Numerical Simulation of the Failure Process of Rocks

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Abstract

The initiation, development and coalescence of fractures have been studied by various researchers. The current consensus is that such complex process is however not yet fully understood. In the present study, a numerical model, based on finite element approach, has been developed to simulate the fracture process when the specimen is under compression. The heterogeneous nature of the rocks is also taken into account by assuming spatial variations in the properties of the rocks be defined by the Weibull function. The numerical simulation shows that failure is mainly a process of tensile fractures developed in highly stressed shear bands. The study has provided insight on the development of fractures, especially in the post-peak range.

Key Words: Fracture, Acoustic Emission, Finite Element, Weibull

1. Introduction

As it is well known that rocks are polycrystalline and polyphase materials, the failure process is closely related to the behaviors of their mineral contents and the fractures in the rocks [2]. Though experimental studies have been carried out to investigate the modes of failure of rocks for decades [3,5,8,9,11,16,21,27-30], the details of the failure mechanisms, including the micro-fracture initiation, propagation, coalescence, axial splitting, shearing etc, are not yet fully understood and still remain a subject of considerable scientific interests.

Numerical approach is useful for simulating the rock failure behavior under various stress fields and the effects of various rock parameters. Many numerical methods, such as finite element, boundary element, finite difference and discrete element methods, have been developed to simulate the stress-strain behavior of rocks or rock masses. As the anisotropic properties and discontinuities play a dominant role in the deformation and collapse behavior of rock structures, [14] simulated the fracture propagation using a laminate model. They introduced an additional set of lamina, which was orientated according to the maximum principal stress at onset of failure and associated with a softening stress-strain law. Many modeling works on rock failure [4,7,18] described the deformation behavior of rock before failure. Lattice model [10] had been used in simulating concrete and sandstone laboratory scale specimens subjected to tensile as well as combined tensile and shear loading by Van Mier and his collaborators [17,19,20,25,26]. In the lattice model, the continuum was discretised into a network of brittle breaking beam elements. Heterogeneity was implemented by assigning different limit strengths to each of the beam elements in a lattice. The influence of microstructure was thus introduced in this model and the development of fracture could be modelled. The correct patterns could be found for different loading cases.
In view of the random nature of the rocks, lattice and bonded particle models have been combined with statistical theory to account for the variability in the rock properties. On the other hand, a finite element package (RPFA) has been developed for the study of the rock failure process by assuming the strength and stiffness of the rocks be defined spatially by the Weibull function. Varying the coefficients of the Weibull functions, one can simulate rocks with different degrees of homogeneity. This paper reports the applications of the package in the simulation of the behavior of rocks under compression.

2. Brief Outline of The Rock Failure Process Analysis Package

Numerical simulation is currently a popular approach used for modeling deformation behavior of rock before failure and RPFA is one of the packages developed for such purpose. The package considers the deformation of an elastic material containing randomly distributed micro-features. As load is applied, the fractures will grow, interact and coalesce. As a result, the behavior becomes nonlinear reflecting the growth of micro-fractures as well as macro-fractures. As the details of the RPFA can be found in published literatures [12,22-24], only the pertinent points will be given here.

To simulate the heterogeneous nature of the rocks, the strength as well as modulus parameters are assigned to each element according to the Weibull function which are defined in terms of the characteristic strength and modulus. For example, the element strength ($\sigma$) is:

$$\varphi = \frac{m}{\sigma_o} \left( \frac{\sigma}{\sigma_o} \right)^{m-1} \exp \left( -\left( \frac{\sigma}{\sigma_o} \right)^m \right)$$  \hspace{1cm} (1)

where $\sigma_o$ is the mean compressive strength and $m$ is the homogeneity index. $\varphi$ is the density distribution function. According to the definition, a larger $m$ implies that the material is more heterogeneous. In the package, it is assumed that the elastic modulus will also follow the same distribution.

It is also further assumed that the material properties will be reduced when an element reaches its failure strength. The failure envelope is defined by the Mohr-Coulomb criterion with a tension cut-off, which is taken to be 10% of the element strength. Due to the weakening of the failed elements, it is possible to simulate the strain-softening phenomenon which is observed in experimental studies.

