MHD Non-Newtonian Power Law Fluid Flow and Heat Transfer Past a Non-Linear Stretching Surface with Thermal Radiation and Viscous Dissipation

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Abstract

Non-Newtonian magneto-hydro dynamic boundary layer flow of an electrically conducting power law fluid flowing over a non-linear stretching surface in the presence of thermal radiation, taking into account the viscous dissipation effects is investigated. By using quasi-linearization technique first linearize the non linear momentum equation and then the coupled ordinary differential equations are solved numerically by an implicit finite difference scheme. The numerical solution is found to be dependent on several governing parameters. A systematic study is carried out to illustrate the effects of various parameters on the fluid velocity and the temperature distribution in the boundary layer through graphs. The results for the local skin-friction coefficient and the local Nusselt number are tabulated and discussed and found to be in good agreement with earlier published results.


1. Introduction

In modern technology and in industrial application, non-Newtonian fluids play an important role. Schowalter [1] was the first one, who formulated the boundary layer flow of a non-Newtonian fluid and established the conditions for the existence of a similarity solution. Acrivos et al. [2] considered the problem of natural convection heat transfer to power-law fluids for different geometric configuration. The various aspects of the stretching sheet problem involving Newtonian/non-Newtonian fluids have been extensively studied by Sakiadis [3], Crane [4], Gupta [5], Sahoo [6], and Chen [7]. These research works do not however consider the situation where hydro magnetic effects arise.

Mahapatra [8] analyzed the MHD flow of a power-law fluid past an infinite porous flat plate subjected to suction or blowing. The study of magneto hydrodynamic flow and heat transfer over a stretching sheet may find its applications in polymer technology related to the stretching of plastic sheets. In view of this, the study of MHD flow of Newtonian/non-Newtonian flow over a stretching sheet was carried out by many researchers like Sarpakaya [9], Cortell [10], Emad [11], Kishan [12], Chaim [13], and Ishak [14]. All the above mentioned investigators confined their analyses to MHD flow and heat transfer over a linear stretching sheet. However, the intricate flow and heat transfer problem over a non-linearly stretching sheet with the effects of internal heat generation/absorption is yet to be studied. This has applications to engineering processes involving nuclear power plants, gas turbines and many others has been studied by Cortell [15], Chen [16] and Nandeppananar [17].

The main concern of the present paper is to study the effect of variable thermal conductivity on the power-law
fluid flow and heat transfer over a non-linearly stretching sheet in the presence of a transverse magnetic field by taking into account viscous dissipation and radiation effects. Because of the intricacy, the influence of power-law index parameter, magnetic parameter, non-linear velocity and temperature exponent and heat source/sink make the momentum and energy equations coupled and highly non-linear partial differential equations. These equations are in turn solved numerically by an implicit finite-difference scheme along with Gauss-Seidel method with the help of C-programming.

2. Mathematical Formulation

Consider steady laminar two-dimensional boundary layer flow of a viscous incompressible and electrically conducting fluid obeying power-law model in the presence of a transverse magnetic field $B_0$. $x$-axis is taken along the direction of the flow and $y$-axis normal to it. The continuous stretching sheet is assumed to have a non-linear velocity and prescribed temperature of the form $U(x) = bx^m$ and $T_w(x) = T_0 + T_w$, respectively, where $b$ is the stretching constant, $x$ is the distance from the slot; $T_0$ is ambient temperature, $A$ is a constant whose value depends upon the properties of the fluid. Here, $m$ and $r$ are the velocity and temperature exponents, respectively. The governing boundary layer equations are

\[
\frac{\partial u}{\partial x} + \frac{v}{\partial y} = 0 \tag{2.1}
\]

\[
\frac{u}{x} + \frac{v}{y} = -\frac{1}{\mu} \left( \frac{\partial u}{\partial y} \right) - \frac{\sigma B_0}{\rho} u \tag{2.2}
\]

