Production of Chilled Air by Melting Ice in Cool-Thermal Discharge Systems

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Abstract

A mathematical model of cool-thermal discharge systems with melt removal has been developed and studied theoretically to simulate the outlet temperature of the chilled air produced during on-peak power-consumption hours. Equations have been derived for estimating the thickness of the airflow velocity, the thickness of melted ice and the total amount of cool thermal flux during the operating process with a specified inlet air temperature or outlet air temperature. Numerical examples have been illustrated in which either the inlet air temperature or outlet air temperature was specified, and the results of cool-thermal discharge fluxes have been also discussed.

Key Words: Cool Thermal Discharge, Chilled Air, Moving Boundary

1. Introduction

The cool-thermal storage and discharge systems are the processes to produce ice during off-peak period in the evening and to obtain chilled air in the daytime peak, respectively, with air flowing over melting ice. Therefore, the electric utility during nighttime off-peak period can be utilized sufficiently so as to increase the economic benefit of power generation facilities. In addition, power supply companies offer a cheaper rate to encourage energy consumer during off peak hours, the danger of power shutdown due to insufficient supply of electricity could hence be avoided as well as lots of the electricity expense could be saved. Recently, several researchers proposed the cool-thermal storage system of the vacuum freezing method to increase both the electricity utility profitability and energy efficiency \[2,3,12,18]\).

Although there are only a few of literature on applications of cool-thermal storage and discharge systems exists \[1,4,6,7,9,11,14,16,17,19]\), the theoretical analysis on a new device of cool-thermal storage \[20,21\] and cool-thermal discharge \[22\] has been studied. Moreover, the cool-thermal discharge system with convective boundary conditions \[23\] also has been developed.

The adjustable outlet temperature in cool-thermal discharge systems with melt removal for the space cooling is highly expected. Here a new model proposed is a cool-thermal discharge system with outlet temperature adjusted so as to facilitate the practical application in our daily life. The ice layer temperature is at the melting point initially, and the melt was removed immediately once the ice melting occurs with the ambient air flowing through the open duct, as shown in Fig. 1. Therefore, the airflow rate must be regulated continuously to produce the desirable outlet temperature as well as to afford more flowing air in the duct due to the removal of melt as the operation proceeds.
The cool-thermal discharge system, as shown in Figure 1, is the ice layer with infinite thickness, length L and width B. The depth from the ice layer surface to the insulated plate (or the height of the open duct) is of $W << L$. The ice layer was assumed to be initially at its melting point $sT$ with no temperature gradient. Constant physical properties and no density change on melting were assumed. The air of velocity $bu$ flowing through the open duct with the inlet and outlet temperatures are $ifT$, and $ofT$, respectively. The flux of energy absorption or the flux of cool thermal discharge at the free surface of the ice layer varies hourly during the discharging period due to the changing of the removal of melt and the thickness of the open duct as functions of time.

By taking a differential energy balance within the open duct, the system is governed by the following equation:

$$\frac{dT}{dz} + \frac{h(T - T_s)}{\rho Wc_p u_b} = 0 \quad (1)$$

The conditions are:

$$T(z = 0) = T_{f,i} \quad (2)$$

$$T(z = L) = T_{f,o} \quad (3)$$

integration of Eq. (1) with the use of Eqs. (2) and (3) yields

$$u_b = \frac{hLT_{ln}}{\rho c_p W(T_{f,i} - T_{f,o})} \quad (4)$$

in which

$$T_{ln} = \frac{(T_{f,i} - T_s) - (T_{f,o} - T_s)}{\ln \left( \frac{T_{f,i} - T_s}{T_{f,o} - T_s} \right)} = \ln \left( \frac{T_{f,i} - T_s}{T_{f,o} - T_s} \right)$$

The following correlation for fully developed turbulent flow of air between two parallel plates with one side heated and the other side insulated, can be derived from Kays’ data [12]:

$$Nu_m = 0.0158 \ Re^{0.8} \quad \text{Re} > 2100. \quad (6)$$

The Reynolds number is defined as

$$Re = \frac{\rho u_b D_e}{\mu} = \frac{\rho u_b (2W)}{\mu} \quad (7)$$

and $D_e$ denotes the equivalent diameter of the air passage, $D_e = 2W$. Substitutions of Eqs. (6) and (7) into Eq. (4) give

$$u_b = \left[ \frac{0.01375kLT_{ln}}{\rho c_p W^{0.8} \left(T_{f,i} - T_{f,o}\right)^{1.2}} \right]^{5} \quad (8)$$

or

$$W = \frac{0.01375kLT_{ln}}{\rho c_p W^{0.8} \left(T_{f,i} - T_{f,o}\right)} \cdot u_b^{0.6} \quad (9)$$