Acoustic emission activities generated during the failure process are also simulated. When an element fails, strain energy will be released. In the analysis, it is assumed that such energy will be converted into acoustic energy. Therefore, each failed element will emit an acoustic signal. The number of failed elements at each loading step can be determined and the amount of energy released by each element can be computed. Following the growth of the failed elements, one can trace the path of development of the fractures and the process of fracture initiation, development and coalescence can be easily visualized.

3. Numerical Simulation

A specimen with a homogeneity index equal to 1.5 is used as an example to validate the package. The length and width of the specimen are 150mm and 100mm respectively. It is discretised into 150x100 elements. Other pertinent parameters adopted in the analysis are tabulated in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Compressive strength</td>
<td>200 MPa</td>
</tr>
<tr>
<td>Mean Elastic Modulus</td>
<td>60000 MPa</td>
</tr>
<tr>
<td>Tension cutoff</td>
<td>10% of the mean compressive strength</td>
</tr>
<tr>
<td>Friction angle</td>
<td>30°</td>
</tr>
</tbody>
</table>

The specimen is loaded along the longitudinal direction by applying an external displacement at constant rate of 0.002mm/step. In each step, the stress states of some elements may reach the failure states. Such elements are considered to be damaged and they are weakened accordingly. The stress and deformation are then adjusted instantaneously to reach the equilibrium state. At locations with increased stresses due to stress redistribution, the stress state may exceed the critical value and further failures will occur. The process is repeated until there is no new failure of element and the equilibrium is satisfied. External displacement is then increased for the next step.

The numerical results are shown in Figure 1. The strain is plotted as a function of loading step (Figure 1(a)). Such a format has been used to allow comparison with the instantaneous values of two important parameters derived from the simulation. The two parameters are the AE counts (event rate) and the associated energy release (Figures 1(b) and (c)).
Figure 1. AE parameters vs loading step: (a) nominal stress; (b) AE event rate and AE event accumulation; and (c) AE energy and accumulation (simulated with RFPA2D)
The event rate shows the following features:

a) During the initial deformation or linear elastic phase (marked with symbol 'a' in the curves), small amount of energy has been released even though some AE events have occurred.

b) The rate of AE events increases significantly in the inelastic phase (point 'b' to 'd'). It is noted that such AE events are generated by micro-fractures that result in non-linear deformation behavior.

It can be seen that a large number of AE events occurred before reaching the peak load (maximum strength). Although more than 50% of the AE events occurred before reaching peak load, it is important to note that less than 20% of the acoustic energy is dissipated during this stage (Figure 1). Results depicted in Figures 1(a) and (b) also show that there is a good correlation between the stress-strain curve and event rate. Note that every large stress drop (point 'e', 'f', 'g' and 'i') in the stress-strain curve (Figure 1(a)) corresponds to a high event rate (Figures 1(b)). The amount of energy released is also larger (Figures 1(c)). It is also found that the AE released energy seems to have a closer relationship with stress drop. Although the AE event counts at point 'i' are just half of that at point 'f', the released AE energy at point 'i' is nearly the same as that at point 'f'. It implies that more energy is released per event. Note that the maximum stress drop occurs at point 'i'.

Figure 2 presents the locations of AE events that have occurred at each loading stage. Each circle represents an AE event and the diameter of the circle represents the relative magnitude of the released AE energy. A higher released energy is indicated by a circle of larger diameter. The locations of AE events at 56% of the peak stress are plotted in Figure 2(a). Note that the events are distributed throughout the specimen, reflecting the statistically uniform deformation during this portion of the simulation. It is very difficult to predict where the macro-fracture will initiate at this stage. While a few events still scatter throughout the specimen, the events are mainly clustered near the zone that seems to be the potential nucleation site in the future failure process (Figure 2(d)). However, it is interesting to find that the next site for more active AE events is not in this predicted potential site but in a zone approximately perpendicular to the fracture zone formed in stage 'd' as shown in Figure 2(e). It is believed to be the result of the stress redistribution or stress migration from the stress released area to the surrounding micro-seismic inactive zone. This zone has developed quickly along the diagonal line from the upper left corner to the bottom right corner (Figure 2(f)). During these stages, the event counts decrease in number throughout the other regions of the specimen. Eventually, the sites at stage 'd' and stage 'e' will combine and lead to the final rupture of the specimen. The AE event plots also show that the AE events along the narrow active zone have dissipated most of the acoustic energy.