\[
\frac{\partial T}{\partial x} + \frac{v}{y} \frac{\partial T}{\partial y} = \frac{\alpha}{\rho c_p} (T - T_w) - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + \mu \left( \frac{\partial u}{\partial y} \right)^2 \tag{2.3}
\]

where $\nu$ is kinematic viscosity of the fluid, $n$ is power-law index, $\rho$ is fluid density and $c_p$ is specific heat at constant pressure. The first term on the right hand side of the Equation (2.2), is the shear rate $\frac{\partial u}{\partial y}$ has been assumed to be negative throughout the boundary layer. The radiative heat flux is $q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$. Assuming the temperature difference within the flow is such that $T^4$, may be expanded in a Taylor series about $T_w$ and neglecting higher order terms, we get $T^4 \approx 4T_w^3 - 3T_w^4$, $\sigma^*$ is Stefan-Boltzmann constant and $k^*$ is the mean absorption coefficient. $T$ is temperature of fluid and $\alpha$ is the thermal diffusivity of the fluid. Thus the relevant boundary conditions are:

\[
u(x, 0) = U(x), \ \psi(x, 0) = 0, \ \ T(x, 0) = T_w(x) \tag{2.4}
\]

\[
u(x, y) \to 0, \ \ T(x, y) \to T_w, \ \ \text{as} \ \ y \to \infty \tag{2.5}
\]

The momentum and energy equations can be transformed to set of ordinary differential equations by the following transformation

\[
\eta = \frac{y}{x}(Re_{_w})^{-1}, \ \ \theta(\eta) = \frac{T - T_w}{T_w - T_w}, \ \ \psi(\eta, \xi) = \frac{U}{\alpha} \xi (Re_{_w})^{-1} f(\eta) \tag{2.6}
\]

where $\eta$ is similarity variable $\psi(x, y)$ is stream function $f$ and $\theta$ are dimensionless similarity function and temperature, respectively. The velocity $u$ and $v$ are given by

\[
u = \frac{\partial u}{\partial y}, \ \ v = -\frac{\partial u}{\partial x} \tag{2.7}
\]

The local Reynolds number is defined by $Re_{_w} = \frac{U^2 x^n}{\nu}$.

The equations (2.2) & (2.3) transform into the coupled non-linear ordinary differential equations of the form

\[
\nu(-f)^{n-1} f^m - mf^{n-2} + \left( \frac{2mn - m + 1}{n + 1} \right) f f' - M f' = 0 \tag{2.8}
\]

\[
(1 + R)\theta' + Pr \left( \frac{2mn - m + 1}{n + 1} \right) f f' + Pr(5\theta - rf f') \tag{2.9}
\]

\[
+ Ec(f')^2 = 0
\]

The boundary conditions (2.4)–(2.5) now become

\[
f(\eta) = 0, \ f'(\eta) = 1, \ \theta(\eta) = 1, \ \text{at} \ \ \eta = 0 \tag{2.10}
\]

\[
f'(\eta) \to 0, \ \theta(\eta) \to 0, \ \text{as} \ \ \eta \to \infty
\]
where $M = \sigma B_0^2 / \rho b$ is the magnetic parameter, $Pr = Npe^{\frac{2}{n+1}}$ is the modified Prandtl number for power-law fluids, $Npe = \frac{c_p \rho U_x}{k}$ is the convectional Peclet number, $S = \frac{Q}{\rho c_p b}$ is heat source/sink, $Ec = \frac{\mu U^2}{\alpha (T_w - T_\infty)}$ is local Eckert number, $R = \frac{16 \alpha}{k^2 \kappa} T_\infty$ is the radiation parameter. Here, primes denote the differentiation with respect to $\eta$. The skin-friction coefficient $C_f$ and the local Nusselt number $Nu_x$, which are defined as

$$C_f = \frac{2 \tau_w}{\rho U^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}$$