3. Cool Thermal Discharge

The total amount of cool-thermal discharge flux required to ice melting can be calculated from

$$\rho c_p u_b BW(T_{f,i} - T_{f,o}) = \rho ice Q_m BL \frac{dX}{dt} \quad (10)$$

where

$$\frac{dX}{dt} = \frac{dW}{dt} = \frac{dW}{dt} \cdot \frac{du_b}{dt} \quad (11)$$

Differentiating Eq. (9) with respect to $u_b$, one may obtain

$$\frac{dW}{du_b} = -\frac{1}{6} \left[ \frac{0.01375kLT_{ln}}{\rho c_p W^{0.8} \left(T_{f,i} - T_{f,o}\right)} \right] \cdot u_b^{-2} \quad (12)$$

Substitutions of Eqs. (9), (11) and (12) into Eq. (10) give

$$\frac{du_b}{dt} = -6 \rho c_p u_b^2 (T_{f,i} - T_{f,o}) \quad (13)$$

Integration Eq. (12) from $t = 0$ to $t = t$, gives the time history of the airflow rate $u_b$ as follows:

$$u_b = \frac{u_{b0} P_{ice} L}{6u_{b0} \rho c_p (T_{f,i} - T_{f,o}) B + \rho ice Q_m L} \quad (14)$$

Note that $u_{b,0}$, the initial velocity of the airflow rate, is obtained from Eq.(8) with $W = W_i$

$$u_{b,0} = \left[ \frac{0.01375kLT_{ln}}{\rho c_p W^{0.8} \left(T_{f,i} - T_{f,o}\right)^{1.2}} \right]^{5} \quad (15)$$
Substitution of Eq. (8) into Eq. (10) yields the thickness of melted ice
\[
\frac{dX}{dt} = \frac{dW}{dt} = 4.915 \times 10^{-10} \left( \frac{L}{c_p \mu (T_{f,i} - T_{f,o})} \right)^4 \left( \frac{kT_{in}}{W} \right)^5,
\]
(16)

Also, the time history of the height of open duct was calculated by integrating Eq. (16) from \( t = 0 \) to \( t = t \)
\[
W = X + W_i = \left( \frac{2.949 \times 10^2}{\rho_{ice} Q_m} \left( \frac{L}{c_p k(T_{f,i} - T_{f,o})} \right)^4 \left( kT_{in} \right)^5 t + W_i^6 \right)^{1/6},
\]
(17)

Therefore, time history of the cool-thermal discharge flux \( q \) is also calculated from Eq. (16) as follows:
\[
q = \rho_{ice} Q_m \frac{dX}{dt} = 4.915 \times 10^{-10} \left( \frac{L}{c_p \mu (T_{f,i} - T_{f,o})} \right)^4 \left( \frac{kT_{in}}{W} \right)^5.
\]
(18)

### 4. Numerical Examples

We consider three cases for the ambient temperatures, 32, 33 and 34 °C, and three cases for the outlet air temperature, 24, 25 and 26 °C. Here, we assign the following numerical values:
\[
t_0 = 10 h, \ T_i = 273 K, \ p = 1 \ atm, \ D_c = 2W = 0.2 m, \ L = 10 m.
\]
The physical properties of air at 1 atm and 20 °C are [5]: \( \rho = 1.16 \text{kg/m}^3 \), \( c_p = 1.0048 \text{kJ/kg-K} \), \( k = 0.0951 \text{kJ/m-h-K} \) and \( \mu = 0.0656 \text{kg/m-s} \). The physical properties of ice at 1 atm and 0 °C are [10]: \( Q_m = 334 \text{kJ/kg} \) and \( \rho_{ice} = 917 \text{kg/m}^3 \). Substituting these values into the appropriate equations, results for \( u_b, X \) and \( q \) have been calculated and those of \( u_b, X \) and \( q \) are shown in Figs. 2-7.

The calculation procedure will be described briefly as follows. First, the Reynolds numbers \( Re \) were estimated from Eq. (7). Next, the height of the open duct was calculated from Eq. (17). Finally, the total amount of cool-thermal discharge flux \( q \) required with \( T_{f,i} \) or \( T_{f,o} \) specified from Eq. (18).

### 5. Discussion and Conclusion

The mathematical formulation of cool-thermal discharge systems with melt removal from ice melting has been developed, and the airflow velocity and the total amount of cool-thermal discharge flux were calculated with the analysis of energy balances in the air flowing duct. The present study is to estimate how large airflow velocity, \( u_b \), is needed under specified inlet air temperature, \( T_{f,i} \), or outlet air temperature, \( T_{f,o} \), which is adjusted for cool-thermal discharge systems to produce the chilled air.

