS_t = S_t dZ$. We simply have

$$
g_x(t, x) + \frac{1}{2} g_{xx}(t, x)(e^{-\gamma} \phi - x)^2 \sigma^2 = 0, \quad 0 \leq t \leq T \text{ and } x \in \mathbb{R}
$$

where $\phi_t = \frac{e^{r(t-t)}}{(r-\gamma)T}$. Therefore, for $0 \leq t \leq T$, the price $C_{t,T}(K)$ at time $t$ of continuously averaged Asian call on a continuously dividend-paying asset with payoff (4) at time $T$ is

$$
C_{t,T}(K) = e^{rT} S_t g \left( t, \frac{V_t}{e^{rT} S_t} \right)
$$

where $g(t, x)$ satisfies (37) and $V_t$ is given by (20), which retrieves the result developed by Vécéré in [6]. If there are no paying dividend ($\gamma = 0$), that is, in the geometric jump-diffusion model, $dS_t = S_t (r dt + \sigma dB_t + dJ_t)$, where $J_t = \sum_{j=1}^{N_t} Y_j$. We simply have

$$
g_x(t, x) + \frac{1}{2} g_{xx}(t, x)(e^{-\gamma} \phi - x)^2 \sigma^2 + \int_0^T \left[ g_x(s, X_s)(e^{-\gamma} \phi - X_s) \left( \frac{z}{1+z} \right) \right] \lambda(1+z) h(\phi) ds = 0
$$

for $0 \leq t \leq T$ and $x \in \mathbb{R}$

where $\phi_t = \frac{1-e^{-(r-\gamma)T}}{rT}$. Therefore, for $0 \leq t \leq T$, the price $C_{t,T}(K)$ at time $t$ of continuously averaged Asian call with payoff (4) at time $T$ is

$$
C_{t,T}(K) = S_t g \left( t, \frac{V_t}{S_t} \right)
$$

where $g(t, x)$ satisfies (39) and $V_t$ is given by

$$
V_t = \frac{1-e^{-(r-\gamma)T}}{rT} S_t + \frac{e^{-(r-\gamma)T}}{T} \int_0^T S_t ds - e^{r(T-t)} K
$$

which recovers the result developed by Lin in [15]. If there are no jump ($\lambda = 0$) and no paying dividend ($\gamma = 0$),
that is, in the geometric Brownian model, \( dS_t = S_t \, dt + \sigma S_t \, dB_t \). We simply have

\[
g(t, x) + \frac{1}{2} \sigma^2 \phi' \phi - \frac{x^2}{2} \phi'' = 0, \quad 0 \leq t \leq T \text{ and } x \in \mathbb{R}
\]

(42)

where \( \phi = \frac{1}{r \gamma} e^{-\gamma t} \). Therefore, for \( 0 \leq t \leq T \), the price \( C_{t,r}(K) \) at time \( t \) of continuously averaged Asian call with payoff (4) at time \( T \) is

\[
C_{t,r}(K) = \mathbb{E}_t \left[ \left( \frac{V_t}{S_t} \right) \right] \tag{43}
\]

where \( g(t, x) \) satisfies (42) and \( V_t \) is given by (41), which recovers the result described by Shreve in [16].

4. Concluding Remarks

Asian option contracts are more complicated to analyze than their European counterparts, because the value of an Asian option is strongly path-dependent. The pricing problem for these options has been widely studied and generalized since Geman and Yor [5]. Recently, Vécer [6] studies pricing of Asian options (written on an underlying stock without jumps) on a continuously dividend-paying asset. The explicit pricing formulas for the value functions are still not available. In this paper we consider the pricing problems of these options in the jump-diffusion models. By the change of numéraire technique, we derive a partial integro-differential equation (PIDE) whose solution leads to the prices of Asian options with paying continuous dividend yields when the underlying asset follows the dynamics of a jump-diffusion model. In particular, if there is no jump, we recover the results developed by Vécer [6]. The results presented in this paper were formulated for Asian call options. However, using put-call parity we obtain immediately Asian put options. The purpose of this article is to present an analytical approach for finding a PIDE for Asian call options in jump-diffusion models with continuously paying dividend. As we know, the advantage of analytical solution is more accurate than approximation methods. In practice, a practitioner has to obtain the solution of the PIDE for the prices of options. However, in general an exact or closed-form solution to (35) can not be expected. Practitioners can only rely on numerical approximation; in the option pricing context, the most common way to discretize the differential operators has been finite difference methods (see Achdou and Pironneau [17] and Tavella and Randall [18]), and the treatment of the integral term associated with jumps in models is more challenging. More recently, option pricing in jump-diffusion models has provided many numerical methods for solving PIDEs–however, the numerical issue for solving PIDEs and building convergence numerical schemes for computing such solutions is still a subtle problem in mathematics as it is not always possible in financial problems to solve a PIDE with any real precision in an acceptable period of time (for a deeper discussion of this question, see Cont and Tankov [19] and the references therein; and further, a more detailed account of numerical methods for PIDEs can be found in Duffy [20]). In future research on this problem, it would be of interest to give a numerical analysis to find an efficient numerical method that approximates the solution of the corresponding PIDE derived in the proposed model and compare with the market price of Asian options and other approximated formula (for example, Rogers and Shi [12], Kemna and Vorst [21], and Grant et al. [22]). Furthermore, it would be very interesting to compare with the efficiency of the proposed method and some approximations.

Acknowledgements

The author would like to thank the referees for their constructive suggestions and comments. This research was supported by the National Science Council, Taiwan, ROC, under Grant NSC 98-2115-M-156-002.

References


Manuscript Received: Mar. 30, 2012
Accepted: Aug. 17, 2012