(2.11)

where wall shear stress $\tau_w$, & heat transfer from the sheet $q_w$ are:

$$\tau_w = \mu_0 \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}$$

(2.12)

with $\mu_0$ is dynamic viscosity and $k$ is thermal conductivity. Using

The equations (2.6)–(2.7), we obtain

$$C_f = \left( \frac{-2 \tau_w}{\rho \beta x^2} \right)_{y=0} = 2 [-f''(0)] [\text{Re}_x]^{-\frac{1}{m+1}},$$

$$Nu_x = [\text{Re}_x]^{-\frac{1}{m+1}} \theta'(0)$$

(2.13)

3. Numerical Procedure

The system of transformed governing non-linear coupled differential Equations (2.8) and (2.9) with the boundary conditions (2.10) is solved numerically using implicit finite-difference scheme. The transformed non-linear differential equations are first linearized by Quasi-linearization technique discussed by Bellman and Kalaba [18].

4. Results and Discussions

The computations have been carried out for various values of magnetic field parameter $M$, power-law index $n$, Eckert number $Ec$, velocity exponent parameter $m$, temperature exponent parameter $r$, Modified Prandtl number $Pr$ and heat source/sink parameter $S$. Figure 1(a)–(c) demonstrates the dimensionless velocity profile $f'$ for the different values of magnetic field parameter $M$. The effect of magnetic field parameter results in flatter- ing of velocity profile $f'$ in all the cases. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is due to fact that variation of $M$ leads to the variation of the Lorentz force due to the magnetic field and the Lorentz force produces more resistance to transport phenomena.

Figure 2(a)–(c) represents the horizontal velocity profile $f'$ with $\eta$ for different values of velocity exponent parameter $m$. We observe that the effect of increasing values of velocity exponent parameter $m$ is to reduce the momentum boundary layer thickness, which tends to

![Figure 1. Velocity profiles for different values of M for m = 0.1, Pr = 1, r = 0, S = 0, Ec = 0.](image)
zero as the space variable $\eta$ increases from the boundary surface. Physically $m < 0$ implies that the surface is de-celerated from the slot $m = 0$ implies the continuous mo-mentum of a flat surface and $m > 0$ implies that the surface is accelerated from the extended slit. The velocity profile $f'$ decreases with the increase of stretching sheet parameter $m$ is observed from the figures. The graphs of temperature profiles $\theta$ for different values of stretching sheet parameter $m$ in the presence of magnetic field parameter $M$ is shown in Figure 3(a)–(c) respectively. The effect of increasing values of stretching sheet parameter $m$ is to decrease temperature profiles $\theta$ is observed from these figures. However, the effect of stretching sheet parameter $m$ is more in the presence of magnetic field parameter.

In Figure 4(a)–(c) the temperature profiles $\theta$ has been plotted for different values of radiation parameter $R$. It is shown from the figure that the increase in radiation parameter $R$ leads to increase in temperature profiles. This result is expected because the presence of thermal radiation works as a heat source and so the quantity of heat added to the flow causes the motion of the fluid to accelerate. The thermal boundary layer thickness increases for pseudo-plastic fluids with the influence of radiation parameter $R$ is significantly increasing when compared with other fluids.

Figure 5(a)–(c) portrays the dimensionless temperature for different values of modified Prandtl number. The temperature of the fluid decreases with the increase of $Pr$, because thermal boundary layer thickness decreases due to increase in $Pr$. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Fluids with lower $Pr$ possess higher thermal conductivities (and thicker thermal boundary layer structures) so that heat can diffuse from the sheet faster than for higher $Pr$ fluids (thinner boundary layers). Hence Prandtl number can be used to increase the rate of cooling in conducting fluids. The effect is more in Pseudo plastic fluids in comparison to its effect in dilatent fluids.