Figure 3 shows the simulated initiation, propagation and coalescence of fractures in different loading steps. It can be seen that the onset of failure in the specimen subjected to uniaxial compressive loading is first indicated by the formation of a large number of isolated fractures (Figure 3(a)). Such local fracturing characterizes the relief of stress concentration produced by the mechanical inhomogeneities in the specimen. The micro-fractures begin to form clusters at stage 'c' and 'd', and become clearly localised in stage 'e'. The number of micro-fracture sites (AE events) indicates that micro-fracture interaction can cause micro-fractures to propagate and coalesce when as few as 10-15% of the site are fractured. This is quickly followed by the development of two macroscopic fracture zones during the post-peak regions (stages 'f' and 'g'). Fracture strips divided by thin columns (bridges) form along the diagonal of the specimen and their propagations create critical arrays of fractures. It is worth noting that although most fractures are distributed along these two conjugated zones but most of them are tensile fractures orientated parallel to the applied stress. The buckling or shearing of the bridges lead to fracture linkage en echelon and forming of fault. Figure 3(h) shows that there is a highly stressed area between the two macro-fracture zones. Finally, the interior macroscopic fracture zones are interconnected to form a V-shape open fracture (Figure 3(i)). Although the fractures are in a tensile failure mode, it is important to note that most of fractures are distributed in a highly stressed shear zone. The larger stress drop shown in Figure 1(a) and higher energy release shown in Figure 1(c) at point 'i' also provides evidence to indicate that most of the tensile fractures in this stage are distributed in a highly stressed shear deformation zone. Since the fractures in this diagonal line become dominant, the cones are not developed at the two ends of the specimen and the specimen ultimately fails in two parts along a diagonal planes. As a result, a big block in the bottom right corner breaks away from the specimen (Figure 3(j)).
Figure 2. Plots of simulated AE locations during each step. Each circle represents one AE event and its relative magnitude.

Figure 3. Plots of simulated failure process. The gray color in plots represents the Young’s modulus of the elements.
As mentioned above, there is a strong relationship between the sharp increase of AE events and the stress drop. The reason for this stress drop is in fact the jump phenomenon of fracture propagation. As a fracture propagates, a stress relaxation occurs and the fracture propagation stops. As the load increases, the concentration of stress in the fracture tip attains the limiting strength and the fracture propagation starts again. The process continues until interactions among fractures begins to develop and more complex failure patterns are formed. As a result, there will be big stress drops Figure 1 (points 'a'-'c') and Figures 3(e) – 3(i).

From this simulation, a better understanding of the mechanics of material strength can be approached. Comparing the stress-strain curve (Figure 1) and the failure process (Figure 3), it is found that the specimen does not fail abruptly when the maximum strength is reached. The stress-strain curve becomes non-linear when micro-fractures develop in the specimen (Figures 3(a)-(c)). At point 'e', the specimen reaches its maximum strength. Although the loading bearing capacity drops dramatically from point 'f' to 'h', it is not necessary representing the collapse of the specimen. The specimen rapidly loses its loading bearing capacity but the fractures have not yet propagated thoroughly until point 'i'. Subsequently, a distinct transition occurs and the loading capacity begins to decrease at a much slower rate until it reaches its residual strength (Figure 1) which is about 25% of its maximum strength. It is higher than the residual strength set-up in the individual element constitutive law (10% of the maximum element strength).