The airflow velocity in the open duct and the thickness of melted ice in the ice layer may be estimated from Eqs. (14) and (17), respectively, while the total amount of cool-thermal discharge flux is calculated from Eq. (18). The most important assumptions in this work are that the ice layer is initially at its melting point and that the volume changed due to ice melting is neglected. Moreover, according to the standard defined in Central Weather Bureau, Taiwan, R.O.C., wind velocity less than 0.3 m/s is of no wind. It is not applicable to real systems in the laminar regime of the airflow velocity being less than 0.2 m/s, then the turbulent regime was assumed in this study with the use of Eq. (6).

Figures 2-4 show that the airflow velocity, the thickness of melted ice and the total amount of cool-thermal discharge flux taken as illustrations with the use of the inlet air temperature \( T_{f,i} \) as a parameter for a specified \( T_{f,o} \) while Figs. 5-7 show those numerical results with outlet air temperature \( T_{f,o} \) as a parameter for a specified \( T_{f,i} \).
Figure 2. Time history of the airflow velocity with inlet air temperature as a parameter.

Figure 3. Time history of the thickness of melted ice with inlet air temperature as a parameter.

Figure 4. Time history of the cool thermal flux with inlet air temperature as a parameter.

Figure 5. Time history of the airflow velocity with outlet air temperature as a parameter.
In the present study, we considered the variation of air temperature decreasing from $T_{f,i}$ to $T_{f,o}$ along the flow channel. The logarithmic mean values, Eq. (5), were taken for estimating the rate of convective heat transfer between the flowing air and the free surface of ice layer.

It is shown in Figs. 2 and 5 that the cooling passage keeps expanding with time as the operation proceeds, and then the airflow velocity decreases with time. These results can be checked by Eq. (8) as well as by Eq. (14). It is also found in Fig. 2 that the airflow velocity $u_b$ increases as the inlet air temperature decreases for a specified $T_{f,o}$. The higher inlet air temperature, the longer residence time of air inside the cooling passage for a specified outlet air temperature. Under such an operation condition, larger temperature differences were achieved, and hence the airflow velocity would be even lower for a specified outlet air temperature. However, the higher inlet air temperature associated with the smaller airflow velocity results in the lower mean convective heat transfer coefficient. Therefore, the thickness of melted ice decreases with increasing inlet air temperature, as shown in Fig. 3.

The total amount of cool-thermal discharge flux is also calculated from Eq. (18) with the use of $X(t)$. The result of the total amount of cool-thermal discharge flux with the use of the inlet air temperature $T_{f,i}$ as a parameter for a specified $T_{f,o}$, as shown in Fig. 4, decreases with increasing inlet air temperature accordingly.

It may be seen from Figs. 5-7 that the airflow velocity, the thickness of melted ice and the total amount of cool-thermal discharge flux increase with outlet air temperature $T_{f,o}$ for a specified $T_{f,i}$. The temperature difference of outlet air temperature and inlet air temperature increases with decreasing outlet air temperature for a specified inlet air temperature, and hence the airflow velocity increases with the outlet air temperature for a shorter residence time inside cooling passage. In other words, for obtaining lower outlet air temperature under the same inlet air temperature, lower airflow velocity (longer residence time) is needed. Besides, the average convection heat transfer coefficient decreases with the airflow velocity, therefore, the ice melting thickness and cool thermal discharge flux decrease.
with $T_{f,o}$ for a specified $T_{f,i}$, as shown in Figs. 6 and 7.

**Nomenclature**

- $B$: cooling channel width (m)
- $c_p$: specific heat of air (kJ/kg K)
- $D_e$: the equivalent diameter of cooling passage (m)
- $h$: average convective heat transfer coefficient (kJ/m$^2$ K)
- $k$: thermal conductivity of air (kJ/m K)
- $L$: cooling channel length (m)
- $m_{Nu}$: Nusselt number ($hD_e/k$)
- $Pr$: Prandtl number ($\rho \alpha / \mu$)
- $Q_m$: heat of melting (kJ/kg)
- $Re$: Reynolds number ($\rho u_b D_e / \mu$)
- $T_{f,i}$: the inlet temperature of air (K)
- $T_{f,o}$: the outlet temperature of air (K)
- $T_s$: ice temperature at the free surface (K)
- $T_{ln}$: defined by Eq. (5)
- $t$: time (h)
- $t_0$: total operating time per day (h)
- $u_b$: gas flowing velocity (m/s)
- $W$: the height of the cooling passage (m)
- $X$: thickness of the water layer (m)

**Greek letters**

- $\rho$: air density (kg/m$^3$)
- $\rho_{ice}$: ice density (kg/m$^3$)
- $\mu$: air viscosity (kg/m – h)

**References**


Accepted: Jun. 20, 2001