![Figure 2](image1.png)

**Figure 2.** Velocity profiles for different values of $m$ for $M = 0$, $Pr = 1$, $\tau = 0$, $S = 0$, $Ec = 0$.

![Figure 3](image2.png)

**Figure 3.** Temperature profiles for different values of $m$ for $M = 1$, $Pr = 1$, $\tau = 0$, $S = 0$, $Ec = 0$. 
In Figure 6(a)–(c) it is observed that the dimensionless temperature profile $\theta$ increases with increase of Eckert number $Ec$. This is due to the fact that the heat energy is stored in the fluid due to the frictional heating. So, we can say that the strong frictional heating slows down the cooling process and in this case the study suggests that the rapid cooling of the sheet can be made possible if the viscous dissipation can be made as small as possible. The effect of viscous dissipation is more in pseudo plastic fluids in comparison with other fluids.

From the Figure 7(a)–(c) it is noticed that the dimensionless temperature $\theta(\eta)$ decreases for increasing strength of the heat sink and due to increase of heat source strength the temperature increases. So, the thickness of thermal boundary layer reduces for increase of heat sink parameter, but it increases with heat source parameter. This result is very much significant for the flow where heat transfer is given prime importance.

Figure 8(a)–(c) reveal the influence of temperature exponent parameter $r$. The effect of increasing values of the temperature exponent parameter $r$ is to decrease $\theta(\eta)$. The effect of power law index $n$ on $f'$ and $\theta$ are plotted in Figures 9(a) and (b) respectively. It can be seen from the figures that with increase of power law index $n$ values the velocity profiles $f'$ increases. The dimensionless temperature profiles decrease with increase of power law index $n$ is noticed from Figure 9(b).

From the Table 1 it is observed that with increase in magnetic field parameter $M$ is to increase $f'(0)$ values. The effect of power-law index $n$ for fixed values of mag-
Figure 6. Temperature profiles for different values of $Ec$ for $M = 0$, $m = 0.1$, $r = 0.1$, $S = 0$, $Pr = 1$.

Figure 7. Temperature profiles for different values of $S$ for $M = 1$, $m = 0.1$, $r = 0.1$, $Pr = 1$, $Ec = 0$.

Figure 8. Temperature profiles for different values of $r$ for $M = 1$, $m = 0.1$, $S = 0$, $Pr = 1$, $Ec = 0$. 
netic parameter reduces $-f''(0)$. In Table 2 the temperature gradient ($-\theta'(0)$). From the table we notice that the effect of increasing values of power-law index $n$ is to decrease the wall temperature gradient whereas reverse trend is seen for magnetic field parameter $M$ when $S = 0$. The increasing values of temperature exponent parameter $r$ and velocity exponent parameter $m$ is to reduce values of $-\theta'(0)$. This result has significance in industrial applications to reduce expenditure on power supply in stretching sheet when the sheet just by increasing magnetic field parameter $M$. The effect of heat source/sink to decrease wall temperature gradient.

5. Conclusions

In summary the present study describes the power law fluid flow and heat transfer over a non-linearly stretching sheet in the presence of transverse magnetic field by taking into account viscous dissipation effects and heat source/sink parameter. From the numerical results and above discussions it can be found that

(1) The effect of $m$ is to decrease the temperature profile $f''$ and profile $\theta$: (a) Pseudo plastic fluids ($n = 0.8$); (b) Newtonian Fluids ($n = 1$); (c) Dilatent Fluids ($n = 1.6$).

(2) The effect of increasing values of magnetic parameter $M$ is to decelerate the velocity profile $f'$.

(3) The effect of temperature exponent parameter $r$ and modified Prandtl number is to decrease the temperature profiles $\theta$.

(4) The viscous dissipation effect is to increase the temperature profiles.

(5) The heat source/sink parameter is reduces the temperature profiles for $S < 0$ (sink) and increase the temperature profiles for $S > 0$ (source) when compared with $S = 0$.

(6) The effect of power law index parameter $n$ is to increase the velocity profile $f'$ and reduces the temperature profiles $\theta$.

Table 1. Numerical values of the skin friction coefficient

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Table 2. Numerical values of Nusslet number for different values of the physical parameters

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References


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