In order to capture the main features of the failure behavior of the minerals in the granite specimens, an artificial simulation to the grain level is necessary. Instead of modeling any specific specimen, a typical granite specimen consisting of the major features of the granite are established for numerical simulation. Based on statistical analysis results of a number of specimens [15], the size as well as the shape of the grains are chosen in such a way that they resemble those of the actual one as closely as possible. The feldspar is irregular in shape whereas the quartz and biotite are nearly-round and nearly-rectangular respectively. The ratio of quartz: feldspar: biotite in the model is 3:6:1, that is the average mineral content ratio of granite. Furthermore, it is observed that feldspar is the base phase of the granite with insets of quartz and biotite grains. Such arrangement of the grains is also simulated. Strength as well as modulus parameters are assigned to each grain according to the material properties of the mineral that it represents. It is anticipated that there is not only variation in properties between grains of the same minerals but also variation in a grains due to imperfections. Therefore, it is necessary to include such variation in the analysis. As in the normal analysis, the variation is defined by the Weibull function. The strength and modulus parameters used in the analyses are tabulated in Table 2. As the grain boundaries are usually weaker than the grains, element along the grain boundaries are assigned material properties lower than those of the grains. Based on the experimental results [15], reduction factors are assigned and the values are also tabulated in Table 2. The characteristic elastic moduli and Poisson's ratios are based in the published data from Ref [13] whereas the characteristic compressive strengths are deduced from the hardness indices [1,6].

Table 2. Parameters of the simulation materials

<table>
<thead>
<tr>
<th>Mineral</th>
<th>E (GPa)</th>
<th>C (MPa)</th>
<th>C/T</th>
<th>m</th>
<th>Φ (degree)</th>
<th>B (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>90</td>
<td>200</td>
<td>20</td>
<td>3</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>Feldspar</td>
<td>70</td>
<td>180</td>
<td>5</td>
<td>30</td>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>Biotite</td>
<td>20</td>
<td>120</td>
<td>10</td>
<td>30</td>
<td>30</td>
<td>5</td>
</tr>
</tbody>
</table>

E: Characteristic elastic modulus
C: Characteristic compressive strength
C/T: Compressive strength/Tensile strength
m: Heterogeneity index
Poisson's ratio = 0.3
Φ: Friction angle
B: Weakening coefficient of grain boundary

Figure 4 shows the initial state of the simulation specimen. Note that each grain is represented by a number of elements. As a regular mesh is used in the analysis, the exact number of elements used to represent a grain will depends on the grain size. The gray scale represents the modulus of the grains: a stiffer grain is represented by a lighter color. Therefore, the biotite grains are the darkest while the quartz grains have the lightest color.
Numerical Simulation of the Failure Process of Rocks

It is noted that intragranular fractures first develop in quartz boundary and then in feldspar at a later stage (Figure 5). Taking this factor into account, the quartz is assumed to have a smaller heterogeneity index than feldspar and biotite so that the probability of the existence of weaker elements in quartz grain is increased. As quartz also starts to fracture at a lower load and the fractures are mainly due to tension failure, the \( \text{compressive strength/tensile strength} \) ratio \( \text{('C/T')} \) for quartz should be larger. A \( \text{('C/T')} \) ratio of 20 is assigned to quartz, while for feldspar and biotite the values are 5 and 10 respectively.

The loading is applied in steps by controlling the stroke rate. The rate is assumed to be 0.00005mm/step until the specimen ruptures. To minimize the end effect due to the platen, the modulus of the platen is taken to be the same as that of the feldspar, which is the major component of the granite.

It must be pointed out that the anisotropic behavior of feldspar and biotite is exactly be modeled but it is believed that the effects are actually be accounted for by adopting a lower \( \text{('C/T')} \) ratio.

Although it is not intended to simulate exactly all the minute details of the fracture process, the model, however, should be able to reproduce the major fracture features observed during the laboratory testing. The computed fracture patterns of the specimen at various stages of loading are depicted in Figure 5. As failed elements will be assigned a very low modulus, they will appear as black patches. As a result, one can easily locate the failed elements and trace the development of fractures when the specimen is loaded. From the figures, the following points are noted:

- Fractures will first open along the boundaries of the quartz and biotite grains or within the quartz grains. The load level is approximately 53% of the peak load.
- The boundary fractures will grow into the quartz grains as the load is increased. The fracture orientation is mainly along the loading direction. The fractures are unstable but limited to within the grain.
- As it is more difficult for fractures to propagate in the feldspar element, fractures starting from quartz grain will be halted when it encounters a feldspar grain. It will slow down the rate of propagation.
- The fractures will propagate into the feldspar grains when the load is very close to the peak load.
- In the pre-peak range, the fractures are mainly intragranular.
• Intergranular fractures appear in the post-peak range. Fractures in the quartz grains further develop into macro-fractures when the specimen is further loaded in the post-peak range.

Figure 6 shows the acoustic emission activities and stress-strain curves of the simulation specimen. The results show that the number of activities increases when the loading is approximately 65% (Point 'A') of the peak load. Though the value is slightly higher than that obtained in the test of actual specimen [10], the agreement can still be considered to be satisfactory. Comparing the stress-strain curve and the simulation results, one may also conclude that opening of new fractures in quartz will lead to a drop in modulus and the non-linear behavior of granite in the pre-peak range is mainly due to such fractures.

Figure 6. Stress strain curve and acoustic events

The result shows that the sequence of the major fractures and acoustic emission activities predicted by the numerical model shown in Figures 5 and 6 are in good agreement with the observed ones.

Figure 7 shows the results obtained by varying the mineral contents. In the analyses, the quartz content is increased whereas the feldspar content is decreased. The amount of biotite remains unchanged. The results indicate that the failure patterns are very similar in all cases, though the peak load decreases (Table 3). It is not unexpected as the amount of 'weak' mineral increases, and therefore, the strength should decreases accordingly. As the quartz is the major mineral in Case 5 (60%), it is highly possible that the quartz will become the base phase whereas the feldspar and biotite grains are insets. Therefore, such a case is also considered and parametric study has been carried out by varying the arrangement of the grains. Figure 8 shows that the failure pattern is very different from those observed in the previous study. Though the fractures still start in the quartz and grow into the quartz grains, the fractures concentrate in the quartz grains and the feldspar grains are basically not fractured. Depending on the arrangement of the grains, the fractures are not necessary vertical. It is also interested to note that the peak strength remains almost unchanged.

Analyses are also conducted to study the effects of grain size. Without varying other parameters, the grain size for each mineral is reduced. Five cases are generated and the average grain sizes for each case are tabulated in Table 4. Results show that reduction in grain size will increase the load level at which significant fractures within grains start to develop (Figure 9). As this load level increases, the peak load also increases (Table 4). Furthermore, the failure mode appears to be more brittle. The results seem to indicate that rocks having the same mineral contents, the one with the finer grains will tend to be stronger and more brittle. Of course, there are other factors affecting the behavior of rocks and these must be taken into account before drawing definite conclusions on their behavior.

Table 3. Variation of peak load with quartz content

<table>
<thead>
<tr>
<th>Quartz Content (%)</th>
<th>Peak Load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>247</td>
</tr>
<tr>
<td>20</td>
<td>209</td>
</tr>
<tr>
<td>30</td>
<td>186</td>
</tr>
<tr>
<td>50</td>
<td>179</td>
</tr>
<tr>
<td>60</td>
<td>142</td>
</tr>
</tbody>
</table>

Table 4. Variation of peak load with average grain size

<table>
<thead>
<tr>
<th>Average Grain Size (mm)</th>
<th>Peak Load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>225</td>
</tr>
<tr>
<td>2.0</td>
<td>188</td>
</tr>
<tr>
<td>2.4</td>
<td>179</td>
</tr>
<tr>
<td>2.5</td>
<td>171</td>
</tr>
<tr>
<td>3.0</td>
<td>159</td>
</tr>
</tbody>
</table>
Figure 7. Results for parametric study: Effects of quartz content
Figure 8. Results for parametric study: Effects of different arrangement of the grains (quartz as base phase; feldspar content = 60%)
Figure 9. Results for parametric study: Effects of grain size
4. Conclusions

In this paper, attempts have been made to characterize the progressive failure leading to collapse in rock specimens, in particular for granite, under uniaxial compression. Although the simulations are based on two-dimensional models, the phenomenological approach shows in detail the nucleation, propagation, interaction and coalescence of micro-fractures within the rock specimens. The simulation results are able to provide insight on the mechanisms of rock failure and improve the understanding of the failure process. The study also shows that AE events should be a useful indicator of various stages of failures.

References


